SEM 파라메타 측정에 대한 MLE 기법과 POF 기법의 성능비교

김 덕 년†

요 약

본 논문은 전송 잡음 환경 하에서 전자파 산란 물체의 식별을 위하여 사용하는 파라메타 측정 기법에 관한 연구이다. 최대 유사측정(Maximum Likelihood Estimation : MLE) 기법은 물체 식별에 변형하여 응용되면 종래 잘 알려져 사용되어온 함수군속(Pencil of Functions) 기법보다 더 좋은 측정결과를 가진다는 것을 본 논문은 보여주고 있다. MLE 기법을 포함하여 파라메타 식별을 위한 도구로서 지금까지 여러 제안기법들이 있었으나, 본 논문에서는 샘플 데이타의 길이에 관계없이 목표 시스템의 파라메타 양에만 관계하는 최소 단위의 메트릭스 연산이 사용됨을 보여주므로 잡음이 상재하는 추출 데이타로부터 목표식별에 가장 강한 장점이 있다.

Preformance Comparison of MLE Technique with POF(Pencil of Functions) Method for SEM Parameter Estimation

Doug Nyun Kim†

ABSTRACT

Parameter estimation techniques are discussed for the complex frequency analysis of an electromagnetic scatterer. The paper suggests how the Maximum Likelihood estimation technique can be applied for this purpose. Experiments on hypothetical data sets demonstrate that the Maximum Likelihood technique is better than the Pencil of Functions technique. Although there have been several techniques including MLE suggested as tools of the parameter estimation, the proposed method has strong advantanges under the noise-contaminated sample data environment because it uses minimal dimension of system matrix that stands totally independent of the length of extracted data set.

1. Introduction

Signal processing techniques have been a basic tool for estimating the Singularity Expansion Method (SEM) [1] parameters from the measured transient response data produced by an electromagnetic scatterer.

The parametric Auto Regressive Moving Average (ARMA) modeling for the transient response leads to a set of homogeneous equations for the parameters which are essential in finding the poles of the scatterer. This approach is often called the Prony method [2, 3], which provides an extremely accurate solution when the data is noise-free. However under a noisy environment, the Prony method loses its reliability in the estimation of the SEM parameters [3]. Efforts have been made to improve the noise-handling capability of the Prony method and/or to take a totally different approach to the estimation technique. Special attention has been paid to the Pencil of Functions (POF) [4, 5] method which outp-

† 정 회 원 : 한국전자통신연구소 지상시스템 연구부 실장
논문접수 : 1994년 9월 22일, 심사완료 : 1994년 11월 10일

erforms the modified Prony method [6].

The POF method utilizes the special properties of the family of signals, which are produced at each stage by processing the original data sequence through an appropriate string of filters. The family of signals taken from the filter series has special properties that are utilized for pole extraction.

That is, the 'linear-dependency' among the family of signals incorporates to find the system parameters without iteration [4]. However, the POF method needs to find a noise covariance matrix which is dependent upon the choice of the filter. Unfortunately the optimum selection of the filter for the purpose of enhancing the signal-to-noise ratio is not known. Mackay and McCowen[5] chose a new form of the filter by which the performance of the POF method was greatly improved. It should be noted that new filter isn ot dependent upon data. The POF method can be proved to be less sensitive to noise than in the second Prony method [2]. Aside from the estimation techniques discussed above, the Maximum Likelihood (ML) estimation technique has been investigated by many researchers [7, 8, 9, 10, 11, 12], since it provides a unique solution that is unbiased and consistent as the number of trials grows without bounds.

In fact, the ML estimator will provide the minimum attainable variance estimate which is well known as the Cramer-Rao (C-R) lower bound. It has been pointed out [13] that the likelihood function can be maximized by the ML solution explicitly neither in closed form nor even in recursive form. Hence many iterative methods have been suggested as approaching tools for the ML solution. The proposed ML technique distinguishes itself from other Maximum Likelihood approaches simply because it firstly suggests and derives the recursive formula that utilizes only K x K matrix inversion, where K is the order of system to be identified. Traditional ML approaches require (N-K) x (N-K) matrix inversion, where N is length of sampled data that is normally contaminated by noise. What it means is that traditional ML method may be constrained its application to smaller sampled data set. On the contrary the proposed scheme takes the sample data length with no limit and it is quite indepenedent of data length.

In section 2 the ML algorithm is described in a unified framework, and in section 3 the ML approach is compared with the POF method by simulation. It is the objective of this paper to demonstrate that the ML estimation technique is better than the modified POF method.

