□ 論 文 □

도시경제모형과 교통
-토지이용모형을 결합한 대도시통합모형-

AN INTEGRATED URBAN MODEL : A COMBINATION OF AN URBAN ECONOMIC MODEL AND TRANSPORTATION-LAND USE MODELS

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국문요약

도시확산과 지방분권화의 시대적 추세는 도시경제 및 교통-토지이용모형의 구조에도 영향을 미치고 있다. 첫째, 도시의 공간적 확산과 고밀도화로 인한 도시내 제반 경제.사회활동이 이전보다는 훨씬 다양해지고 복잡해지고 있다. 들째, 지방분권화로인해 대도시내 작은지역단위별 주민들이 자신의 지역문제에 대한 관심과 요구들이 증가하고 있다. 또한 대부분의 도시계획 및 정책은 대도시내 공간적으로 뿐만아니라 서로 다른 이익집단에 상이한 영향을 미치는 것으로 파악되고 있다. 이러한 요인으로 인해 대도시 모형은 1) 도시내 제반활동의 흐름을 체계적으로 분석할 수 있어야하며, 2) 도시계획 및 정책의 효과를 도시내 세분화된 공간별, 업종별로 분석할 수 있어야 한다.

본 연구는 Wilson의 모형에 도시내 중요한 경제활동인 통근과 shopping변수를 추가하여 Wilson의 모형이 대도시모형으로 이용될 때 발생하는 문제를 해결하였다. 또한 모형을 구성함에 있어 Matrix Inversion 과 Non-linear Programming Model의 두가지 방법을 도입하고 그 장단점을 비교하였다. Matrix Inversion의 경우 해 (Solution)을 찾기가 쉬워 실질적이고 승수계산과정에서 각 Round마다 승수의 크기를 알 수 있는 반면 Programming을 통한 모형의 경우 제약식의 도입이 용이하고 목적함수의 형태를 바꾸는 것이 가능하므로 이론적 기틀을 세우는데 유리한 접근방법으로 분석되었다.

1. INTRODUCTION

Although research on input−output models has focused on the areas of regional, interregional, and multiregional input−output models, there has been little discussion of metropolitan input−output models. Major reasons for the limited discussion of the metropolitan input−output models are i) the different factors characterizing the metropolitan economy such as free flows of people, commodity and services and ii) implementation problems resulting from a detailed sectoral and zonal system. In particular, most urban policies require highly detailed urban economic models in terms of sector and space because of different consequences of a policy on various business groups and on various zones.

As Richardson and Gordon (1989) mentioned, few urban models satisfy both sectoral and spatial disaggregation. The regional input−output model can deal with inter−sectoral linkages, but it is aspatial. Conversely, many metropolitan spatial allocation models such as the Lowry type models can disaggregate the metropolitan region into as many zones as possible, but can deal only with two types of industries: basic and non−basic.

Alternatively, the interregional and multiregional model may be regarded as the most ideal and detailed form of completely connected regional input−output model (Blair and Miller, 1983). However, the detailed requirement of urban models in terms of sector and space creates problems of operationability and these models are seldom implemented in the real world.

Another problem in applying multiregional and interregional input−output analysis to urban models is that interregional and multiregional models have limitations to model the flows of people (such as commuting). Commuting is one of the important factors that is not captured in interregional input−output models. The different locations of consumption and production activities imply unequal economic impacts over space. The structure of a metropolitan model should, therefore, be different from that of the standard regional model (Richardson, 1985).

The purpose of this study is to develop an integrated urban model model which combines an input−output model with a Lowry−type model. Wilson's (1970) synthetic model of the regional input−output model and the gravity type model will be extended into an urban input−output model by incorporating a journey from home to work matrix (work trip matrix) and a journey from home to shop matrix (non−work trip matrix) as well as a commodity flow matrix.

The next section reviews research on applying regional economic models and spatial interaction models into the urban context. Wilson's model (1970) and Macqill's model (1977) will be reviewed because the proposed model in this paper is an extension of those models. The contribution of the new model to the field of urban economics will be presented in the conclusion.

