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## Modeling of an isolated intersection using Petri Network

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### ABSTRACT

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The development of a mathematical modular framework based on Petri Network theory to model a traffic network is the subject of this paper. Traffic intersections are the primitive elements of a transportation network and are characterized as event driven and asynchronous systems. Petri networks have been utilized to model these discrete event systems; further analysis of their structure can reveal information relevant to the concurrency, parallelism, synchronization, and deadlock avoidance issue. The Petri-net based model of a generic traffic junction is presented. These modular networks are effective in synchronizing their components and can be used for modeling purposes of an asynchronous large scale transportation system. The derived model is suitable for simulations on a multiprocessor computer since its program execution safety is secured. The software pseudocode for simulating a transportation network model on a multiprocessor system is presented.

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### 1. Introduction

While transportation traffic networks are event driven and asynchronous, their evolution in time depends on the complex interactions of the timing of the various discrete event, such as the arrival or the departure of cars at the individual traffic junctions and the initiation or completion of a signal timing pattern. Their state history is piece-wise constant and changes only at discrete time instants.

Petri networks(PNs) [Pet81, Rei85] have been utilized to model discrete event systems composed of separate, interacting components [DA85, MCB84, Gar85, BM86, MKMH86, EFRV86]. Each component of an asynchronous transportation network, a traffic junction, has its own state of being. The state of an asynchronous transportation network, a traffic junction, has its own state of being. The state of a component is an abstraction of the necessary information to describe its (future)

actions. The components(traffic junctions) exhibit concurrency and parallelism, since activities associated with one traffic junction may occur simultaneously with various activities(passing through cars) in other junctions. This concurrent nature activity in our system creates many difficult modeling problems. Due to the traffic junction state interactions(cars moving from one junction to another), it is necessary to synchronize the occurring events.

A mathematical representation of the system can be provided through Petri Network theory. The derived PN can be analyzed and reveal information about the structure and the dynamic behavior of the transportation network. Furthermore, this information can be used to evaluate and suggest further changes to improve the system behavior (i.e. avoid traffic congestion, rerouting, signal timing determination, etc).

In this paper, Petri network theory has been applied to provide the PN-model of an isolated traffic junction within a transportation network. The individual junction is characterized as a discrete event system, while the incoming/outgoing cars and the signal timing represent its state. A modular PN model has been constructed, which furthermore facilitates the construction of the modeling of large transportation networks. Its proper construction resolves the parallelism and the deadlock avoidance situation between its interacting components. Moreover, due to its effectiveness to synchronize the actions of its components, the proposed PN-model is well suited for traffic simulations on a

multiprocessor system.

## 2. Mathematical Notation and Preliminaries

In this section, we present the basic background and concepts for the development of Petri Networks that will be used throughout this article. The terminology pertinent to PNs will be introduced in this section for clarity and for self containment purposes of this article. For a more comprehensive and formal introduction to Petri nets we refer the reader to one of the existing books [Pet81, Rei85], or survey papers [Mur89].

### 2.1 Petri Network Terminology

A Petri network [Mur89] is a directed, weighted, bipartite graph consisting of two sets of nodes called transitions and places, where arcs can emanate from a place to a transition or the reverse.

A PN structure comprises of the set of finite places  $P = \{p_1, p_2, \dots, p_n\}$ , the set of transitions  $T = \{t_1, t_2, \dots, t_m\}$ , an input function  $I : T \rightarrow P^*$  a mapping from transitions to bags of places, and an output function  $O : T \rightarrow P^*$  a mapping from transitions to bags of places. The set of places and transitions are disjoint  $P \cap T = \emptyset$ , and the number of occurrences of the place  $p_i$  in the corresponding bag  $I(t_j)$  is denoted by  $\#(p_i, I(t_j))$ . In the PN graphical representation places are drawn as circles and transitions as bars, while the input and output functions are represented by directed arcs from places to transitions and vice-versa, respectively.