2. Maximum Likelihood Estimation

Consider the measured data fi for i = 0, 1, 2, ... , L - 1 :

$$f_i = h_i + e_i \tag{1}$$

Where hi is an impulse response of the K-th order system and e_i is the zero mean white Gaussian noise process at the i-th sampling instant.

One wishes to estimate the system poles $\{p_j\}$. Let r_j be the residue associated with the j-th pole p_j, then at the i-th sampling instant,

$$h_i = \sum_{j=1}^{k} r_j (P_j)^i \tag{2}$$

In matrix form, equation (1) becomes

$$\underline{f} = C(\underline{p})\ \underline{r} + \underline{e} \tag{3}$$

Where

$$C(\underline{p}) = \begin{bmatrix} 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \\ p_1 & p_2 & p_3 & \cdot & \cdot & \cdot & p_k \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_1^{L-1} & p_2^{L-1} & & \cdot & \cdot & \cdot & p_k^{L-1} \end{bmatrix} \quad (4a)$$

$$\underline{f} = [\ f \ \ f1 \ . . . \ fL\text{-}1 \ \ ]t, \quad (4b)$$
$$\underline{p} = [\ p1 \ p2 \ . . . \ pk \ \ ]t, \quad (4c)$$
$$\underline{r} = [\ r1 \ r2 \ . . . \ rk \ \ ]t, \quad (4d)$$
$$\underline{e} = [\ e0 \ e1 \ . . . \ eL\text{-}1 \ \ ]t. \quad (4e)$$

Where "t" denotes the matrix transpose operation.

The matrix C(p) is of the form of the Vandermonde matrix of rank k. It is well known that minimizing the norm $||e||^2$ is equivalent to maximizing the likelihood function [14].

The optimum parameter estimates for{r, p} are obtained by minimizing $||e||^2$ with respect to p(or C(p)) associated with the relationship

$$r = \{\ C\ (\underline{p})^t\ C(\underline{p})\ \}^{-1}\ C(\underline{p})^t\ \underline{f}. \quad (5)$$

In other words, the estimation of C(p) is equivalent to solving the nonlinear least squares problem in the minimum norm sense,

$$\min\ ||\ \underline{e}\ ||^2, \quad (6a)$$

where

$$\underline{e} = [I - C(\underline{p})\ \{C(\underline{p})^t C(\underline{p})\ \}^{-1} C(\underline{p})^t\ ]\ \underline{f}, \quad (6b)$$

and I is the identity matrix. Solving min $||e||^2$ as in the form of (6) directly is not feasible. Efforts have been made recently to replace equation (6) with more tractable forms [9, 10].

The technique introduced in [10] replaces the matrix C(p) with matrix consisting of the characteristic polynomial coefficients and employs an iterative method to solve the nonlinear equations.

However, the dimensionality of the matrix involved in each iteration is L-K which may be quite large in some applications. The system poles satisfy the equation

$$P_J^K\ B(p_j) = 0 \ \text{for}\ k = K, K+1, \ldots, L-1 \ \text{and}\ j = 1, 2, \ldots, K, \quad (7)$$

where $B(z) = \sum_{i=0}^{k} z^{-i}$ is termed the characteristic polynomial whose roots are the system poles. Define the LxL lower triangular matrix U

$$U = \begin{bmatrix} b_0 & & & & & & \\ b_1 & b_0 & & & & 0 & \\ \cdot & \cdot & b_0 & & & & \\ \cdot & \cdot & & \cdot & & & \\ b_k & b_{k-1} & \cdots & b_0 & & & \\ & b_k & b_{k-1} & \cdots & b_0 & & \\ & & \cdot & \cdot & & \cdot & \\ & 0 & & \cdot & \cdot & & \cdot \\ & & & b_k & b_{k-1} & \cdots & b_0 \end{bmatrix}_{LxL.} \quad (8)$$

Note that U is nonsingular and its inverse matrix $V = U^{-1}$ is always existing. Matrix V is shown to be also lower triangular matrix and can be denoted by

$$V = \begin{bmatrix} v_0 & & & & \\ v_1 & v_0 & & 0 & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdots & \cdot & \\ \cdot & \cdot & \cdot & & \cdot \\ v_{L-1} & & & v_1 & v_0 \end{bmatrix} \quad (9)$$

Let Ud and Vr be the matrices consisting of last L-K rows of U and last L-K columns of V, respectively, and Uu and V_l be the matrices consisting of first K rows of U and first K columns of V, respectively. Then the LxL matrices, U and V, can be partitioned as follows:

$$U = \begin{bmatrix} U_u \\ \hline U_d \end{bmatrix} \begin{matrix} \}K \\ \}L\text{-}K \end{matrix} \quad (10)$$
$$\quad L$$

$$V = [\, \underset{K}{V_l} \mid \underset{L\text{-}K}{Vr} \,] \; \}L . \quad (11)$$