2. Literature Review

Even though a few researchers have attempted to combine regional input-output models with other urban models (Moses (1960), Wilson (1970), Hewings (1985), and Kim et al (1983)), there have been very limited attempts to develop a metropolitan input-output model. Here, I will review several researches on combining the regional input-output model with the spatial allocation model.

2.1 A. G. Wilson

Wilson (1970) derived four types of linkage between input-output and commodity flow models: 1) the unconstrained gravity input-output model, 2) the production constrained gravity input-output model, 3) the attraction constrained gravity input-output model, and 4) the production-attraction constrained gravity input-output model. This review explores the unconstrained gravity input-output model because the other three models have the same functional forms except the equation(s) added into constraints such as demand or (and) supply constraints. Basically his model is a modification of the Leontief-Strout model (1963) by integrating gravity and input-output models using entropy-maximizing principles. Wilson's unconstrained gravity input-output model has the following objective function and constraints.

$$\text{Max } S = -\sum_{ijm} X_{ij}{}^{m} \ln X_{ij}{}^{m} \quad (2.1)$$

st.

$$\Sigma_j X_{ji}{}^{m} = \Sigma_n (a_{mn}{}^{i}\, \Sigma_j X_{ij}{}^{n}) + y_i{}^{m} \quad (2.2)$$

$$\sum_{ij} c_{ij}{}^{m} X_{ij}{}^{m} = C^{m} \quad (2.3)$$

where $X^{m}{}_{ij}$ = the flow of commodity m between zone i and j; $a^{i}{}_{mn}$ = input-output coefficients defining the requirements of sector m's output per unit output of sector n in zone i; $Y^{m}{}_{i}$ = the exogenously pre-specified demand for m in zone i; $c^{m}{}_{ij}$ = the average cost of movement of m between zone i and j; and C^{m} = total transportation cost for the commodity m.

Wilson's contribution is the integration of a gravity model and an input-output model with the entropy maximization principle. In his non-linear programming model, interregional (zonal) commodity flows are determined, given interindustrial linkages and interregional transportation costs, while minimizing transportation cost or maximizing entropy. While his model can explain interindustrial linkages over space, it still falls short of providing an urban context. Since his model ignores commuting and shopping trips, his model cannot explain consumption patterns over space. Consumption patterns are one of the important factors in economic impact analysis not only because the allocation of induced impacts should be made based on money flows for the consumption activities but because the largest part of the exogenous sector is household consumption in the urban area (Artle, 1961)

2.2 S. M. Macgill

Macgill (1977) presents the Lowry model (1964) as an input−output model. Since the Lowry model is regarded as a metropolitan economic model which is a combination of the economic base model and the spatial interaction model, Macgill's formal presentation of the Lowry model as input−output model can be stated as the first attempt to build a metropolitan input−output model.

By extending the spatially aggregated model into the spatially disaggregated input−output Lowry model, Macgill fully incorporates trip matrices (work and nonwork trip flows) into the input−output Lowry model representation. This section presents an input−output transaction matrix by assuming two sectors and two zones, even though she assumed two nonbasic sectors and three zones.

(Table 2.1) Macgill's Spatially Disaggregated Model

	HH1	HH2	NB11	NB12	NB21	NB22	BAS	TOT
HH1	0	0	NB1T111	NB1T112	NB2T211	NB2T212	BTB1*	XH1
HH2	0	0	NB1T121	NB1T122	NB2T221	NB2T222	BTB2*	XH2
NB11	S111	S112	0	0	0	0	0	XR11
NB12	S121	S122	0	0	0	0	0	XR12
NB21	S211	S212	0	0	0	0	0	XR21
NB22	S221	S222	0	0	0	0	0	XR21

Table 2.1 shows the transaction matrix of the spatially disaggregated input−output Lowry model representation. HH, NB, and BAS in the table mean the household, nonbasic, and basic sectors. The first and second subscripts are sector and zone, respectively. Of initial interest are 1) the endogenous transactions in the table, and 2) the exogenously pre−specified basic employment. The first two rows present labor input from the household to nonbasic and basic industries by zone. NB_k and B present the wage rate for nonbasic industry k and basic industry, respectively. $T1_{ij}$, $T2_{ij}$, and TB_{ij} are the work trip flow matrix which can be presented as the singly constrained model (destination constrained model) as follows:

$$T_{ij} = \frac{E_j^k f(c_{ij})}{\sum_j f(c_{ij})} \qquad (2.8)$$

where E^k_j means the number of employed by type k (basic or nonbasic) in zone j, and the other part of equation is the probability matrix of commuting.