A generalized marked Petri network  $M$  is a five-tuple  $C=(P, T, I, O, H)$  with its associated network state(marking)  $\mu$ ,  $M = (C, \mu)$ . The network state(marking)  $\mu$  at the  $k$  th event is a vector assignment of positive integers  $N$  (tokens) to the network places  $\mu(k) = [\mu_1(k), \mu_2(k), \dots, \mu_n(k)] : P \rightarrow N$  represented by drawing black dots(each one representing one token) inside the corresponding places.  $H()$  is the inhibition function, a mapping from transitions to sets of places, indicated graphically by small circle-headed arc directed toward a transition from every place contained in its inhibition bag, and the marking by drawing black dots inside the places each one representing one token.

The dynamic behavior of Petri nets is facilitated through the utilization of the incidence matrix  $D$ (used extensively in marked graph theory), which describes the directed connections of the transitions and the places. For a Petri net with  $n$  places and  $m$  transitions, the incidence matrix  $D^+_{m \times n} - D^-_{m \times n}$ , where the element  $[i, j]$  of the  $D^+(D^-)$  matrix is defined as  $D^+[j, i] = \#(p_i, O(t_j))$  ( $D^-[j, i] = \#(p_i, I(t_j))$ ). Similarly the inhibition incidence matrix  $D_{m \times n}$  is related to the inhibition function  $H$  with its  $[i, j]$  element defined as 1(0) if there is one (zero) inhibition arc from the  $j$ th places to the  $i$ th transition.

The execution(time evolution) of a PN is controlled by its current token marking. A Petri net executes by firing transitions. A transition fires by removing tokens from its input places and creating new tokens which are distributed to its output places.

A transition can fire when is enabled, or

when : 1) each of its input places has at least as many tokens in it as arcs from the place to the transition, and 2) zero tokens are in all of its inhibitor input places.

Using the incidence matrices, a transition  $t_j$  is enabled if the following conditions are satisfied

where  $e[j]$  is the unit  $m$ -vector which is zero everywhere except in its  $j$ th component, and  $( )$ .

The result of firing transition  $t_j$  is a new marking  $\mu(k+1)$ , where

### 3. Traffic Intersection Petri Network Model

In this section, we present a live bounded Petri Network model of an isolated traffic intersection. To further illustrate and clarify the major components in the structure of this PN, we will initially focus on an "elementary" traffic junction, where no left turns are allowed. The basic Petri network modules will be analyzed and the overall PN operation will be elucidated. In the final part of this section, we will manifest the complete model of a junction with left turns allowances.

#### 3.1 Petri network model of a traffic junction with no left turn allowance

The schematic of a traffic junction in which

only straight through and right turns are allowed is depicted in Figure 3.1. The signal cycle is divided into two periods of "effective red green" for each phase, the first

corresponding to the halted traffic and the second to the traffic having the right of way. In this elementary problem formulation we inherently made the assumptions that:

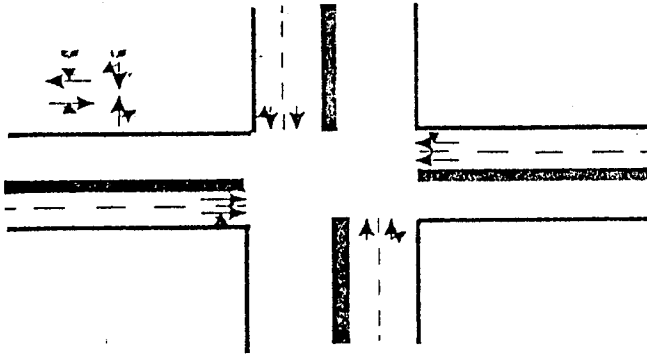


Figure 3.1 Traffic Junction with no Left Turn Allowance.

- there is no cycle devoted to the right of way to pedestrians,
- only one lane exists in each direction,
- the incoming and outgoing traffic bounds have infinite capacity,
- the time allocated for every incoming automobile to cross this intersection is identical, and
- no devoted lane exists for right turns.

According to these presumptions during a signal timing period frame, the automobiles can be advanced through the intersection in two distinctive traffic patterns. To exemplify these patterns, when the right of way is allocated to the North-South (N-S) traffic bounds and the East-West (E-W) traffic bounds are halted, the possible patterns (routes) that the automobiles are permitted to sustain are for the N-S(S-N) bound either N-to-S (S-to-N) traffic, or the N-to-W (S-to-E) traffic.

