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ACCRETIVE OPERATORS IN A PROBABILISITIC NORMED SPACES

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1. Introduction and Preliminaries

Throughout this paper, the definitions and properities related to probabilistic normed spaces are followed as in [2]. Let $\mathcal{R}$ be the set of all real numbers. A mapping $F : \mathcal{R} \to [0,1]$ is called a distribution function on $\mathcal{R}$ if it is nondecreasing and left continuous with $\inf F = 0$ and $\sup F = 1$. We denote by L the set of all distribution functions on $\mathcal{R}$.

A mapping $\Delta : [0,1] \times [0,1] \to [0,1]$ is said to be a triangle norm (briefly, a T-norm) if

(1) $\Delta(0,0) = 0$ and $\Delta(a,1) = a$ for every $a \in [0,1]$,
(2) $\Delta(a,b) = \Delta(b,a)$ for every $a,b \in [0,1]$,
(3) $\Delta(a,b) \geq \Delta(c,d)$ for every $a,b,c,d \in [0,1]$ with $a \geq c$ and $b \geq d$,
(4) $\Delta(\Delta(a,b),c) = \Delta(a,\Delta(b,c))$ for every $a,b,c \in [0,1]$.

Let X be a real linear space and $F : X \to L$. For $x \in X$, we denote $F(x)$ by F_x. A triplet (X, F, Δ) is called a probabilistic normed space (briefly, a PN-space) if

(1) $F_x(0) = 0$ for every $x \in X$,
(2) $F_x = H$ if and only if $x = 0$, where $H \in L$ with $H(t) = 1$ for every $t > 0$, and $H(t) = 0$ for every $t \leq 0$,
(3) $F_{rx}(t) = F_x(t/|r|)$ for every $x \in X$, $r \in \mathcal{R}$ with $r \neq 0$, and $t \in \mathcal{R}$,
(4) $F_{x+y}(s+t) \geq \Delta(F_x(s), F_y(t))$ for every $x,y \in X$ and $s,t \in \mathcal{R}$.

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Let $x \in X$, $\epsilon > 0$ and $\lambda \in (0,1)$. Then an (ϵ, λ)-neighborhood of x, denoted by $N_x(\epsilon, \lambda)$, is defined by $N_x(\epsilon, \lambda) = \{y \in X \mid F_{x-y}(\epsilon) > 1-\lambda\}$. The family $\{N_x(\epsilon, \lambda) \mid x \in X, \epsilon > 0, \lambda \in (0,1)\}$ of neighborhood induces a topology on X satisfying the first axiom of the countablity and a Hausdorff topology on X with a continuous T-norm Δ.

Let (X, F, Δ) be a PN-space with a continuous T-norm Δ. A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if every $\epsilon > 0$ and $\lambda \in (0,1)$, there exists a positive integer N such that $F_{x-x_n}(\epsilon) > 1-\lambda$ for every $n \geq N$. We denote $x_n \to x$ or $\lim_{n\to\infty} x_n = x$. A sequence $\{x_n\}$ is said to be a Cauchy sequence if for every $\epsilon > 0$ and $\lambda \in (0,1)$, there exists a positive integer N such that $F_{x_n - x_m}(\epsilon) > 1-\lambda$ for every $n, m \geq N$. A PN-space with a continuous T-norm Δ is said to be complete if every Cauchy sequence in X is convergent to some point in X.

For a PN-space (X, F, Δ) with a continuous T-norm Δ, if $x_n \to x$ in X then $\underline{\lim}_{n\to\infty} F_{x_n}(t) = F_x(t)$ for every $t \in \mathcal{R}$.

The concept of an accretive operator in a PN-space was introduced by Zhang-Chen ([3]). One may refer to Barbu ([1]) for an accretive operator in a Banach space.

Let (X, F, Δ) be a PN-space and $A : X \to 2^X$ an operator with domain $D(A) = \{x \in X \mid Ax \neq 0\}$ and range $R(A) = \cup\{Ax \mid x \in D(A)\}$. We may identify A with its graph. A is said to be accretive in X if every $[x_1, y_1]$, $[x_2, y_2] \in A$, $r > 0$, and $t \in \mathcal{R}$,

$$F_{x_1 - x_2}(t) \geq F_{x_1 - x_2 + r(y_1 - y_2)}(t)$$

and A is said to be m-accretive in X if A is accretive in X and $R(I + rA) = X$ for every $r > 0$, equivalently, by [3], A is accretive in X and $R(I + rA) = X$ for some $r > 0$.

