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W–REGULAR CONVERGENCE OF R^i–CONTINUA

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1. Introduction and basic definitions

In the course of study of dendroids, Czuba [3] introduced a notion of R^i-continua which is a generalization of R-arc [1]. He showed a new class of non-contractible dendroids, namely of dendroids which contain an R^i-continuum. Subsecequently Charatonik [2] attempted to extend the notion into hyperspace $C(X)$ of metric continuum X. In so doing, there were some oversights in extending some of the results relating R^i-continua of dendroids for metric continua. In fact, Proposition 1 in [2] is false (see example C below) and his proof of Theorem 6 in [2] is not correct (Take Example 4 in [4] with $K = [e, e']$ as an R^1-continuum of X and work it out. Then one sees that $K \notin \mathcal{K}$ as he claimed otherwise.).

The aims of this paper are to introduce a notion of w-regular convergence which is weaker than 0-regular convergence and to prove that the w-regular convergence of a sequence $\{X_n\}_{n=1}^{\infty}$ to X_0 of subcontinua of a metric continuum X is a necessary and sufficient for the sequence $\{C(X_n)\}_{n=1}^{\infty}$ to converge to $C(X_0)$, and also to prove that if a metric continuum X contains an R^i-continuum with w-regular convergence, then the hyperspace $C(X)$ of X contains R^i-continuum.

Let (X, d) be a compact metric space. Let $2^X = \{A \subset X : A$ is nonempty and closed $\}$ and let $C(X) = \{A \in 2^X : A$ is connected$\}$. For each $A \in 2^X$ and $\varepsilon > 0$, let $N(\varepsilon, A) = \{x \in X : d(x, a) < \varepsilon$ for some $a \in A\}$. If $A, B \in 2^X$, let $H(A, B) = \inf\{\varepsilon > 0 : A \subset N(\varepsilon, B)$ and $B \subset N(\varepsilon, A)\}$; we call H the Hausdorff metric for 2^X (or $C(X)$) induced by d. The spaces 2^X and $C(X)$, with the Hausdorff metric, are called hyperspaces of X.

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We adopt the following notations: if $x \in M \subset X$, let $C(M) = \{A \in C(X) : A \subset M\}$, $T(x,M) = \{A \in C(M) : x \in A\}$, and let $M^* = \{\{x\} : x \in M\}$.

Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of subsets of a space X. Let LiA_n be the set of all $x \in X$ such that if U is a neighborhooh of x, then $U \cap A_n \notin \emptyset$ for infinitely many n. If $LiA_n = LsA_n = A$, then we say that the sequence $\{A_n\}_{n=1}^{\infty}$ converges to A, written $LtA_n = A$ or $A_n \to A$. It is known [7] that if $\{A_n\}_{n=1}^{\infty}$ is a sequence in the hyperspace 2^X (or $C(X)$) of the metric continuum X, then $A_n \to A$ if and only if $H(A_n, A) \to 0$ as $n \to \infty$. For further properties of these definitions, we refer to [9].

2. Convergence of $\{C(X_n)\}_{n=1}^{\infty}$

Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of subcontinua of a metric continuum X which converges to X_0. One may ask under what condition imposed on the sequence so that $\{C(X_n)\}_{n=1}^{\infty}$ converges to $C(X_0)$. In [9], 0-regular convergence was given. However, this condition is sufficient but not necessary. We provided here one simple condition, call it w-regular convergence, which is both necessary and sufficient.

DEFINITION 2.1 [9]. A sequence $\{X_n\}_{n=1}^{\infty}$ of subsets of a metric continuum X is said to convergence 0-regularly to X_0 provided that the following two conditions are satisfied:

(a) $X_n \to X_0$ as $n \to \infty$;
(b) Given $\varepsilon > 0$, there exist $\delta(\varepsilon) > 0$ and a positive integer N such that if $n \geq N$, then any two points of X_n less than δ apart lie together in a connected subset of X_n of dianeter less than ε.

THEOREM 2.2 [6]. *If $\{X_n\}_{n=1}^{\infty}$ is a sequence of subcontinua of a metric continuum X such that $X_n \to X_0$ 0-regularly, then $\{C(X_n)\}_{n=1}^{\infty}$ converges to $C(X_0)$ with respect to the Hausdorff metric.*

DEFINITION 2.3. A sequence $\{X_n\}_{n=1}^{\infty}$ of subsets of a metric continuum X is said to converge h-regularly to X_0 if it satisfies the following two conditions:

(a) $X_n \to X_0$;

(b) Given $\varepsilon > 0$ and $A \in T(x, X_0)$, there exist $\delta > 0$ and a positive integer N such that each point $y \in X_n \cap V(x)$, where $V(x)$ is the δ-neighborhood of x, has an element $B \in T(y, X_n)$ such that $H(A, B) < \varepsilon$ for each $x \in X$ and for $n \geq N$.