The proposed PN model of the simplified traffic junction appears in Figure 3.2. The number of incoming automobiles are represented as tokens which are deposited in the "incoming" places (IPs)  $p_0$ ,  $p_1$ ,  $p_2$ , and  $p_3$  for the N, W, S and E bounds respectively. The outgoing automobiles are similarly represented by the tokens of the outgoing places (OPs)  $p_{24}$ ,  $p_{25}$ ,  $p_{26}$ , and  $p_{27}$  for the N, W, S, and E outgoing bounds.

The signal timing is adjusted according to the distinct tokens deposited in the four "controller places" (CPs)  $p_{20}$ ,  $p_{21}$ ,  $p_{22}$ , and  $p_{23}$ . At every time instant only two tokens reside in the CPs. A token placed in  $p_{20}$ , and  $p_{22}$  ( $p_{22}$  and  $p_{23}$ ) indicates the right of way for the N-S (E-W) bound. At the beginning of the timing cycle places  $p_{20}$ , and  $p_{22}$  contain one "bound" token. The existence of the token indicates the right of way to the N-S bound. After the allocated time interval for the N-S

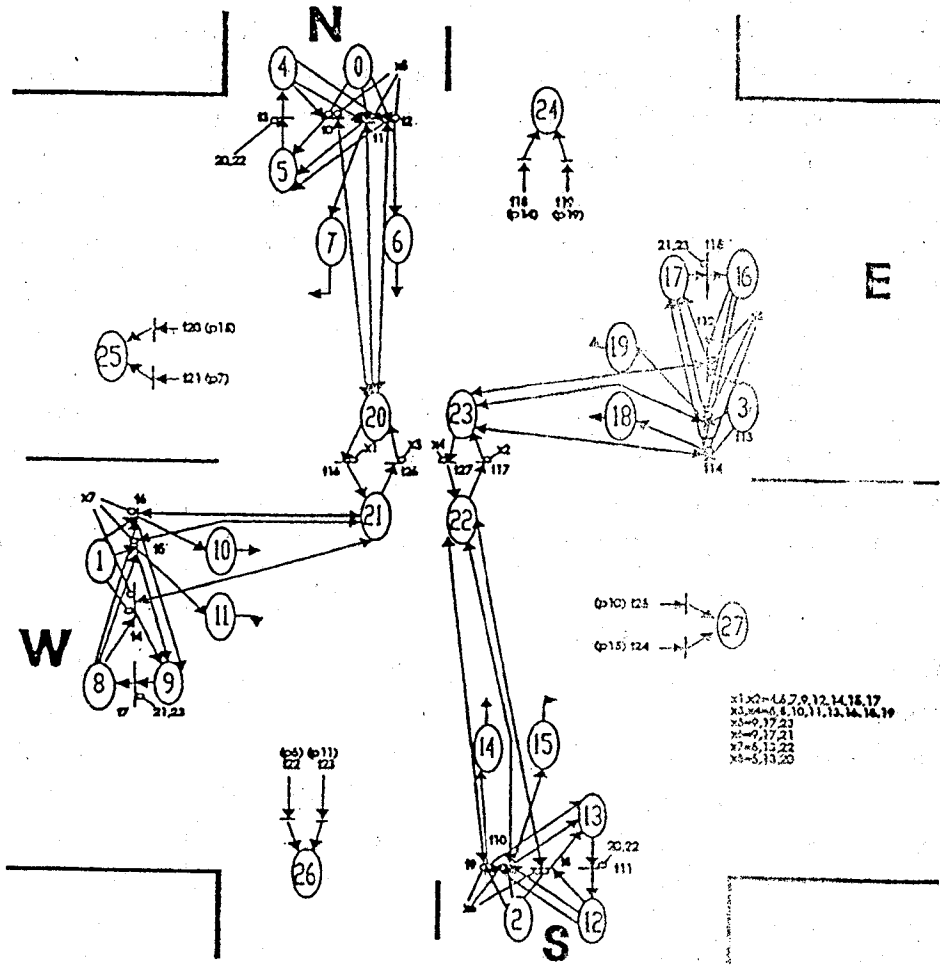


Figure 3.2 : Elementary Traffic Junction Petri Network Model

bound, specified by the number of tokens<sup>1</sup> in the "traffic duration light places" (LPs)  $p_4$  and  $p_{12}$ , transitions  $t_{16}$  and  $t_{17}$  fire and the two "bound" tokens move into places  $p_{21}$  and  $p_{23}$ . In effect, this gives the right of way to the E-W bound and disables the N-S bound. imilarly, the bound tokens will remain in  $p_{21}$  and  $p_{23}$  throughout the duration of the E-W bound specified by the number of tokens in  $p_8$  and  $p_{16}$ . At the end of the cycle the bound and the traffic

duration tokens are deposited into their original places resulting thus in a cyclic marking

$$(\mu_{20}, \mu_{21}, \mu_{22}, \mu_{23}) = (1, 0, 1, 0) \rightarrow (0, 1, 0, 1) \rightarrow (1, 0, 1, 0) \rightarrow \dots$$

The remaining places in Figure 3.2 correspond to buffer places, in which tokens are temporarily deposited prior to reaching at their designated places.

<sup>1</sup>  $\mu_4 = \mu_{12}$  and  $\mu_8 = \mu_{16}$  at the beginning of cycle.

