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FUZZY PRE-IRRESOLUTE MAPPINGS

JIN HAN PARK AND BAE HUN PARK

1. Introduction and preliminaries

Weaker forms of fuzzy continuity have been considered by many authors [1, 2, 6, 8, 9] using the concepts fuzzy semiopen sets [1], fuzzy regularly open sets [1] and fuzzy preopen sets [2]. J. H. Park et al. [11] showed that fuzzy precontinuity and fuzzy almost continuity due to Mukherjee and Sinha [8] is equivalent concepts.

In Section 2 of this paper we define and study fuzzy pre-irresolute mapping which is stronger than fuzzy precontinuous, and show that the concepts of fuzzy continuous and fuzzy pre-irresolute mappings are independent. In Section 3, we introduce and study concepts of fuzzy pre-separation axioms of fuzzy topological spaces.

Throughout this paper, by (X, τ) (or simply X) we mean a fuzzy topological space in Chang's [3] sense. A fuzzy point in X with support $x \in X$ and value α $(0 < \alpha \leq 1)$ is denoted by x_α. For a fuzzy set A in X, $\mathrm{Cl}A$, $\mathrm{Int}A$, $1 - A$ and $(A)_0$ will respectively denote the closure, interior, complement and support of A, whereas the constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X, respectively. A fuzzy set A of X is said to be q-coincident with a fuzzy set B, denoted by $A\mathrm{q}B$, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [7]. It is known [7] that $A \leq B$ if and only if A and $1 - B$ are not q-coincident, denoted by $A\bar{\mathrm{q}}(1 - B)$. For definitions and results not explained in this paper, the reader is referred to [1, 2, 7] in the assumption they are well known. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd' and 'fts', respectively.

DEFINITION 1.1 [1,2]. A fuzzy set A in X is said to be

(a) fuzzy semiopen (fuzzy semiclosed) if $A \leq \mathrm{Cl}\mathrm{Int}A$ (resp. $\mathrm{Int}\mathrm{Cl}A \leq A$),

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(b) fuzzy preopen (fuzzy preclosed) if $A \leq \text{IntCl}A$ (resp. $A \geq \text{ClInt}A$).

THEOREM 1.1 [2]. (a) *An arbitrary union of fuzzy preopen sets is a fuzzy preopen set,*

(b) *any intersection of fuzzy preclosed sets is a fuzzy preclosed set.*

THEOREM 1.2 [2]. *Let X and Y be fts's such that X is product related to Y. Then the product $U \times V$ of a fuzzy preopen set U in X and a fuzzy preopen set V in Y is a fuzzy preopen set in the fuzzy product spaces $X \times Y$.*

DEFINITION 1.2 [11]. A fuzzy set A in a fts X is said to be fuzzy pre-q-nbd (fuzzy pre-nbd) of fuzzy point x_α if there exists a fuzzy preopen set B such that $x_\alpha \text{q} B \leq A$ (resp. $x_\alpha \in B \leq A$).

THEOREM 1.3 [11]. *A fuzzy set A is a fuzzy preopen if and only if for each fuzzy point $x_\alpha qA$, A is a fuzzy pre-q-nbd of x_α.*

DEFINITION 1.3 [2]. Let A be any fuzzy set of a fts X. Then fuzzy pre-closure (pCl) and pre-interior (pInt) of A are defined as follows:

$$pClA = \bigwedge\{B \mid B \text{ is fuzzy preclosed and } A \leq B\},$$
$$pIntA = \bigvee\{B \mid B \text{ is fuzzy preopen and } B \leq A\}.$$

THEOREM 1.4 [11]. *Let A be a fuzzy set in X and x_α be a fuzzy point in X. Then $x_\alpha \in pClA$ if and only if for each fuzzy pre-q-nbd U of x_α, UqA.*

THEOREM 1.5. *Let A be a fuzzy set in a fts X. Then A is fuzzy semiopen set if and only if $pClA = ClIntA$.*

Proof. Let A be a fuzzy semiopen set in X. Then $\text{pCl}A$ is fuzzy preclosed and so $\text{ClInt}A \leq \text{ClIntpCl}A \leq \text{pCl}A$. Since A is fuzzy semiopen set, $\text{pCl}A \leq \text{pClClInt}A = \text{ClInt}A$. Hence $\text{pCl}A = \text{ClInt}A$.