LEMMA 2.4 [6,9]. *Let A be a metric continuum. For each $\varepsilon > 0$, there is a finite set $F = \{a_1, a_2, \ldots, a_n\} \subset A$ such that*

(1) *$H(F, A) < \varepsilon$, and*
(2) *the distance between any two consecutively indexed points of F is less than ε.*

LEMMA 2.5. *If the sequence $\{X_n\}_{n=1}^{\infty}$ of subcontinua of a metric continuum X converges 0-regularly, then it converges h-regularly.*

Proof. Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of subcontinua of a metric space X which converges to X_0 0-regularly. Let $A \in T(x, X_0)$ and $\varepsilon > 0$. Since $X_n \to X_0$ 0-regularly, there exist δ, $0 < \delta < \varepsilon$, and a positive integer N_1 such that, for each $n > N_1$, if $p, q \in X_n$ such that $d(p, q) < \delta$, then there is a subcontinuum of X_n containing p and q, denoted by $B_n(p, q)$, such that the diameter of $B_n(p, q)$ is less than $\frac{\varepsilon}{2}$.
Since $X_n \to X_0$, there is a positive integer N_2 such that if $n > N_2$, then $H(X_n, X_0)$
$< \frac{\delta}{3}$.

Since A is a subcontinuum, we let $F = \{a_1, a_2, \ldots, a_n\} \subset A$ such that $H(F, A) < \frac{\delta}{6}$ and $d(a_s, a_{s+1}) < \frac{\delta}{6}$ for each $s = 1, 2, \ldots, t-1$, by Lemma 2.4. Let $F' = F \cup \{x\}$. Then $H(F', A) < \frac{\delta}{6}$, and $d(x, a_i) < \frac{\delta}{6}$ for some $a_i \in F$. Let $N = \max\{N_1, N_2\}$ and let V be the $\frac{\delta}{6}$-neighborhood of x in X and $y \in V \cap X_{n_0}$ for some $n_0 > N$.

For each $a_s \in F$, choose $y_s \in X_{n_0}$ such that $d(a_s, y_s) < \frac{\delta}{3}$. Then $d(y_s, y_{s+1}) < d(y_s, a_s) + d(a_s, a_{s+1}) + d(a_{s+1}, y_{s+1}) < \delta$. And $d(y, y_i) < d(y, x) + d(x, a_i) + d(a_i, y_i) < \delta$. Then we have subcontinua $B_{n_0}(y, y_i)$, $B_{n_0}(y_s, y_{s+1})$ of diameter less than $\frac{\varepsilon}{2}$. Let $B_{n_0} = B_{n_0}(y, y_i) \cup_{s=1}^{t-1} B_{n_0}(y_s, y_{s+1})$. Then $B_{n_0} \in T(y, X_{n_0})$. And one can easily verify that $H(B_{n_0}, A) < \varepsilon$.

DEFINITION 2.6. The sequence $\{X_n\}_{n=1}^{\infty}$ of subsets of a metric continuum X is said to converge to X_0 *w*-regularly if it satisfies the followings:

(1) $X_n \to X_0$, and
(2) Given $\varepsilon > 0, x \in X_0$, and $A \in T(x, X_0)$, there are a $\delta > 0$ and a positive integer N such that, whenever the δ-neighborhood V of $x \in X$ intersects $X_n, n \geq N$, then there is a point $y \in V \cap X_n$ having an element $B \in T(y, X_n)$ with $H(A, B) < \varepsilon$.

THEOREM 2.7. *h-regular convergence of a sequence of subcontinua of a metric continuum implies w-regular convergence.*

REMARK 2.8.