Since $UV = I$,

$$U_d \; V_l = 0_{(L\text{-}K)xK}. \quad (12)$$

Equation (7) can be written in matrix form ,

$$U_d \; C(\underline{p}) = 0_{(L\text{-}K)xK}. \quad (13)$$

And

$$\text{rank} \{ C(p) \} = \text{rank} \{V_l\} = K ,$$
$$\text{rank} \{ U_d \} = L\text{-}K . \quad (14)$$

Note that the column space of U_d^t is orthogonal to both the column space of $C(\underline{p})$ and that of V_l. The space spanned by the columns of the matrix $C(\underline{p})$ is the column space of V_l.

Thus there exists a nonsingular KxK matrix T with complex elements such that

$$C(\underline{p}) = V_l \; T. \quad (15)$$

By substituting equation (15) into equation (6b), one obtains

$$\underline{e} = [I - V_l \, \{V_l^t V_l\}^{-1} \, V_l^t] \, \underline{f}. \quad (16)$$

Let the orthogonal projector $P(V_l)$ onto V_l be $V_l^t \, (V_l^t V_l)^{-1} \, V_l$. Then,

$$\| \, \underline{e} \, \|^2 = \underline{f}^t \, \{ \, I - P \, (V_l) \, \} \underline{f}. \quad (17)$$

and the ML estimator now is to find the optimum $\underline{b} = [\, b_0 \; b_1 \ldots b_K]$ by solving min $\| \, e \, \|^2$ with the given data vector $\underline{f}$.

An iterative method can be used. Using the estimated $\underline{b}$ obtained from the previous iteration, the matrix V is formed with the elements which can be recursively calculated by the relationship

$$v_0 = 1/b_0$$
$$v_j = \sum_{i=1}^{j} b_i v_{j-i} \qquad 1 \le j \le K,$$
$$\sum_{i=1}^{K} b_i v_{K-i} \qquad K + 1 \le j. \quad (18)$$

Only KxK matrix inversion is involved in calculating $P(V_l)$ at each iteration. Thus less numerical difficulties are encountered. The iterative procedure will continue until the convergence criterion is satisfied. System poles are found by calculating the roots of the characteristic polynomial estimate, and associated system residues are obtained by equation (5). This is called the Iterative Preprocessing Algorithm (IPA) and is documented in [11, 12].

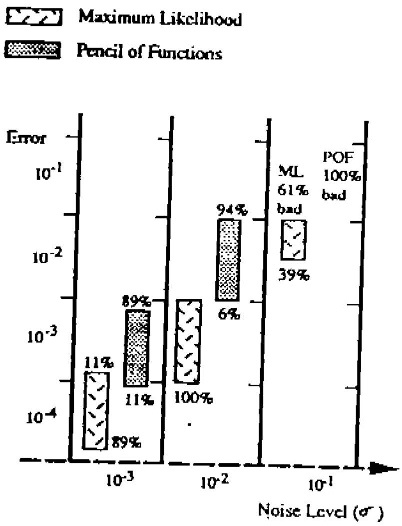
3. Simulation Results

The same example was chosen as in [5]. The poles and residues were:

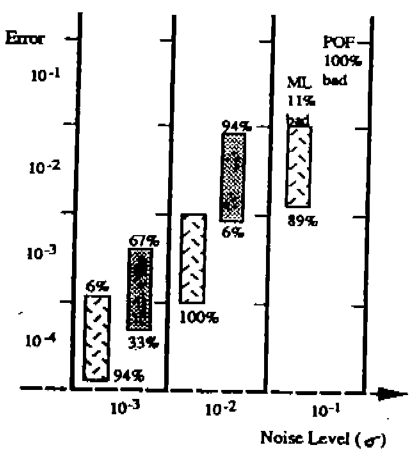
poles	residues
$s_{1,2} = -.1 \pm j1.5$	$r_{1,2} = .4 \mp j.2$
$s_{3,4} = -.2 \pm j1.8$	$r_{3,4} = 1. \mp j.5$
$s_{5,6} = -.3 \pm j1.0$	$r_{5,6} = 2. \mp j1.$

System order N = 6, Sampling Time T=.5

Three different levels of noise were used. i.e.,σ = 0.001, 0.01, 0.1. For each noise level, 18 independent trials were performed. For each trial different noisy data is generated and the result of estimation is compared with true values of system poles and the distribution of estimation errors are shaded in falling boundary marked with upper and lower percentile. In most cases the number of iterations required for convergence was less than 5. (Fig. 1) compares the ML estimation method to the POF technique when the data length=50. When σ = .001, 89% of the test results shows that the ML method can estimate only within 10^{-4} order of error while the POF technique can estimate only within 10^{-3} order. The estimation capabilities of the two techniques become distinct as the noise increases. With 10^{-1} noise level the ML method still estimates the 39% of the whole tests within 10-2 order of error, but the POF method cannot. For another comparison the data length was extended to 100. It is found that both estimation techniques have improved capabilities of estimation especially for the low noise level (10^{-3}) (Fig. 2). Note that even under the heavy noise (σ = .1), the ML technique is capable of estimating the poles of the given example within 10^{-2} order of error for about 90% of tests.