Conversely, let A be a fuzzy set with $\text{pCl}A = \text{ClInt}A$. Then $A \leq \text{pCl}A = \text{ClInt}A$ and hence A is fuzzy semiopen.

2. Fuzzy pre-irresolute mappings

DEFINITION 2.1 [2]. A mapping $f : X \to Y$ is said to be fuzzy precontinuous if $f^{-1}(V)$ is a fuzzy preopen set for each fuzzy open set V in Y.

THEOREM 2.1. *For a mapping $f : X \to Y$ the following are equivalent:*

(a) *f is fuzzy precontinuous.*

(b) *$ClIntf^{-1}(B) \le f^{-1}(ClB)$ for each fuzzy set B in Y.*

(c) *$f(ClIntA) \le Clf(A)$ for each fuzzy set A in X.*

Proof. (a)⇒(b): Let B be a fuzzy set in Y. Then by Theorem 3.7 of [11], $f^{-1}(\mathrm{Cl}B)$ is fuzzy preclosed set in X. Since $\mathrm{ClInt}A \le A$ for each fuzzy preclosed set A in X, $\mathrm{ClInt}f^{-1}(B) \le \mathrm{ClInt}f^{-1}(\mathrm{Cl}B) \le f^{-1}(\mathrm{Cl}B)$.

(b)⇒(c): Straightforward.

(c)⇒(a): Let V be a fuzzy closed set in Y. By hypothesis, we have

$$f(\mathrm{ClInt}f^{-1}(V)) \le \mathrm{Cl}f(f^{-1}(V)) \le \mathrm{Cl}V = V,$$
$$\mathrm{ClInt}f^{-1}(V) \le f^{-1}(f(\mathrm{ClInt}f^{-1}(V))) \le f^{-1}(V).$$

Then $f^{-1}(V)$ is a fuzzy preclosed set and hence by Theorem 3.7 of [11], f is fuzzy precontinuous.

DEFINITION 2.2. A mapping $f : X \to Y$ is said to be fuzzy pre-irresolute if $f^{-1}(V)$ is a fuzzy preopen set in X for each fuzzy preopen set V in Y.

Clearly a fuzzy pre-irresolute mapping is fuzzy precontinuous, but the converse is not true by the following example.

EXAMPLE 2.1. Let U_1, U_2, U_3 and U_4 be fuzzy sets in unit interval I defined as follows:

$$U_1(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 2x - 1 & \frac{1}{2} \le x \le 1; \end{cases} \qquad U_3(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{4} \\ \frac{1}{3}(4x - 1) & \frac{1}{4} \le x \le 1; \end{cases}$$

$$U_2(x) = \begin{cases} 1 & 0 \le x \le \frac{1}{4} \\ -4x+2 & \frac{1}{4} \le x \le \frac{1}{2} \\ 0 & \frac{1}{2} \le x \le 1; \end{cases} \quad U_4(x) = \begin{cases} 8x & 0 \le x \le \frac{1}{8} \\ -\frac{1}{3}(8x-4) & \frac{1}{8} \le x \le \frac{1}{2} \\ 0 & \frac{1}{2} \le x \le 1. \end{cases}$$

Consider fuzzy topologies $\tau_1 = \{0_I, U_3, 1_I\}$ and $\tau_2 = \{0_I, U_1, U_2, U_1 \vee U_2, 1_I\}$. Define $f : (I, \tau_1) \to (I, \tau_2)$ by $f(x) = \frac{1}{2}x$ for each $x \in I$. Then f is a fuzzy precontinuous but not fuzzy pre-irresolute.