The third to last rows in the table show the shopping trips household in each zone make to purchase nonbasic industries 1 and 2. Like Romanoff's (1974) presentation of the economic base model in a matrix form, all intermediate transactions between nonbasic and basic and among nonbasic industries are zero because no interdependence among nonbasic and between nonbasic and basic are assumed in the Lowry model. Moreover, the vector of basic sectors in the Table 2.1 is consistent with Romanoff's presentation. Since none of nonbasic industries output is consumed as final demand within the region or as exports to outside the region, all the cells representing the basic industries' purchase from nonbasic sectors are zero.

A significant modification to the overall model mechanism was made in order to overcome these limitations. A full set of interactions both within and between the basic and nonbasic sectors is accommodated into the input−output representation of the Lowry model, and all the cells with zero coefficients in the Table 2.1 are filled with non−zero coefficients.

Macgill presents the full extension of the Lowry model as an input−output model in a different way by incorporating the entropy maximizing concept. Unlike the endogenous variable of the previous matrix presentation of the model, X^m_j (total output in sector m in zone j), the maximum entropy representation of the extended model estimates the flow of commodity m between zone i and j, X^m_{ij} . From Wilson's (1970) entropy maximizing methods, the maximum entropy formulation of the Lowry model extensions depends on the expression of the underlying assumptions of that model in the form of constraining equations (Macgill, op. cit.). The entropy maximization of the extensions of the Lowry model has the same equational forms as Wilson's model has.

In summary, the multipliers of Macgill's Lowry input−output model accommodating all sectoral interactions are not very different conceptually from the so−called Type II multipliers in the input−output model, because both models have endogenous household consumption sector. One argument against Macgill's model, however, is that the multipliers from Macgill's model is not the same as those from the closed input−output model. Coefficients of the household sector in Macgill's model (shopping and labor input) are computed by using the wage rate and expenditure pattern, while coefficients of the household sector in the closed input−output model are computed directly from the transaction matrix. Therefore, Macgill's model cannot take account of the Type II multiplier effects as in the closed input−output model.

Another problem of Macgill's model is its applicability to the real world. In her model, zone− and sector−specific commuting and shopping flow information is ideal in Isard's sense but unrealistic, in particular, if zone and sector are highly detailed.

An alternative to overcome Macgill's limitations will be suggested in the following section.

3. An Integrated Urban Model

The characteristics of an urban economy, compared with a regional economy, are 1) high dependence of urban economic growth on indigenous demand (household consumption), 2) a high degree of industrial linkage (agglomeration economy), and 3) free flows for commuting and shopping.

This section introduces a new urban model which includes the three above characteristics for urban economic models. The Integrated Urban Model (hereafter the IUM) is an extension of the combination of the regional input−output model and the gravity type model with the entropy maximization principle, modeled by Wilson. The IUM incorporates the Lowry type models within an input−output framework relieving several unrealistic assumptions in Macgill's input−output representation of Lowry model. The major difference of the IUM from Wilson's model is that the IUM has three types of flow matrices (work trips, non−work trips, and commodity flows) within the model, while Wilson's model has focused on estimating Leontief−Strout type commodity flows.

The IUM will be presented in two ways: 1) a matrix inverse multiplier model, and 2) a non−linear mathematical programming model.

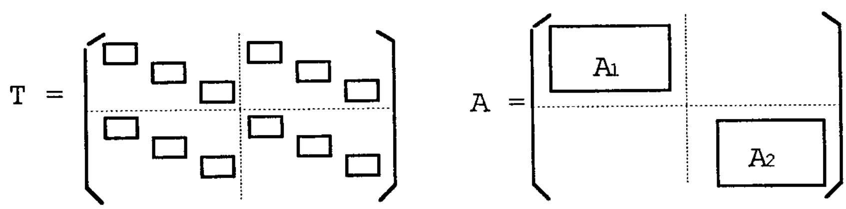
3.1 The Integrated Urban Model as a Matrix Inverse Multiplier Model

The IUM is presented as a matrix inverse multiplier model in this section with three entropy maximization models as submodels. This section mainly consists of two parts: 1) estimating net indirect multipliers, and 2) estimating net induced multipliers. For calculating net indirect multipliers, interzonal trade flows will be estimated via an entropy maximization model. Given the trade flows, a multizonal input−output model will be introduced. For calculating the net induced multipliers, interzonal trip flows (commuting and shopping trips) will be estimated via two submodels with the entropy maximization principle and the multizonal input−output model will be adjusted via the trip matrices.