The network's principal module for each bound corresponds to a subnetwork of five places. For the  $N$  traffic bound places  $p_0, p_4, p_5, p_6$  and  $p_7$  constitute this cardinal set of places<sup>2</sup>. Transitions  $t_1$  and  $t_2$  can fire only when tokens exist in places : 1)  $p_{20}$  and  $p_{22}$ , 2)  $p_0$ , 3)  $p_4$ , and 4) zero tokens reside in  $p_{13}, p_{17}$  and  $p_2$ . 3. The satisfaction of these terms equivalently implies that : 1) the right of way has been allocated to the N-S bound (from the  $p_{20}, p_{22}$  "bound" tokens), 2) there is at least one automobile awaiting to cross the intersection from the N-bound (being accounted as a token at place  $p_0$ ), 3) sufficient time remains for the automobile to cross this intersection (expressed by the existence of tokens in  $p_4$ , and 4) the buffer place  $p_5$  (for the E(W) bound is free of token (network bookkeeping) which is necessary for the well-posed PN behavior. The firing of transition  $t_1$  ( $t_2$ ) indicates that this automobile will migrate to the east (south) section. Stochastical weights can be assigned to the arcs coupling place  $p_0$  with these two transitions, reflecting the distribution of automobiles that will either advance straight through the intersection or proceed with a right turn. The firing of transition  $t_0$  exhibits the absence of automobiles from the N-bound to cross this intersection, while the N-S bound has the right of way. The traffic light tokens are deposited to the buffer place  $p_5$  prior to being rerouted through the firing of  $t_3$  to their initial place  $p_4$ .

### 3.2 Petri network model of a generic traffic junction

The Petri network of an isolated traffic intersection with a dedicated lane for each bound for assigned left turns appears in Figure 3.3 : the associated  $D^+$ ,  $D^-$ , and  $D^0$  incidence matrices appear in Appendix I. The network graph is similarly symmetric, with the  $(p_0, p_1), (p_2, p_3), (p_4, p_5)$  and  $(p_6, p_7)$  pairs of places accounting for the incoming places (IPs) for the N, W, S, and E traffic bounds respectively. Contrasting with the four assigned IPs allocated for the "elementary traffic junction", we signify that the first (second) element of each pair corresponds to the incoming vehicle place for a left (straight through and right) turn.