(1) h-regular convergence does not imply 0-regular convergence.
(2) w-regular convergence does not imply h-regular convergence.
(3) $C(X_n) \to C(X_0)$ does not imply h-regular convergence of $X_n \to X_0$

We illustrate the remark by the following two examples:

EXAMPLE A. Let $X = [0,1] \times [0,1]$. Let $p_0 = (0,0)$ and $q_0 = (1,0)$ and let $X_0 = p_0q_0$ denote the line segment between p_0 and q_0. For each positive integer n, let $p_n = (0, \frac{1}{n})$ and $q_n = (1, \frac{1}{n})$. For each even positive integer $n = 2m$, let $X_m = p_{n-1}q_{n-1} \cup q_{n-1}q_n \cup p_nq_n$ if m is odd, and let $X_m = p_{n-1}q_{n-1} \cup p_{n-1}p_n \cup p_nq_n$ if m is even. Then $X_m \to X_0$ h-regularly but not 0-regularly at either p_0 or q_0.

EXAMPLE B. We give a sequence of figures Z which converges to an arc. Then the convergence of the sequence is h-regular but not 0-regular. Let $p_0 = (0,0)$ and $s_0 = (1,0)$, and let $X_0 = p_0s_0$. For each positive integer n, let $p_n = (0, \frac{1}{2n-1}), q_n = (\frac{3}{4}, \frac{1}{2n-1}), r_n = (\frac{1}{4}, \frac{1}{2n})$, and $s_n = (1, \frac{1}{2n})$. Let $X_n = p_nq_n \cup q_nr_n \cup r_ns_n$ for each n. Let $x = (\frac{1}{4}, 0)$ and $A = p_0x$. Then the convergence $X_n \to X_0$ is w-regular but not h-regular at x. Also it is easily seen that $LtC(X_n) = C(X_0)$.

THEOREM 2.9. *If the sequence $\{X_n\}_{n=1}^{\infty}$ of subsets of a metric continuum X converges to X_0 w-regularly, then the sequence $\{C(X_n)\}_{n=1}^{\infty}$ converges to $C(X_0)$.*

Proof. Since $LtX_n = X_0$, $X_0^* \subset LiC(X_n) \subset LsC(X_n) \subset C(X_0)$. Let $A \in C(X_0), a \in A$, and let $\varepsilon > 0$. Since $X_n \to X_0$ is w-regular, there exist δ-neighborhood V of a and a positive integer N such that $V \cap X_n \neq \emptyset$ and a point $y \in V \cap X_n$ having an element $B \in T(y, X_n) \subset C(X_n)$ with $H(A, B) < \varepsilon$ for all $n > N$. Thus $A \in LiC(X_n)$ and hence $C(X_0) \subset LiC(X_n)$. Therefore we have $C(X_0) = LtC(X_n)$.

THEOREM 2.10. *The sequence $\{X_n\}_{n=1}^{\infty}$ of subcontinua of a metric continuum X converges to X_0 w-regularly if and only if the sequence $\{C(X_n)\}_{n=1}^{\infty}$ converges to $C(X_0)$.*

Proof. Suppose $X_n \rightarrow X_0$ is w-regular. It suffices to show that $C(X_0) \subset LiC(x_n)$. Let $A \in C(X_0)$, and $\varepsilon > 0$ be given. Let $a \in A$. The w-regular convergence implies that there is $\delta > 0$ and N such that the δ-neighborhood V of a intersects X_n for all $n > N$ and there is a point $a_n \in X_n$ having an element $A_n \in T(a_n, X_n)$ such that $H(A, A_n) < \varepsilon$ for each $n \geq N$. Thus $A \in LiC(X_n)$. Hence we have $LtC(X_n) = C(X_0)$.

Now suppose $LtC(X_n) = C(X_0)$. Let $\varepsilon > 0$ and $A \in T(a, X_0)$. Let $\{A_n\}_{n=1}^{\infty}$, $A_n \in C(X_n)$, be a sequence which converges to A. Let N be an integer such that $H(A, A_n) < \varepsilon$ for all $n \geq N$. Let $\delta = \varepsilon$, and let V be the δ-neighborhood of a. Then $V \cap A_n \neq \emptyset$ for all $n \geq N$. So we pick a point $a_n \in A_n$ for each $n \geq N$. Then these satisfy w-regular convergence condition.

3. R^i-continua in $C(X)$

In [1], it was proven that if a metric space X contains a proper subset A which is homotopically fixed, then X is not contractible. Subsequently Czuba [4] proved that any R^i-continua of a dendroid is homotopically fixed. But it can be verified that it holds for all metric continua. In [2], there were some attempts to generalize R^i-continua of dendroids for metric continua X [2, Proposition 1] and extending them to hyperspaces $C(X)$ [2, Theorem 6, Corollary 7, and Corollary 17]. (The statement of Corollary 17 remains true by [8]).