In section 2, we are concerned with properities of accretive operators and their resolvents in a PN-space. Section 3 contains some results of convergence of resolvents of accretive operators in a PN-space.

2. Properities of Accretive Operators

Let (X, F, Δ) be a PN-space and A be accretive in X. We put $J_r = (I+rA)^{-1}$ and $A_r = \frac{1}{r}(I-J_r)$ for every $r > 0$. Then $D(J_r) = R(I+rA)$, $R(J_r) = D(A)$ and $D(A_r) = D(J_r)$ for every $r > 0$.

First, we consider the properities of J_r.

LEMMA 1. *Let A be accretive in X. Then J_r is single-valued and*

$$F_{J_r x - J_r y}(t) \geq F_{x-y}(t)$$

for every $x, y \in D(J_r)$, $r > 0$ and $t \in \mathcal{R}$.

Proof. Let $x, y \in D(J_r)$, $r > 0$ and $t \in \mathcal{R}$. Suppose $y_1, y_2 \in J_r x$. Since A is accretive in X,

$$\begin{aligned} F_{y_1-y_2} &\geq F_{y_1-y_2+r(\frac{1}{r}(x-y_1)-\frac{1}{r}(x-y_2))}(t) \\ &= F_0(t) = H(t). \end{aligned}$$

Hence $F_{y_1-y_2}(t) = H(t)$ and thus $y_1 = y_2$. There exists $[x_1, y_1]$, $[x_2, y_2] \in A$ such that $x = x_1 + ry_1$ and $y = x_2 + ry_2$ and thus $J_r x = x_1$, $J_r y = x_2$. Since A is accretive in X,

$$F_{J_r x - J_r y}(t) = F_{x_1 - x_2}(t) \geq F_{x_1 - x_2 + r(y_1 - y_2)}(t) = F_{x-y}(t).$$

PROPOSITION 2. *Let A be accretive in X.*

(1) *Suppose $\Delta(a, a) \geq a$ for every $a \in [0, 1]$. Then*

$$F_{\frac{1}{n}(J_r^n x - x)}(t) \geq F_{J_r x - x}(t)$$

for every $x \in D(J_r)$, $r > 0$, $t \in \mathcal{R}$ and $n = 1, 2, \cdots$.

(2) *$\frac{r}{p}x + \frac{p-r}{p}J_p x \in D(J_r)$ and $J_p x = J_r(\frac{r}{p}x + \frac{p-r}{p}J_p x)$ for every $x \in D(J_p)$, $p > 0$ and $r > 0$.*

(3) *$F_{J_p x - J_r y}(t) \geq F_{\frac{r}{p+r}(x - J_r y) - \frac{p}{p+r}(y - J_p x)}(t)$ for every $x \in D(J_p)$, $y \in D(J_r)$, $p, r > 0$ and $t \in \mathcal{R}$.*

Proof. (1) Let $x \in D(J_r)$, $r > 0$, $t \in \mathcal{R}$ and $n = 1, 2, \cdots$. By assumption and Lemma 1,

$$\begin{aligned}
&F_{\frac{1}{n}(J_r^n x - x)}(t) = F_{J_r^n x - x}(nt) \\
&\geq \Delta(F_{J_r^n x - J_r^{n-1} x - x}(t), F_{J_r^{n-1} x - x}((n-1)t)) \\
&\geq \Delta(F_{J_r^n x - J_r^{n-1} x}(t), \Delta(F_{J_r^{n-1} x - J_r^{n-2} x}(t), \cdots\cdots, \\
&\qquad \Delta(F_{J_r^2 x - J_r x}(t), F_{J_r x - x}(t)) \cdots\cdots) \\
&\geq \Delta(F_{J_r x - x}(t), \Delta(F_{J_r x - x}(t), \cdots, \Delta(F_{J_r x - x}(t), F_{J_r x - x}(t))) \cdots) \\
&\geq F_{J_r x - x}(t).
\end{aligned}$$