The IUM has a Leontief−Strout−type multiregional input−output model as its basic component. The multiregional input−output model can be written as follow:

$$X = (I - TA)^{-1} * TF \qquad (3.1)$$

where X=total output; T=trade flow coefficients1; and F=final demand. The inverse matrix represents output multipliers. If we assume two zones and three sectors in each zone, the T and A matrices will be presented as follow:



(Figure 3.1) The Structure of Matrices of Trade Flow and Technical Coefficients where A1 and A2 are technical coefficients for zone 1 and 2 (3*3 in each matrix). The dimension of the A matrix in the above equations depends on the number of zones and sectors. For example, if there are n zones and m sectors, the dimension of A matrix is $(n*m)^2$.

3.1.1. Estimation of Interzonal Trade Flow

The ideal form of information for the spatial allocation of indirect impacts is an Isard−type commodity and service flow which is both zone−specific and sector−specific (T^{lm}_{ij}, where i,j=origin and destination sectors and l,m=origin and destination zones). However, zone− and sector−specific trade flows are neither available in reality nor operable. This study uses a Leontief−Strout type trade flow matrix (T^{lm}_{i}) which is zone−specific in a given sector (i) in order to build a metropolitan input−output model.

Given the data on zonal supply and demand, two more data set are required to estimate intraurban commodity flows: 1) the observed travel times among zones, and 2) total travel cost. The entropy maximization model will be used in order to estimate intraurban commodity flows.

The entropy maximization model has following objective fuction and constraints:

$$\text{MAX} \quad -\sum_{ij} T_{ij}^{m} \ln T_{ij}^{m} \qquad (3.6)$$

st.

$$\sum_{j} T_{ij}^{m} = O_i^{m} \qquad (3.7)$$

$$\sum_{i} T_{ij}^{m} = D_j^{m} \qquad (3.8)$$

$$\sum_{i}\sum_{j} T_{ij}^{m} c_{ij}^{m} = C^{m} \qquad (3.9)$$

where O^m_i and D^m_j are total supply and demand in zone i in sector m. C^m represents total travel cost in sector m. If the above non−linear programming model is solved, the variable T^m_{ij} is estimated.

3.1.2 Estimation of Net Indirect Multipliers

With the trade flow matrix (T^m_{ij}) estimated via the entropy maximization model, the regional input−output model can be converted into the multizonal input−output model which will be used as a basic component for the IUM.

$$X = (I - TA)^{-1} * TF \qquad (3.10)$$

The above multizonal input−output model can generates three types of outputs depending on the types of multipliers as follow:

$$X_d = I * FD \qquad (3.11)$$

$$X_o = (I - T_oA_o)^{-1} * FD_O \qquad (3.12)$$

$$X_c = (I - T_cA_c)^{-1} * FD_C \qquad (3.13)$$

where I=identity matrix; FD=final demand; A_o=technical coefficients of open I−O model; A_c=technical coefficient of closed I−O model; X_d=direct outputs; X_o=direct plus indirect outputs; X_c=total outputs which include direct, indirect, and induced output; and T_o and T_c are trade flow matrices for the open and closed models.

Given Equations 3.11, 3.12, and 3.13, net indirect multipliers can be computed by subtracting one equation from another as follow:

$$X_{id} = [(I - T_oA_o)^{-1} - I] * FD_O \qquad (3.14)$$

and X_{id} is net indirect. Net indirect multipliers represent net multiplier effects from intermediate demand.