THEOREM 2.2. *For a mapping $f : X \to Y$ the following are equivalent:*

(a) *f is fuzzy pre-irresolute.*

(b) *$f^{-1}(B)$ is fuzzy preclosed in X for each fuzzy preclosed set B in Y.*

(c) *$pClf^{-1}(B) \le f^{-1}(pClB)$ for each fuzzy set B in Y.*

(d) *$f(pClA) \le pClf(A)$ for each fuzzy set A in X.*

(e) *$f^{-1}(pIntB) \le pIntf^{-1}(B)$ for each fuzzy set B in Y.*

Proof. (a)⇔(b): Clear.

(b)⇒(c): Let B be a fuzzy set in Y. By (b), $f^{-1}(\mathrm{pCl}B)$ is fuzzy preclosed and so $\mathrm{pCl}f^{-1}(B) \le f^{-1}(\mathrm{pCl}B)$.

(c)⇒(d) and (d)⇒(c) can be easily seen.

(c)⇒(e): Let B be any fuzzy set in Y. By (c), we have

$$1 - \mathrm{pInt}f^{-1}(B) = \mathrm{pCl}f^{-1}(1-B) \le f^{-1}(\mathrm{pCl}(1-B)) = 1 - f^{-1}(\mathrm{pInt}B).$$

Thus $f^{-1}(\mathrm{pInt}B) \le \mathrm{pInt}f^{-1}(B)$.

(e)⇒(a): Let B be any fuzzy preopen set in Y. Then $B = \mathrm{pInt}B$. By (e), we have $f^{-1}(B) = f^{-1}(\mathrm{pInt}B) \le \mathrm{pInt}f^{-1}(B)$. Then $f^{-1}(B)$ is a fuzzy preopen set and hence f is fuzzy pre-irresolute.

THEOREM 2.3. *A mapping $f : X \to Y$ is fuzzy pre-irresolute if and only if for each fuzzy point x_α in X and each fuzzy pre-nbd V of $f(x_\alpha)$, there exists a fuzzy pre-nbd U of x_α such that $f(U) \le V$.*

Proof. The proof is easy and hence omitted.

THEOREM 2.4. *A mapping $f : X \to Y$ is fuzzy pre-irresolute if and only if for each fuzzy point x_α in X and each fuzzy preopen pre-q-nbd*

V of $f(x_\alpha)$, there exists a fuzzy preopen pre-q-nbd U of x_α such that $f(U) \le V$.

Proof. Let x_α be a fuzzy point in X and V be a fuzzy preopen pre-q-nbd of $f(x_\alpha) = f(x)_\alpha$. Since $V(f(x)) + \alpha > 1$, there exists a positive real number β such that $V(f(x)) > \beta > 1 - \alpha$, so that V is a fuzzy preopen pre-nbd of $f(x)_\beta$. By Theorem 2.3, there exists a fuzzy preopen set U containing x_β such that $f(U) \le V$. Now, $U(x) \ge \beta$ implies $U(x) > 1 - \alpha$ and thus U is a fuzzy preopen pre-q-nbd of x_α.

Conversely, let V be a fuzzy preopen set and $x_\alpha \in f^{-1}(V)$. Let m be a positive integer such that $1/m \le f^{-1}(V)(x)$. For any positive integer $n \ge m$, we put $\alpha_n = 1 + 1/n - f^{-1}(V)(x)$. Then $0 < \alpha_n \le 1$ for all $n \ge m$. Now, we have

$$\begin{aligned} V(f(x)) + \alpha_n &= V(f(x)) + 1 + \frac{1}{n} - f^{-1}(V)(x) \\ &= 1 + \frac{1}{n} > 1 \end{aligned}$$