(2) Let $x \in D(J_p)$, $p > 0$ and $r > 0$. There exists $[x_1, y_1] \in A$ such that $x = x_1 + py_1$.

$$\begin{aligned}
&\frac{r}{p}x + \frac{p-r}{p}J_p x = x_1 + ry_1 \in D(J_r) \quad \text{and} \\
&J_r(\frac{r}{p}x + \frac{p-r}{p}J_p x) = J_r(x_1 + ry_1) = x_1 = J_p x.
\end{aligned}$$

(3) Let $x \in D(J_p)$, $y \in D(J_r)$, $p, r > 0$ and $t \in \mathcal{R}$. Putting $q = \dfrac{p-r}{p+r}$ by (2),

$$\begin{aligned}
&\frac{q}{p}x + \frac{p-q}{p}J_p x \in D(J_q) \text{ and } \frac{q}{r}x + \frac{r-q}{r}J_r y \in D(J_q), \\
&J_p x = J_q(\frac{q}{p}x + \frac{p-q}{p}J_p x) = J_q(\frac{r}{p+r}x + \frac{p}{p+r}J_p x), \\
&J_r y = J_q(\frac{q}{r}y + \frac{r-q}{r}J_r y) = J_q(\frac{p}{p+r}y + \frac{r}{p+r}J_r y).
\end{aligned}$$

By Lemma 1,

$$\begin{aligned}
F_{J_p x - J_r y}(t) &= F_{J_q(\frac{r}{p+r}x + \frac{p}{p+r}J_p x) - J_q(\frac{p}{p+r}y + \frac{r}{p+r}J_r y)}(t) \\
&\geq F_{\frac{r}{p+r}(x - J_r y) - \frac{p}{p+r}(y - J_p x)}(t).
\end{aligned}$$

Next we consider the properties of A_r.

PROPOSITION 3. *Let A be accretive in X.*

(1) *Suppose $\Delta(a,a) \geq a$ for every $a \in [0,1]$. Then*

$$F_{A_r x - A_r y}(t) \geq F_{\frac{2}{r}(x-y)}(t)$$

for every $x, y \in D(J_r)$, $r > 0$ and $t \in \mathcal{R}$.

(2) *$A_r x \in AJ_r x$ for every $x \in D(J_r)$ and $F_{A_r x}(t) \geq \sup_{y \in Ax} F_y(t)$ for every $x \in D(A) \cup D(J_r)$, $r > 0$ and $T > 0$.*

Proof. (1) Let $x, y \in D(J_r)$, $r > 0$ and $t \in \mathcal{R}$. Then by Lemma 1,

$$\begin{aligned} F_{A_r x - A_r y}(t) &= F_{\frac{1}{r}(x-y) - \frac{1}{r}(J_r x - J_r y)}(t) \\ &\geq \Delta(F_{\frac{1}{r}(x-y)}(\frac{t}{2}), F_{J_r x - J_r y}(\frac{rt}{2})) \\ &\geq \Delta(F_{\frac{2}{r}(x-y)}(t), F_{\frac{2}{r}(x-y)}(t)) \\ &\geq F_{\frac{2}{r}(x-y)}(t). \end{aligned}$$

(2) Let $x \in D(J_r)$, $r > 0$. By definition, $A_r x \in AJ_r x$. Let $x \in D(A) \cap D(J_r)$, $r > 0$ and $t > 0$. Suppose $[x, y] \in A$. By Lemma 1,

$$\begin{aligned} F_{A_r x}(t) &= F_{x - J_r x}(rt) = F_{J_r(x+ry) - J_r x}(rt) \\ &\geq F_{x+ry-x}(rt) = F_y(t). \end{aligned}$$

Thus $F_{A_r x}(t) \geq \sup_{y \in Ax} F_y(t)$.

We are going to consider the maximum accretivity.

DEFINITION 4. Let $A, B : X \to 2^X$ be operators. B is said to be an extension of A if $D(A) \subset D(B)$ and $Ax \subset Bx$ for every $x \in D(A)$. We denote it by $A \subset B$.

DEFINITION 5. A is said to be maximal accretive operator in X if A is an accretive operator of X and for every accretive operator B of X with $A \subset B$, $A = B$.