3.1.3 Estimation of Net Induced Multipliers

Estimating net induced multipliers is somewhat more complicated because induced output must be traced via household expenditure patterns. To address induced household expenditure patterns, two trip matrices are employed: the JHW (Journey from home to work matrix) and the JSH (journey from services to home). The two trip matrices are also obtained via the entropy maximization model used in estimating commodity flows.

Induced consumption expenditures are traced back from the work place to the residential site via the JHW matrix and from the residential place to the shopping place through JSH matrix (Richardson and Gordon, 1989). The following matrix notation succinctly shows the net induced multipliers:

$$X_{iu} = JSH * JHW * [(I - T_c A_c)^{-1} - (I - T_o A_o)^{-1}] * FD_C \qquad (3.15)$$

where X_{iu}=zonal induced output in sector k; and JSH and JHW are journey−to−work and journey−to−shop matrices. Because obtaining zone and sector specific trip flows is hardly possible, it is assumed that all the employees within a zone have the same probability to commute and shop from one to another, regardless of sector. The two trip matrices (JHW and JSH) have dimensions of $(n^*m)^2$.

3.2 The Integrated Urban Model As a Non−linear Programming Model

Because the IUM is an extension of Wilson's model, it is worth explaining Wilson's model more specifically. For convenience, Equation 2.2 in Wilson's unconstrained model can be represented in the following matrix notation, if we assume a two zone and three sector model for simplicity:

$$\begin{vmatrix} X11^1 + X21^1 \\ X11^2 + X21^2 \\ X11^3 + X21^3 \\ X12^1 + X22^1 \\ X12^2 + X22^2 \\ X12^3 + X22^3 \end{vmatrix} = \begin{vmatrix} a11^1 & a12^1 & a13^1 & & & \\ a21^1 & a22^1 & a23^1 & & & \\ a31^1 & a32^1 & a33^1 & & & \\ & & & a11^2 & a12^2 & a13^2 \\ & & & a21^2 & a22^2 & a23^2 \\ & & & a31^2 & a32^2 & a33^2 \end{vmatrix} * \begin{vmatrix} X11^1 & 0 & 0 & X12^1 & 0 & 0 \\ 0 & X11^2 & 0 & 0 & X12^2 & 0 \\ 0 & 0 & X11^3 & 0 & 0 & X12^3 \\ X21^1 & 0 & 0 & X22^1 & 0 & 0 \\ 0 & X21^2 & 0 & 0 & X22^2 & 0 \\ 0 & 0 & X21^3 & 0 & 0 & X22^3 \end{vmatrix} + \begin{vmatrix} Y1^1 \\ Y1^2 \\ Y1^3 \\ Y2^1 \\ Y2^2 \\ Y2^3 \end{vmatrix}$$

(Table 3.2) The Matrix Notation of Wilson's Unconstrained Gravity Input−output Model (Equation 2.2)

The above matrix notation explains the technical relations of production and consumption of the endogenous sectors (X^m_{ij}).

The IUM will be extended from Wilson's model by introducing three important components in urban modeling: 1) endogenizing the household sector (closed model in the input−output model), 2) introduction of trip matrices (journey to work and journey to shop matrices from the Lowry type model), and 3) split total output into intermediate demand, household consumption, and other final demand. Wilson's unconstrained model will be extended into the following equations by incorporating these three components. Because the household sector is endogenized, an explicit distinction will be made between household and non−household sector in superscript m (m=1,2,3,....n,H; where 1 through n for non−household sectors and H for household sector).

$$S = -\sum_{ijm} (X_{ij}{}^{m} \ln X_{ij}{}^{m} + K_{ij}{}^{m} \ln K_{ij}{}^{m}),$$
$$m \text{ for } X \in 1,2,\ldots n,H; \quad m \text{ for } K \in 1,2,\ldots,n \qquad (3.16)$$

st.