In this section, we will remedy the attempts for a subclass of metric continua.

The following definition was originally given for the class of dendroid.

DEFINITION 3.1 [3]. Let X be a metric continuum. A nonempty proper subcontinuum K of X is called

(1) an R^1-continuum if there exists an open set U such that $K \subset U$ and two sequences $\{C_n^1\}_{n=1}^{\infty}$ of components of U such that $K = LsC_n^1 \cap LsC_n^2$;
(2) an R^2-continuum if there exist an open set U containing K and two sequences $\{C_n^1\}_{n=1}^{\infty}$, $\{C_n^2\}_{n=1}^{\infty}$ of components of U such that $K = LtC_n^1 \cap LtC_n^2$;
(3) an R^3-continuum if there exists an open set U containing K and a sequence $\{C_n\}_{n=1}^{\infty}$ of components of U such that $K = LiC_n$.

In sequel, we denote R^1-continuum, R^2-continuum, and R^3- continuum by $LsC_n^1 \cap LsC_n^2 \subset U$, $LtC_n^1 \cap LtC_n^2 \subset U$, and $LiC_n \subset U$, respectively, as the open set U and the components are given in the definition.

Now Czuba's Proposotion 5 and a part of Corollary 11 in [3] can be stated for metric continua but his Proposition 10 in [3] can not be generalized for metric continua (see Example C below).

PROPOSITION 3.2 [3].

(a) *Each R^2-continuum of a metric continuum X is both R^1 and R^3-continuum.*
(b) *If R^1-continuum of a metric continuum X is a single point, then it is both R^2 and R^3-continuum.*

Proof.

(a) In fact, if $K = LtC_n^1 \subset U$, then $LsC_n^i = LtC_n^i$ for each $i = 1, 2$, so that K is an R^1-continuum. Now define a new sequence $\{D_n\}_{n=1}^{\infty}$ by letting $D_{2n} = C_n^1$ and $D_{2n+1} = C_n^2$. Then it is easy to check that $K = LiD_n \subset U$.
(b) Suppose $K = \{x\} = LsC_n^1 \cap LsC_n^2 \subset U$. For each $i = 1, 2$, choose a convergent subsequence $\{C_{n_k}^i\}_{k=1}^{\infty}$ of $\{C_n^i\}_{n=1}^{\infty}$ whose limit contains x. Then $LtC_{n_k}^i \subset LsC_n^i$ for each $i = 1, 2$, implies that $LtC_{n_k}^1 \cap LtC_{n_k}^2$. The proof that $K = \{x\}$ is an R^3-continuum is the same as in (a).

THEOREM 3.3. *Let $K = LtC_n^1 \cap LtC_n^2 \subset U$ be an R^2-continuum of a metric continuum X such that the convergence of each sequences $\{C_n^i\}_{n=1}^{\infty}$, $i = 1.2$, is w-regular. Then $C(K)$ is a R^2-continuum in $C(X)$.*

Proof. Since $C(U)$ is open in $C(X)$ and each $C(C_n^i)$ is a component of $C(U)$, we let $\mathcal{K} = LtC(C_n^1) \cap LtC(C_n^2)$. Let $K_i = LtC_n^i$ for each $i = 1, 2$. Then $K^* \subset \mathcal{K}$ so that $\mathcal{K} \neq \emptyset$.

Since the convergence is w-regular, we have $C(K_i) = LtC(C_n^i)$ for each i by Theorem 2.9. Let $A \in \mathcal{K}$. Then $A \in C(K_1) \cap C(K_2)$ so that $A \in C(K)$. On the other hand, suppose $A \in C(K)$. Then $A \subset K_1 \cap K_2$ so that $A \in C(K_1) \cap C(K_2)$. This shows that $C(K) = \mathcal{K}$. Since $C(K)$ is connected, it is an R^2-continuum.