PROPOSITION 6. *If A is an m-accretive operator in X, then A is an maximal accretive operator of X.*

Proof. Let B be accretive in X with $A \subset B$. Let $r > 0$ and $t \in \mathcal{R}$. Let $[x, y] \in B$. Since A is m-accretive in X, $x + ry \in R(I + rA)$. There exists $[x_1, y_1] \in A$ such that $x + ry = x_1 + ry_1$. Since B is accretive and $[x_1, y_1] \in B$,

$$F_{x-x_1}(t) \geq F_{x-x_1+r(y-y_1)}(t) = F_0(t) = H(t).$$

Hence $x = x_1$ and thus $y = y_1$. Therefore $[x, y] \in A$, that is, $B \subset A$ and thus $A = B$. Consequently, A is maximal accretive in X.

PROPOSITION 7. *Let A be accretive in X and let $[u, v] \in X \times X$. Then A is maximal accretive in X if and only if*

$$F_{x-u}(t) \geq F_{x-u+r(y-v)}(t)$$

for every $[x, y] \in A$, $r > 0$ and $t \in \mathcal{R}$ implies $[u, v] \in A$.

Proof. Let A be maximal accretive in X. Put $\hat{A} = A \cup [u, v]$. Then $\hat{A}$ is accretive in X and $A \subset \hat{A}$. Since A is maximal accretive in X, $\hat{A} = A$. Hence $[u, v] \in A$. Conversely, let B be accretive in X with $A \subset B$. Let $[u, v] \in B$. Since B is accretive in X, for every $[x, y] \in A$, $r > 0$ and $t \in \mathcal{R}$,

$$F_{x-x_u}(t) \geq F_{x-u+r(y-v)}(t).$$

By assumption, $[u, v] \in A$ and thus $B \subset A$. Hence $A = B$. Therefore A is maximal accretive in X.

PROPOSITION 8. *Let A be accretive in X. Then there exists a maximal accretive operator containing A.*

Proof. Let $\mathcal{B} = \{B :$ accretive in $X \mid A \subset B\}$. Then $(\mathcal{B}, \subset)$ is a partially ordered set. Let $\mathcal{T}$ be a totally ordered set with $\mathcal{T} \subset \mathcal{B}$. It is easy to show that $\mathcal{T}$ has an upper bound. By Zorn's lemma, there exists a maximal element in $\mathcal{B}$. This is a maximal accretive operator of X containing A.

Now we consider the closeedness of accretive operators.

PROPOSITION 9. *Let A be accretive in X. Then the closure $\overline{A}$ of A is also accretive in X.*

Proof. Let $[x_1, y_1], [x_2, y_2] \in \overline{A}$. Then there exists $[x_{1n}, y_{1n}], [x_{2n}, y_{2n}] \in A$ such that $x_{1n} \to x_1$, $x_{2n} \to x_2$, $y_{1n} \to y_1$ and $y_{2n} \to y_2$. Let $r > 0$ and $t \in \mathcal{R}$. Since A is accretive,

$$F_{x_{1n}-x_{2n}}(t) \geq F_{x_{1n}-x_{2n}+r(y_{1n}-y_{2n})}(t).$$

As $n \to \infty$, we have

$$F_{x_1-x_2}(t) \geq F_{x_1-x_2+r(y_1-y_2)}(t).$$

Hence $\overline{A}$ is accretive in X.

PROPOSITION 10. *Let X be complete and Δ be continuous. Let A be accretive in X. If A is closed, then $R(I+rA)$ is also closed for every $r > 0$.*

Proof. Let $z_n \in R(I+rA)$ with $z_n \to z \in X$. Then $\{z_n\}$ is a Cauchy sequence in X. There exists $[x_n, y_n] \in A$ such that $x_n + ry_n = z_n$ and thus $J_r z_n = x_n$. By Lemma 1, for every $t \in \mathcal{R}$

$$F_{x_n-x_m}(t) = F_{J_r z_n - J_r z_m}(t) \geq F_{z_n-z_m}(t).$$

Hence

$$\varliminf_{n,m\to\infty} F_{x_n-x_m}(t) \geq \varliminf_{n,m\to\infty} F_{z_n-z_m}(t) = F_0(t) = 1$$

for every $t > 0$. Thus $\lim_{n,m\to\infty} F_{x_n-x_m}(t) = 1$ for every $t > 0$. Therefore $\{x_n\}$ is a Cauchy sequence in X. There exists $x \in X$ such that $x_n \to x$ and thus $y_n = \frac{1}{r}(z_n - x_n) \to \frac{1}{r}(z - x)$. Since A is closed, $[x, \frac{1}{r}(z-x)] \in A$. Hence $z \in x + rAx \subset R(I+rA)$. Therefore $R(I+rA)$ is closed.