The signal timing for this intersection conforms to the basic four phase pattern, with the controller places (CPs) corresponding to  $p_{11}, p_{16}, p_{20}, p_{25}, p_{29}, p_{37}, p_{38}, p_{43}$ . A typical pattern with the associated CP-marking  $(\mu_{11}, \mu_{16}, \mu_{20}, \mu_{25}, \mu_{29}, \mu_{37}, \mu_{38}, \mu_{43})$  is as follows :

1. the right of way is assigned  $(\mu_{11}, \mu_{16}, \mu_{20}, \mu_{25}, \mu_{29}, \mu_{37}, \mu_{38}, \mu_{43}) = (1, 0, 0, 0, 1, 0, 0, 0)$  to the left turns N-to-E and S-to-W for the N-S bound,
2. the right of way is designated  $(\mu_{11}, \mu_{16}, \mu_{20}, \mu_{25}, \mu_{29}, \mu_{37}, \mu_{38}, \mu_{43}) = (0, 1, 0, 0, 0, 1, 0, 0)$  for straight through N-to-S or S-to-N, and right turns N-to-W or S-to-E for the N-S bound,
3. the right of way is assigned  $(\mu_{11}, \mu_{16}, \mu_{20}, \mu_{25}, \mu_{29}, \mu_{37}, \mu_{38}, \mu_{43}) = (1, 0, 0, 0, 1, 0, 0, 0)$  to the left turns E-to-S and W-to-N for

<sup>2</sup> Similar sets exist for the other bounds as this can easily be verified by the symmetry of the network's graph.