THEOREM 3.4. *$K = LsC_n^1 \cap LsC_n^2 \subset U$ be an R^1-continuum of a metric continuum X having the property that each converging subsequence of $\{C_n^i\}_{n=1}^{\infty}$, $i = 1, 2$, converges w-regularly. Then $\mathcal{K} = LsC(C_n^1) \cap LsC(C_n^2)$ is an R^1-continuum of $C(X)$.*

Proof. Since the continuum K^* is contained in $\mathcal{K}$, $\mathcal{K}$ is nonempty and compact. We show that $\mathcal{K}$ is connected. Let $A \in \mathcal{K}$. Then, for each $i = 1, 2$, there is a sequence $\{A_{n_k}^i\}_{k=1}^{\infty}$, $A_{n_k}^i \in C(C_{n_k}^i)$, such that $A_{n_k}^i \to A$. Then $A \subset LiC_{n_k}^i$ for each $i = 1, 2$. Let $\{D_j^i\}_{j=1}^{\infty}$ be convergent subsequence of $\{C_{n_k}^i\}_{k=1}^{\infty}$ for each $i = 1, 2$. Then $A \subset LiD_j^i = LsD_j^i$ for each $i = 1, 2$. Since the convergence is w-regular, $C(A) \subset LtC(D_j^1) \cap LtC(D_j^2) \subset LsC(C_n^1) \cap LsC(C_n^2)$. Thus the connected set $K^* \cup C(A)$ is contained in $\mathcal{K}$. Therefore, $\mathcal{K}$ is connected and hence is an R^1-continuum of $C(X)$.

REMARK. In Theorem 3.4, we can not say that $\mathcal{K} = C(K)$. In fact, most likely $K \notin \mathcal{K}$ (see [3, Example 4] or Example C below).

THEOREM 3.5. *Let $K = LiC_n \subset U$ be an R^3-continuum of a metric continuum X with the property that each convergent subsequence of $\{C_n\}_{n=1}^{\infty}$ converges w-regularly. Then $\mathcal{K} = LiC(C_n)$ is an R^3-continuum of $C(X)$.*

Proof. Let $\mathcal{K} = LiC(C_n)$. Clearly $K^* = \{\{x\} : x \in X\} \subset \mathcal{K}$. Let $A \in \mathcal{K}$ and let $\{A_n\}_{n=1}^{\infty}$, $A_n \in C(C_n)$, be a sequence which converges to A. Then clearly $A \in C(K)$. We show first that $C(A) \subset LsC(C_{n_k})$

for each subsequence $\{C(C_{n_k})\}_{k=1}^{\infty}$ of $\{C(C_n)\}_{n=1}^{\infty}$. So let $\{C_{n_k}\}_{k=1}^{\infty}$ be any subsequence of $\{C_n\}_{n=1}^{\infty}$. Since $K \subset LsC_{n_k}$, we have $A \subset LsC_{n_k}$. Let $\{D_j\}_{j=1}^{\infty}$ be an convergent subsequence of $\{C_{n_k}\}_{k=1}^{\infty}$. Then $A \subset LtD_j$. Since the convergence is w-regular by our assumption, we have $A \subset C(LtD_j) = LtC(D_j)$. Now $C(A) \subset C(LtD_j)$ and $LtC(D_j) \subset LsC(D_j) \subset LsC(C_{n_k})$. Hence $C(A) \subset LsC(C_{n_k})$ for every subsequence $\{C_{n_k}\}_{k=1}^{\infty}$. Therefore, $C(A) \subset \mathcal{K}$ by [5].

Since $K^* \cup C(A)$ is connected and contained in $\mathcal{K}$ for each $A \in \mathcal{K}$, $\mathcal{K}$ is connected. This proves that $\mathcal{K}$ is an R^3-continuum.

EXAMPLE C (W.J. CHARATONIK). We give an example of an R^1-continuum of a metric continuum X which contains neither R^2- continuum nor R^3-continuum. This is a modified version of the example recently given by W.J. Charatonik to one of the authors.

We construct the example in E^3. If $p, q \in E^3$, the straight line segment between p and q is denoted by pq. Let $a = (2,0,0)$, $q_0 = (1,1,0)$, $r_0 = (-1,1,0)$, $s_0 = (-1,-1,0)$ and $t_0 = (1,-1,0)$. For each positive integer n, let $p_n^+ = (\frac{n+2}{n+1}, \frac{1}{n+1}, 0)$, $P_n^- = (\frac{n+2}{n+1}, \frac{-1}{n+1}, 0)$, $q_n^+ = (\frac{n+2}{n+1}, \frac{n+2}{n+1}, 0)$, $q_n^- = (\frac{n}{n+1}, \frac{n}{n+1}, 0)$, $r_n^+ = (\frac{-(n+2)}{n+1}, \frac{n+2}{n+1}, 0)$, $r_n^- = (\frac{-(n+2)}{n+1}, \frac{n}{n+1}, 0)$, $s_n^+ = (\frac{-(n+2)}{n+1}, \frac{-(n+2)}{n+1}, 0)$, $s_n^- = (\frac{-(n+2)}{n+1}, \frac{-n}{n+1}, 0)$, $t_n^+ = (\frac{n+2}{n+1}, \frac{-(n+2)}{n+1}, 0)$, $t_n^- = (\frac{n}{n+1}, \frac{-n}{n+1}, 0)$.