PROPOSITION 11. *Let A be maximal accretive in X. Then A is closed.*

Proof. Let $[x_n, y_n] \in A$ and $x_n \to u$, $y_n \to v$. Let $r > 0$ and $t \in \mathcal{R}$. Since A is accretive, for every $[x, y] \in A$,

$$F_{x-x_n}(t) \geq F_{x-x_n+r(y-y_n)}(t).$$

As $n \to \infty$,

$$F_{x-u}(t) \geq F_{x-u+r(y-v)}(t).$$

Since A is maximal accretive, by Proposition 7, $[u, v] \in A$. Hence A is closed.

COROLLARY 12. (1) *Let A be m-accretive in X. Then A is closed.* (2) *Let A be maximal accretive in X. Then Ax is closed subset of X for every $x \in A$.*

PROPOSITION 13. *Let X be complete and A accretive in X. Let C be a closed convex subset of X and $p > r > 0$. If $R(I + rA) \supset C$ and $J_rC \subset C$ then $R(I + pA) \supset C$ and $J_pC \subset C$.*

Proof. Let $x \in C$ and $p > r > 0$. Define $S : C \to C$ by $Sz = J_r(\frac{r}{p}x + \frac{p-r}{p}z)$ for every $z \in C$. Let $t \in \mathcal{R}$. By Lemma 1, for every $z_1, z_2 \in C$,

$$\begin{aligned} F_{Sz_1 - Sz_2}(t) &= F_{J_r(\frac{r}{p}x + \frac{p-r}{p}z_1) - J_r(\frac{r}{p}x + \frac{p-r}{p}z_2)}(t) \\ &\geq F_{\frac{p-r}{p}(z_1 - z_2)}(t). \end{aligned}$$

Since $0 < \frac{p-r}{p} < 1$, by [2], there exists $z \in C$ uniquely such that $Sz = z$. It follows that $x \in z + pAz \subset R(I + pA)$. Thus $R(I + pA) \supset C$ and $J_pC \subset C$.

3. Convergence of Resolvents of Accretive Operators

Let (X, F, Δ) be a PN-space and J_r be the resolvent of an accretive operator A in X for every $r > 0$.

PROPOSITION 14. *Let A be accretive in X and Δ be continuous. Then*

$$\lim_{r\to 0+} J_r x = x \quad \text{for every } x \in \cap_{r>0} D(J_r) \cap D(A).$$

Proof. Let $x \in \cap_{r>0} D(J_r) \cap D(A)$ and $t \in \mathcal{R}$. By (2) of Proposition 3, as $r \to 0+$,

$$F_{J_r x - x}(t) = F_{A_r x}(\frac{t}{r}) \geq \sup_{y\in Ax} F_y(\frac{t}{r}) \to 1$$

for every $t > 0$. Thus $\lim_{r\to 0+} F_{J_r x - x}(t) = 1$ for every $t > 0$. Hence $\lim_{r\to 0+} J_r x = x$.

PROPOSITION 15. *Let A be accretive in X and let Δ be continuous with $\Delta(a,a) \geq a$ for every $a \in [0,1]$. Then*

$$\underline{\lim}_{r\to\infty} F_{J_r x/r}(t) = \lim_{r\to\infty} F_{A_r x}(t) = \sup_{y\in R(A)} F_y(t)$$

for every $x \in \cap_{r>0} D(J_r)$ and $t \in \mathcal{R}$.