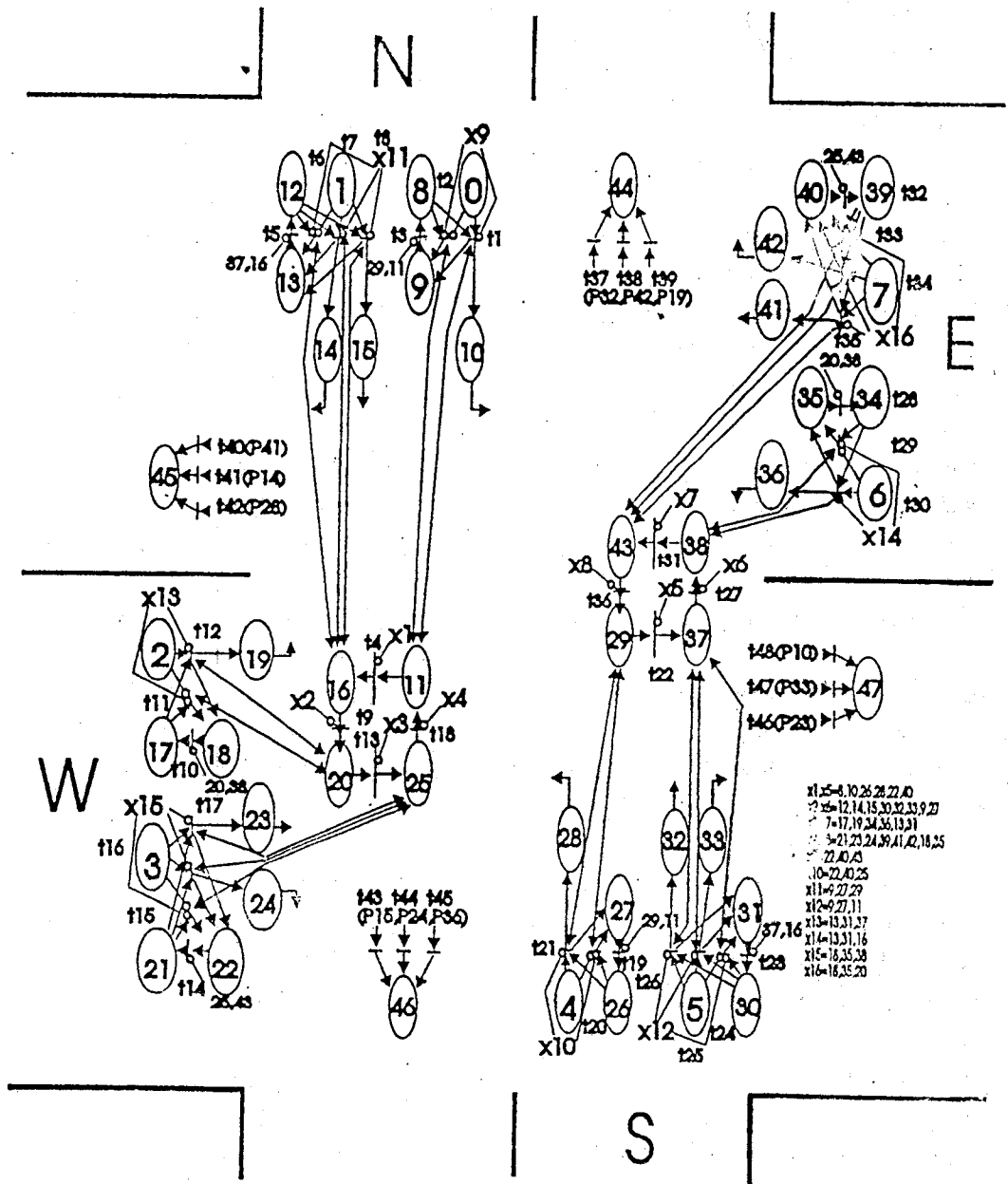


Figure 3.3 : Generic Traffic Junction Petri Network Model

the E-W bound,

4. the right of way is designated  $(\mu_{11}, \mu_{16}, \mu_{20}, \mu_{25}, \mu_{29}, \mu_{37}, \mu_{38}, \mu_{43}) = (0, 1, 0, 0, 0, 1, 0, 0)$  for traight through E-to-W or W-to-E, and right turns E-to-N or W-to-S for the E-W bound.

The cyclic CP-marking attribute will similarly be exploited.

The primary modules for each bound contain two distinct Petri networks. To further exemplify this note, the primary module for the N-bound consists of:

- the network module for the straight through and right turn assignment  $(P_1, T_1) = (\{p_1, p_2, p_{13}, p_{14}, p_{15}\}, \{t_5, t_6, t_7, t_8\})$ , and
- the network module for the left turn assignment  $(P_2, T_2) = (\{p_9, p_8, p_9, p_{10}\}, \{t_1, t_2, t_3\})$ .

The traffic light tokens are deposited for the N-to-S and N-to-W phase into place  $p_{12}$ , and for the left turn procedure into place  $p_8$ . Places  $p_{13}$  and  $p_9$  act as buffers for the traffic light tokens, while places  $p_{14}$ ,  $p_{15}$  and  $p_{10}$  serve as the intermediate buffer places for the outgoing vehicles to the W, S, and E bounds respectively. The network module operation is comparable and similar to the operation of the principal module described in the previous section and will not be further illuminated hereafter. Similar modules exist for the remaining traffic bounds, as this can be verified by the symmetricity of the PN-graph. The essential distinctions of the proposed PN, explicated in Figure 3.3, compared to the ones of the PN for the "elementary" traffic intersection featured in Figure 3.2 are :

- the inclusion of the discrete PN-modules responsible for the left turn operations,
- an increase of the controller places from four to eight combined with an augmentation by four extra transitions  $(t_4, t_{13}, t_2, t_{31})$  to ensure the cyclic PN property,
- additional transitions are now utilized as the input places for the outgoing places.

At this point, we should emphasize that the network complexity, indicated by the total number of places and transitions, has not been reduced and additional buffer places have been inserted to further explicate and corelate the physical interpretation of some places. As an example, consider the network module for the outgoing automobiles towards the S-direction depicted in Figure 3.3. The automobiles from the N, W, and E bounds progress through places  $p_{15}$ ,  $p_{24}$  and  $p_{36}$  to their outgoing place  $p_{46}$  by firing transitions  $t_{43}$ ,  $t_{44}$  and  $t_{45}$  respectively. There is a redundancy in utilizing these three supplementary places and transitions, since the inputs to these places can be linked directly with the outgoing place resulting thus in  $I(p_{46}) = \{t_8, t_{16}, t_{30}\}$ . However, these redundant places  $p_{15}$ ,  $p_{24}$  and  $p_{36}$ , can be utilized to enumerate the number of automobiles attempting a straight through, right turn, or left turn respectively. Valuable information can be extracted by interpreting these numbers, from a statistical point of view, since these are correlated to the automobile queue formations at the traffic intersection. We, therefore decided to maintain these extra places and transitions due to their : 1) physical interpretation, and 2) substantial correlation with the automobile queue formations in the traffic intersection.