Let $S_0 = q_0r_0 \cup s_0t_0 \cup t_0q_0$, and $S_1 = S_0 \cup r_0s_0$. For each positive integer n, considering an ordering in $F_n = \{a, p_n^+, q_n^+, r_n^+, r_n^-, q_n^-, t_n^-, s_n^-, s_n^+, t_n^+, p_n^-, a\}$. Let D_n be the union of all line segments between two consecutive elements as listed in F_n. Let $H_0 = \cup_{n=1} D_n$, and let $X_0 = S_0 \cup H_0$. Let $f : E^3 \longrightarrow E^3$ be the rotation about the line $x = y = 0$ by the angle $\frac{\pi}{2}$. Let $X_i' = f^i(X_0)$, (f^0 = identity, f^2 = the composition of f and f, etc.), for $i = 0,1,2,3$. Let raise X_i' straight upward to the $z = i$ plane for $i = 1,2,3$, so that the resulting continua $\hat{X}_i$, $i = 1,2,3$, together with $X_0 = \hat{X}_0$ are pairwise disjoint.

We now wish to identify only those segments of $f^i(S_0)$, $i = 1,2,3$, in the $z = i$ plane to the those of S_1 so that $f^i(H_0)$ are pairwise disjoint. The identifying relation $\sim$ is defined as follows; let $x \in r_0q_0$, $y \in f(q_0)f(t_0)$, and $z \in f^2(t_0)f^2(s_0)$. Then the three points x, y, and z are identified as one point if and only if their first and second coordinates are the same. For each of other three tripletes of segments,

$\{q_0t_0, f(t_0)f(s_0), f^3(r_0)f^3(q_0)\}$, $\{s_0t_0, f^2(q_0)f^2(r_0), f^3(t_0)f^3(q_0)\}$, and $\{f(q_0)f(r_0), f^2(t_0)f^2(q_0), f^3(s_0)f^3(t_0)\}$, we identify in the sane way. Now let $X = (S_0 \cup X_0 \cup_{i=1}^3 \hat{X}_i)/\sim$ be the quotient space. Then the only R^i-continuum in X is S_1 (which is homeomorphic to the unit circle). Furthermore, S_1 is an R^1-continuum of X which contains neither R^2-continuum nor R^3-continuum.

EXAMPLE D. Let X be the space in Example C and let Y be the space in the Example 4 in [3]. Let $Z = (X \cup Y)/\{a, x\}$ be the quotient space, where a is the point in X and x is a point in Y at which Y is locally connected. Then Z contains the circle S_1 so that Z is not a dendroid. Also Z contains S_1 as an R^1-continuum, the segement ee' in Y as an R^2-continuum, and the singleton subset $\{q\}$ of Y as an R^3-continuum.

References

1. Charatonik, J.J. and Grabowski, Z., *Homotopically fixed arcs and the contractibility of dendroids*, Fund. Math. **100** (1978), 229–237.
2. Charatonik, W.J., *R^i-continua and hyperspaces*, Topology and its applications **23** (1986), 207–216.
3. Czuba, S.T., *R^i-continua and contractibility*, in Proc. International Conf. on Geometric Topology, PWN-Polish Sci. Publ., 1980, pp. 75–79.
4. ———, *R-continua and contractibility of dendroids*, Bull. de L'academie Polonaise des Sciences, ser.des science mathematiques **27** (1979), 229–303.
5. Kuratowski, K., *Topology I and II*, Academic Press, 1968.
6. Nadler, S.B.,Jr., *Concerning completeness of the space of confluent mappings*, Houston J. Math. **2** (1976), 561–580.
7. ———, *Hyperspaces of sets*, Marcel Dekker, New York, 1978.
8. Rhee, C.J. and Hur, K., *On spaces without R^i-continua*, submitted.
9. Whyburn, G.T., *Analytic Topology*, Amer. Math. Soc. Colloq. Publ., vol. 28, Amer. Math. Soc., Providence, R.I., 1942.

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