Proof. Let $x \in \cap_{r>0} D(J_r)$ and $t \in \mathcal{R}$. Put $d_t = \sup_{y\in R(A)} F_y(t)$. By (2) of Proposition 3, $F_{A_r x}(t) \leq \sup_{y\in R(A)} F_y(t) = d_t$. Thus $\overline{\lim}_{r\to\infty} F_{A_r x}(t) \leq d_t$. Let $\alpha \in (0,1)$. By definition of $d_{\alpha t}$, for every $\epsilon > 0$, $d_{\alpha t} - \epsilon < F_{y_0}(\alpha t)$ for some $[x_0, y_0] \in A$. By (1) of Proposition 3,

$$\begin{aligned} F_{A_r x}(t) &= F_{A_r x - A_r x_0 + A_r x_0}((1-\alpha)t + \alpha t) \\ &\geq \Delta(F_{A_r x - A_r x_0}((1-\alpha)t), F_{A_r x_0}(\alpha t)) \\ &\geq \Delta(F_{\frac{2}{r}(x-x_0)}((1-\alpha)t), F_{A_r x_0}(\alpha t)). \end{aligned}$$

Thus for every $t > 0$,

$$\begin{aligned} \underline{\lim}_{r\to\infty} F_{A_r x}(t) &\geq \underline{\lim}_{r\to\infty} \Delta(F_{\frac{2}{r}(x-x_0)}((1-\alpha)t), F_{A_r x_0}(\alpha t)) \\ &\geq \Delta(\underline{\lim}_{r\to\infty} F_{\frac{2}{r}(x-x_0)}((1-\alpha)t), \underline{\lim}_{r\to\infty} F_{A_r x_0}(\alpha t)) \\ &= \Delta(1, \underline{\lim}_{r\to\infty} F_{A_r x_0}(\alpha t)) = \underline{\lim}_{r\to\infty} F_{A_r x_0}(\alpha t) \\ &\geq \sup_{y\in Ax_0} F_y(\alpha t) \geq F_{y_0}(\alpha t) > d_{\alpha t} - \epsilon. \end{aligned}$$

Since ϵ is arbitrary, as $\epsilon \to 0+$, $\underline{\lim}_{r\to\infty} F_{A_r x}(t) \geq d_{\alpha t}$. Since $\lim_{\alpha\to 1-} d_{\alpha t} = d_t$, $\underline{\lim}_{r\to\infty} F_{A_r x}(t) \geq d_t$. Therefore $\lim_{r\to\infty} F_{A_r x}(t) = d_t$. The second equality holds.
Next we consider the first equality. Let $\alpha \in (0,1)$. From

$$F_{J_r x/r}(t) = F_{A_r x - \frac{x}{r}}(t) \geq \Delta(F_{A_r x}(\alpha t), F_{\frac{x}{r}}((1-\alpha)t))$$

we have for every $t > 0$,

$$\begin{aligned} \underline{\lim}_{r\to\infty} F_{J_r x/r}(t) &\geq \underline{\lim}_{r\to\infty} \Delta(F_{A_r x}(\alpha t), F_{\frac{x}{r}}((1-\alpha)t)) \\ &\geq \Delta(\underline{\lim}_{r\to\infty} F_{A_r x}(\alpha t), \underline{\lim}_{r\to\infty} F_{\frac{x}{r}}((1-\alpha)t)) \\ &= \Delta(\underline{\lim}_{r\to\infty} F_{A_r x}(\alpha t), 1) = \underline{\lim}_{r\to\infty} F_{A_r x}(\alpha t). \end{aligned}$$

As $\alpha \to 1-$, $\underline{\lim}_{r\to\infty} F_{J_r x/r}(t) \geq \underline{\lim}_{r\to\infty} F_{A_r x}(t)$. Similarly, $\underline{\lim}_{r\to\infty} F_{A_r x}(t) \geq \underline{\lim}_{r\to\infty} F_{J_r x/r}(t)$. Hence $\underline{\lim}_{r\to\infty} F_{J_r x/r}(t) = \underline{\lim}_{r\to\infty} F_{A_r x}(t)$. The proof is completed.

References

1. V. Barbu, *Nonlinear semigroups and differential equations in Banach spaces*, Edut. Acad. R.S.R., Bucuresti, 1976.
2. B. Schweizer and A. Sklar, *Probabilistic metric spaces*, North-Holland, New York, 1983.
3. S.S. Zhang and Y.Q. Chen, *On the existence of solutions for equations with accretive mapping in probabilistic normed spaces*, Appl. Math. Mech. **11** (1990), 821–828.

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