### 4. Multiprocessor Software Pseudocode Emulation for Modeling of Transportation Networks.

The Advanced Traffic Management System (ATMS) functional component of the IVHS framework [IVH92] encompasses several innovative technologies for the purposes of monitoring and controlling the traffic vehicle flow [AJA85b] on a real-time basis. Real-time control adjustments [AJA85b] demand fast traffic network simulation software packages to test many alternative traffic

routing strategies.

Parallel computers can deliver the potential computational power to support these dynamic real-time decision making requirements. The topology of a transportation network is mapped to the architecture of the parallel computer. The operations of clusters of intersections are simulated by the individual processing elements (PEs) of the computer. Subsequently, the PEs communicate the state of the network to their neighboring PEs and update the control strategy. The overall scheme is depicted graphically in Figure 4.1.

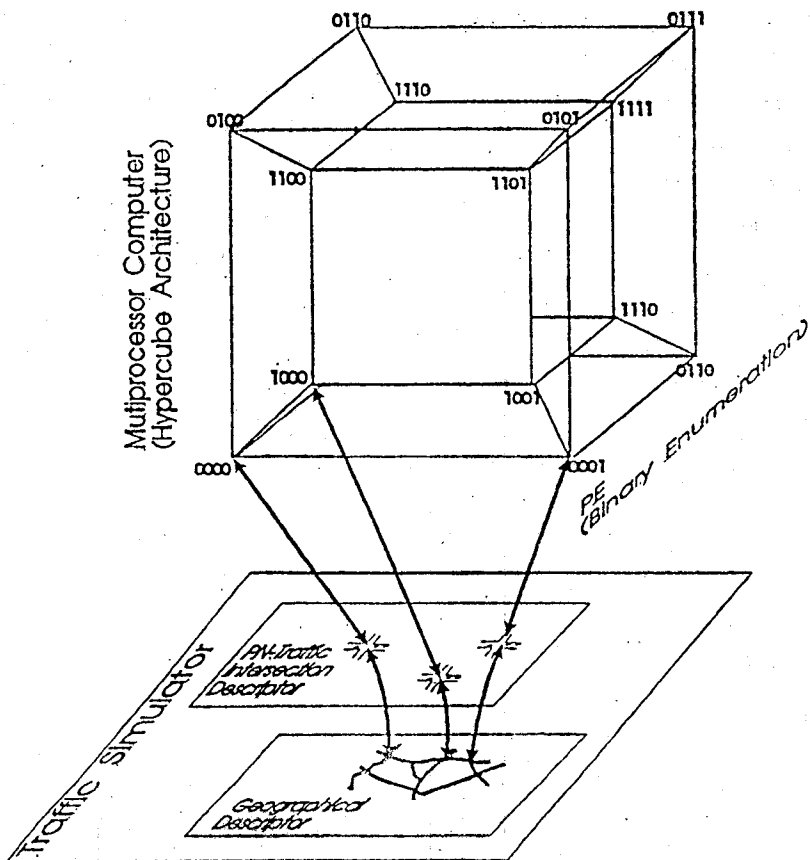


Figure 4.1 Transportation Network simulation on a Parallel Computer

The performance of the parallel compute deteriorates from the communication overhead to synchronize the system entities. This overhead depends on the synchronization requirements between the individual PEs. Maintaining simpler synchronization requirements will thus result in a faster program execution time.

1. the traffic junctions characterized by their spatial coordinates and their traffic routing attributes(left turn permissibility, protected turns, etc),
2. the traffic links typically characterized by their number of lanes, and
3. the moving automobiles within the network.

The modeling process of the interactions between the aforementioned components requires an extensive synchronization of their operations. Typically, an external clock is utilized as the base reference time unit, and all operations are defined with respect to an absolute time basis. The current transportation simulation packages [KA92, YA90] update their state on fixed time intervals. The state is comprised of the location and position of the automobiles in the network. The propagation of the automobiles through the network links and their response upon arriving at an intersection is determined in a sequential manner. The heavy computational load to determine the next state of the system is primarily due to the complexity of the operations within the intersections.