Thus V is a fuzzy preopen pre-q-nbd of $f(x)_{\alpha_n}$ for all $n \ge m$. By hypothesis, there exists a fuzzy preopen set U_n in X such that $x_{\alpha_n} \mathrm{q} U_n$ and $f(U_n) \le V$ for all $n \ge m$. We put $U = \bigvee_{n \ge m} U_n$. Then by Theorem 1.1, U is a fuzzy preopen set in X such that $f(U) = \bigvee_{n \ge m} f(U_n) \le V$. Next we will show that $x_\alpha \in U$. Since $U_n(x) + \alpha_n > 1$ for all $n \ge m$, we have $U(x) > f^{-1}(V)(x) - 1/n$ for all $n \ge m$ which implies $U(x) \ge f^{-1}(V)(x) \ge \alpha$. Thus $x_\alpha \in U$.

THEOREM 2.5. *Let $f : X \to Y$ one-to-one and onto. f is fuzzy pre-irresolute if and only if $\mathrm{pInt} f(A) \le f(\mathrm{pInt} A)$ for each fuzzy set A in X.*

Proof. Let A be any fuzzy set in X. Then clearly $f^{-1}(\mathrm{pInt} f(A))$ is a fuzzy preopen set. By Theorem 2.2, we have

$$\begin{gathered} f^{-1}(\mathrm{pInt} f(A)) \le \mathrm{pInt} f^{-1}(f(A)) = \mathrm{pInt} A, \\ f(f^{-1}(\mathrm{pInt} f(A))) \le f(\mathrm{pInt} A). \end{gathered}$$

Since f is onto, $\mathrm{pInt} f(A) = f(f^{-1}(\mathrm{pInt} f(A))) \le f(\mathrm{pInt} A)$.

Conversely, let B be any fuzzy preopen set in Y. Then $B = \mathrm{pInt} B$. By hypothesis, $f(\mathrm{pInt} f^{-1}(B)) \ge \mathrm{pInt} f(f^{-1}(B)) = \mathrm{pInt} B = B$. This implies that $f^{-1}(f(\mathrm{pInt}f^{-1}(B))) \geq f^{-1}(B)$. Since f is one-to-one, $\mathrm{pInt}f^{-1}(B) \geq f^{-1}(B)$. Hence $f^{-1}(B) = \mathrm{pInt}f^{-1}(B)$.

THEOREM 2.6. *Let X_1, X_2, Y_1 and Y_2 be fts's such that X_1 is product related to X_2, and $f_1 : X_1 \to Y_1$, $f_2 : X_2 \to Y_2$ be mappings. If f_1 and f_2 are fuzzy pre-irresolute, then so is $f_1 \times f_2$.*

Proof. Let $V = \bigvee_{i,j}(G_i \times H_j)$, where G_i's and H_j's are fuzzy preopen sets in Y_1 and Y_2 respectively, be a fuzzy preopen set in $Y_1 \times Y_2$. Using Lemmas 2.1 and 2.3 of [1], we have

$$(f_1 \times f_2)^{-1}(V) = \bigvee_{i,j}(f_1 \times f_2)^{-1}(G_i \times H_j) = \bigvee_{i,j}[f_1^{-1}(G_i) \times f_2^{-1}(H_j)].$$

Since f_1 and f_2 are fuzzy pre-irresolute, $f_1^{-1}(G_i)$ and $f_2^{-1}(H_j)$ are fuzzy preopen sets, and because of Theorems 1.1 and 1.2, it follows that $(f_1 \times f_2)^{-1}(V)$ is a fuzzy preopen set, which implies that $f_1 \times f_2$ is fuzzy pre-irresolute.

THEOREM 2.7. *Let $f : X \to Y$ be a mapping and $g : X \to X \times Y$ be the graph of f. If g is fuzzy pre-irresolute, then f is fuzzy pre-irresolute.*

Proof. It follows from Lemma 2.4 of [1].