$$\Sigma_j X_{ji}{}^m = \Sigma_n (a_{mn}{}^i \; \Sigma_j X_{ij}{}^n) + y_i{}^m \qquad m \in 1,2,\ldots n,H \qquad (3.17)$$

$$\sum_j X_{ij}{}^{m} = \sum_j (IM_{ij}{}^{m} + K_{ij}{}^{m} + Y_{ij}{}^{m}) \qquad m \in 1,2,\ldots n \qquad (3.18)$$

$$\sum_i X_{ij}{}^{m} = \sum_m \sum_j K_{ij}{}^{m}$$
$$m \text{ for } X \in H; \quad m \text{ for } K \in 1,2,\ldots,n \qquad (3.19)$$

$$\sum_{ij} c_{ij}{}^{m} X_{ij}{}^{m} = C^{m} \qquad m \in 1,2,\ldots n,H \qquad (3.20)$$

$$\sum_{ij} d_{ij}{}^{m} K_{ij}{}^{m} = D^{m} \qquad m \in 1,2,\ldots n \qquad (3.21)$$

where X^m_{ij} =the flow of commodity m between zone i and j; a^i_{mn} =input−output coefficients defining the requirements of sector m's output per unit output of sector n in zone i; Y^m_{ij} =the exogenously pre−specified demand for m from zone i to zone j (excluding household consumption); IM^m_{ij} and K^m_{ij} are commodity m consumed for intermediate demand and final demand by zone j, respectively; C^m_{ij} =the average cost of movement of m between zone i and j; C^m =total transportation cost for the commodity m; C^m_{ij} and D^m are the average and total transportation cost for shopping trips respectively.

Equation 3.17 is the Leontief−Strout equation representing zonal supply and demand for the industry m. Unlike Wilson's model Equation 3.17 requires little more information on the pre−determined final demand excluding the household sector. Equation 3.18 shows that total output from zone i to zone j in industry m consists of intermediate demand, household consumption, and other final demand.

Equation 3.19 shows that total income (wages and salary) ($\sum_j X^H_{ij}$ where H= household sector) of the resident in zone i should be equal to the total household consumption by the resident in zone i ($\sum_m \sum_j K^m_{ij}$. Equation 3.20 presents transportation costs for commodity flows (m=1,2,...n) and commuting flows (m=H). Equation 3.21 presents transportation costs for shopping trips.

The above non−linear programming model has two variables to be estimated (X^m_{ij} K^m_{ij}). By the definition given in Equation 3.18, IM^m_{ij} is automatically computed, once these two variables are known. These two variables estimated through the above model contain three important components for the IUM: 1) commodity flows (X^m_{ij} , m=1,2,...

n), 2) commuting flows (X^m_{ij}, m=H), and shopping flows (K^m_{ij}).

3.3 The Comparison between Matrix and programming presentation of the Integrated Urban Model

The difference between the two presentations of the model is that one is a simultaneous equation model and the other is a programming model. In other words, the matrix inverse model is a simultaneous equation model with entropy maximization models as submodels (estimating commodity, commuting and shopping flows), while the programming model has an entropy maximizing objective function in estimating three flows with the input-output multiplier embedded within the constraints.

Macgill argues that advantages of the matrix representation are its simple numerical solution and its ability to monitor individual multiplier rounds giving them a pseudo-dynamic interpretation. In contrast, the advantages of mathematical programming representation are its ability to introduce further constraints or its ability to change the objective criterion.

4. Conclusion

The Integrated Urban Model is significant in several ways and improves upon some limitations of other synthetic models of urban economic models and spatial interaction models. The practical advantage of the model is that it can generate spatially and sectorally disaggregated output and employment impacts of a wide variety of urban economic projects, plans, or policies. In addition, it can distinguish indirect repercussions (impacts on supplying industries) from induced repercussions (impacts from secondary consumption). These advantages of the model improve upon the limitations of aggregate urban models which disguise zonal and sectoral variations in overall output and employment impacts.

The theoretical advantage of the model is that the IUM is a simultaneous model which fully takes account of the spatial and sectoral relationships of multiplier effects (net indirect and induced multipliers) by including commodity flows, work trip flows, and shop trip flows simultaneously. Wilson's and Macgill's models are also regarded as simultaneous models, but their models ignore the spatial and sectoral relationships of net induced multiplier effects. Thus, the IUM resolves the problems of Wilson's, and Macgill's model by dealing with indirect and induced multiplier effects simultaneously.

Note

1. Each cell of trade coefficients means the proportion of commodity m, consumed in zone j and supplied from zone i to total demand in the same industry. Technically, the value of each cell is computed from trade flow matrix by dividing each cell by column-sum.

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