According to the proposed PN-model, the only places that need to be monitored are the input( $p_i$ ,  $i=0, \dots, 7$ ) and output( $p_j$ ,  $j=4, \dots, 47$ ) places of each intersection. The marking of these places indicates the incoming and

outgoing automobiles. The traffic operations within one intersection can be achieved in an asynchronous manner. There is no need for an external clock to synchronize these operations. Instead, the availability of the tokens in the IPs is what determines the sequence of operation in the intersection.

The PN-model imitates that of an asynchronous dataflow computer [Vee86], in which the availability of the operands (tokens) determines the sequence of the operations. The vast speed of dataflow computers [DL90] is due to the elimination of the synchronization requirements between their structural components. A cluster of intersections will be allocated to each PE which will simulate the behavior of the corresponding PNs(equation (2.3)). Then, only the OP markings will be transmitted to its neighboring PEs to coordinate the entire network operation.

The simulation pseudocode for a typical multiprocessor Single Instruction-Multiple Data stream (SIMD) computer is presented in the following flowchart. SIMD computers are similar to array processors in which the PEs perform the same function synchronously over a distributed data stream.

### SIMD Transportation Network Simulation Pseudocode

1. SIMD Initializaion : Perform the transportation network to the SIMD computer mapping by assigning clusters of traffic intersections to individual Processing Elements. Let the  $i$ th PE,  $i=0, \dots, N-1$  be responsible for the traffic operations of the

following intersection cluster  $[i \cdot L, \dots, (i+1) \cdot L]$ .

2. **Pe Initialization** : Download the  $D^+$ ,  $D^-$ , and  $D^o$  matrices to the appropriate PE for the corresponding traffic intersections. This can be achieved by broadcasting all possible PN combinations (i.e. through only, through with right turn permissibility, through with left turn permissibility, etc.) to all PEs. Subsequently, each PE through local processing will select the appropriate  $D$ -matrices to simulate its intersection clusters.
3. **Local PN marking initialization** : Broadcast the CP token-marking for all possible intersection structures. Every PE will initialize its own Petri Networks.
4. **PE - Relative Signal Timing Initialization** : Download sequentially to each PE its LPs. Some of these LPs will be placed into the temporary places. to reflect the relative offset [AJA85a] between the intersection signal timings. Note that this procedure is the most critical one, since improper initialization of the signal timings will result in an erroneous network signal timing representation.
5. **Concurrent Local PE Computations** : The preconditions of PN-markings with respect to the incidence matrices are evaluated from equations (2.1) and (2.2) The time evolution of the PNs is reflected by the concurrent execution from all PEs of equation (2.3). Every PE will indefinitely repeat this step to simulate the dynamic behavior of the transportation network.  
Throughout each iteration : 1) the number of automobiles that are expected to cross

the intersection are represented by the number of tokens in the IPs, and 2) the automobiles that have already traversed through the junction correspond to the OP tokens.

6. **Network State Update** : Each PE broadcasts to its neighboring PEs its OP-marking. This neighbor-to-neighbor communication reduces the communication overhead by avoiding global broadcasting.
7. **Network Link Update Status** : Each PE Simulate the automobile's travel time throughout the link connecting the PE's IPS with the neighboring OPs through a simple delay algorithm.
8. **Transportation Network Bookkeeping** : If needed, the PEs broadcast their IP and OP markings in order to collect statistical data relevant to the network operation (delay, capacity, occupancy, etc.)
9. Proceed to Step 5.

## 5. Conclusion

In this paper the simulation model of transportation network based on Petri network theory was presented. Petri net theory is an effective mathematical tool for analyzing the dynamic behavior of asynchronous discrete event systems.

The traffic network is considered to be comprised of traffic intersections, which are modular event driven systems. The Petri-net based model of a generic traffic intersection was presented. The derived model is suitable for simulations on a multiprocessor computer system, since by being deadlock free and

consistent it provides software safety.

Furthermore, these modular intersection simulation models are effective in synchronizing their components and can be used for modeling

purposes of asynchronous large scale transportation systems. The software pseudocode for simulating a transportation network model on a multiprocessor system was presented.

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