THEOREM 2.8. *Let $f : X \to Y$ and $g : Y \to Z$ be mappings.*

(a) *If f and g are fuzzy pre-irresolute, then $g \circ f$ is fuzzy pre-irresolute.*

(b) *If f is fuzzy pre-irresolute and g is fuzzy precontinuous, then $g \circ f$ is fuzzy precontinuous.*

Proof. Straightforward.

The following Example 2.2 shows that fuzzy continuous and fuzzy pre-irresolute mappings are independent.

EXAMPLE 2.2. Let U_1, U_2 and U_3 be fuzzy sets in $X = \{a, b, c\}$ defined as follows:

$$U_1(a) = 0.4, \quad U_1(b) = 0, \quad U_1(c) = 0;$$
$$U_2(a) = 0, \quad U_2(b) = 0.4, \quad U_2(c) = 0;$$
$$U_3(a) = 0.4, \quad U_3(b) = 0.4, \quad U_3(c) = 0.$$

Consider the fuzzy topologies $\tau_1 = \{0_X, 1_X, U_1, U_2, U_1 \vee U_2\}$ and $\tau_2 = \{0_X, 1_X, U_1, U_3\}$.

(a) If a mapping $f : (X, \tau_2) \to (X, \tau_1)$ defined by $f(a) = b, f(b) = a, f(c) = c$, then f is fuzzy continuous but not fuzzy pre-irresolute.

(b) If a mapping $f : (X, \tau_1) \to (X, \tau_2)$ defined by $f(a) = b, f(b) = a, f(c) = c$, then f is fuzzy pre-irresolute but not fuzzy continuous.

THEOREM 2.9. *If $f : X \to Y$ is fuzzy precontinuous and fuzzy open, then f is fuzzy pre-irresolute.*

Proof. It follows from Theorem 4.3 of [12].

3. Separation axioms

DEFINITION 3.1. A fts X is said to be fuzzy pre-T_0 if for every distinct two fuzzy points x_α and y_β, the following conditions are satisfied:

(a) When $x \neq y$, either x_α has a fuzzy pre-nbd which is not q-coincident with y_β, or y_β has a fuzzy pre-nbd which is not q-coincident with x_α.

(b) When $x = y$ and $\alpha < \beta$ (say), there is a fuzzy pre-q-nbd of y_β which is not q-coincident with x_α.

DEFINITION 3.2. A fts X is said to be fuzzy pre-T_1 if for every distinct two fuzzy points x_α and y_β, the following conditions are satisfied:

(a) When $x \neq y$, x_α has a fuzzy pre-nbd U and y_β has a fuzzy pre-nbd V such that $x_\alpha \bar{q} V$ and $y_\beta \bar{q} U$.

(b) When $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy pre-q-nbd V of y_β such that $x_\alpha \bar{q} V$.

DEFINITION 3.3. A fts X is said to be fuzzy pre-T_2 if for every distinct two fuzzy points x_α and y_β, the following conditions are satisfied:

(a) When $x \neq y$, x_α and y_β have fuzzy pre-nbds which are not q-coincident.

(b) When $x = y$ and $\alpha < \beta$ (say), then x_α has a fuzzy pre-nbd U and y_β has a fuzzy pre-q-nbd V such that $U \bar{q} V$.

Obviously, fuzzy pre-T_2 $\Rightarrow$ fuzzy pre-T_1 $\Rightarrow$ fuzzy pre-T_0. Also, fuzzy T_i axiom [6]$\Rightarrow$ fuzzy pre-T_i axiom, for $i = 0, 1, 2$.

THEOREM 3.1. *A fts X is fuzzy pre-T_0 if and only if for every pair of distinct x_α and y_β, either $x_\alpha \notin pCl(y_\beta)$ or $y_\beta \notin pCl(x_\alpha)$.*

Proof. The proof is easy and hence omitted.

THEOREM 3.2. *A fts X is fuzzy pre-T_1 if and only if for every fuzzy point x_α is fuzzy preclosed in X.*

Proof. The proof is easy and hence omitted.