(Fig. 1) The Performance Comparison Data Length N=50



(Fig. 2) The Performance Comparison Data Length N=100

4. Conclusion

Two SEM parameter estimation techniques are compared. These two techniques: the Pencil of Functions method and the Maximum Likelihood method are applied to the same hypothetical data sets and the test results show that the ML approach has the better performance in handling noisy data. Unlike other Maximum Likelihood method, which requires (N-K) x (N-K) matrix inversion [7, 8, 9, 10], the proposed MLE technique uses minimal number of system matrix dimension and requires only K x K matrix inversion which in return, results in strong and robust findings of system parameters

References

[1] C. E. Baum, "Emerging technolog yfor transient and broad-band analysis and synthesis of antennas and scatterers," Proc. IEEE, Vol. 64, pp. 1598-1616, Nov. 1976.

[2] S. M. Kay and S. L. Marple, "Spectrum analysis-A modern perspective," Proc. IEEE, Vol. 69, No. 11, pp. 1380-1419, Nov. 1981.

[3] K. T. Astrom and P. Eykhoff, "System identification-A survey, "utomatica, Vol. 7, pp. 123-162, 1971.

[4] V. K. Jain, T. K. Sarkar and D. D. Weiner, "Rational Modeling by Pencil-of-Functions Method," IEEE Trans. Acoust. Speech, and Signal Processing, Vol. ASSP-31, No. 3, pp. 564-573, June 1983.

[5] A. J. Mackay & A. McCowen, "An Improved Pencil-of-Functions Method and Comparisons wih with Traditional Methods pf Pole Extraction," IEEE Trans. Antennas & Propagat., Vol. AP-35, No. 4, pp. 435-441, April 1987.

[6] R. Kumaresan and D. W. Tufts, "Estimating the parameters of exponentially damped sinusoids and pole-zero modeling in noise," IEEE Trans. Acoust., Speech, SignalProcessing, Vol. ASSP-30, pp. 833-840, Dec. 1982.

[7] A. G. Evans & R. Fischl, "Optimal Least Squares Time-domain Synthesis of Recursive Digital Filters," IEEE Trans Audio & Electroacoustics, Vol. AU-21, No. 1, pp. 6-65, February 1973

[8] K. Steiglitz, "On the Simultaneous Estimation of Poles and Zeros in Speech Analysis," IEEE Trans. Acoust., Speech, and Signal Processing, Vol. ASSP-25, No. 3, pp. 229-234, June 1977.

[9] R. Kumaresan & A. K. Shaw, "High Resolution Bearing Estimation without Eigendecomposition," in Proc. IEEE IC-ASSP85, Tampa, FL, pp. 576-579, 1985.

[10] Y. Bresler & A. Macovski, "Exact Maximum Likelihood Estimation of Superimposed Exponential Signals in Noise," IEEE Trans. Acoust., Speech and Signal Processing, Vol. ASSP-34, No. 5, pp. 1081-1089, Oct. 1986.

[11] S. Park & J. T. Cordaro, "Preprocessong Techniques for the Estimation of SEM Paramters from Multiple Measurements," Mathematics Note-85, AFWL, Albuquerque, NM, August 1985.

[12] S. Park & J. T. Cordaro, "Maximum likelihood estimation of poles from impulse response data in noise," in Proc. IEEE IC-ASSP87, Dallas, TX, pp. 1501-1504. 1987.

[13] G. F. Franklin and J. D. Powell, Digital Control of Dynamic System, Addison Wesley, June 1980.

[14] S. J. Orfanidis, Optimal Signal Processing, Macmillan, NY, 1985.

김 덕 년

1975년 서울대학교 전기공학과(학사)

1981년 SUNY at StonyBrook 전기공학과(석사)

1988년 Auburn 대학교 전기공학과(박사)

1988년~현재 한국전자통신연구소 책임연구원/실장

관심분야: 성능평가, 다중접속프로토콜, 컴퓨터네트워크, 신호처리