THEOREM 3.3. *A fts X is fuzzy pre-T_2 if and only if for every fuzzy point x_α in X, $x_\alpha = \bigwedge\{pClV \mid V$ is fuzzy pre-nbd of $x_\alpha\}$ and for every $x, y \in X$ with $x \neq y$, there is a fuzzy pre-nbd U of x_1 such that $y \notin (pClU)_0$, where $(pClU)_0$ is support of pClU.*

Proof. Let x_α and y_β be fuzzy points in X such that $y_\beta \notin \{x_\alpha\}$. If $x \neq y$, then there are fuzzy preopen sets U and V containing y_1 and x_α respectively such that $U\bar{q}V$. Then V is a fuzzy pre-nbd of x_α and U is a fuzzy pre-q-nbd of y_β such that $U\bar{q}V$. Hence $y_\beta \notin$ pClV. If $x = y$, then $\alpha < \beta$, and hence there are a fuzzy pre-q-nbd U of y_β and a fuzzy pre-nbd V of x_α such that $U\bar{q}V$. Hence $y_\beta \notin$ pClV.

Finally, for distinct two point x, y of X, since X is fuzzy pre-T_2, there exist fuzzy preopen sets U and V such that $x_1 \in U$, $y_1 \in V$ and $U\bar{q}V$. Since $1 - V$ is fuzzy preclosed set containing U, pCl$U \leq 1 - V$. Hence $y \notin (\text{pCl}U)_0$.

Conversely, let x_α and y_β be distinct fuzzy points in X.

When $x \neq y$, we first suppose that at least one of α and β is less than 1, say $0 < \alpha < 1$. Then there exists a positive real number λ with $0 < \alpha + \lambda < 1$. By hypothesis, there exists a fuzzy pre-nbd U of y_β such that $x_\lambda \notin$ pClU. Then there exists a fuzzy pre-q-nbd V of x_λ such that $V\bar{q}U$. Since $\alpha < 1 - \lambda < V(x)$, V is fuzzy pre-nbd of x_α such that $U\bar{q}V$.

Next if $\alpha = \beta = 1$, by hypothesis there exists a fuzzy pre-nbd U of x_1 such that pCl$U(y) = 0$. Then $V = 1 -$ pClU is a fuzzy pre-nbd of y_1 such that $U\bar{q}V$.

When $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy pre-nbd U of x_α such that $y_\beta \notin$ pClU. Hence there exists a fuzzy pre-q-nbd V of y_β such that $U\bar{q}V$. Therefore, X is fuzzy pre-T_2.

THEOREM 3.4. *Let $f : X \to Y$ be one-to-one mapping.*

(a) *If f is fuzzy precontinuous and Y is fuzzy T_i, then X is fuzzy pre-T_i for $i = 0, 1, 2$.*

(b) *If f is fuzzy pre-irresolute and Y is fuzzy pre-T_i, then X is fuzzy pre-T_i for $i = 0, 1, 2$.*

Proof. We give a proof for $i = 1$ only; the other cases being similar, are omitted. Let x_α and y_β be distinct two fuzzy points in X.

When $x \neq y$, we have $f(x) \neq f(y)$, and by the fuzzy T_1 property of Y, there exist fuzzy nbds U and V of $f(x)_\alpha$ and $f(y)_\beta$ respectively such that $f(x)_\alpha \bar{q} V$ and $f(y)_\beta \bar{q} U$. Since f is fuzzy precontinuous, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy pre-nbds of x_α and y_β respectively such that $y_\beta \bar{q} f^{-1}(U)$ and $x_\alpha \bar{q} f^{-1}(V)$.

When $x = y$ and $\alpha < \beta$ (say), then $f(x) = f(y)$. Since Y is fuzzy T_1, there exists a fuzzy q-nbd V of $f(y)_\beta$ such that $f(x)_\alpha \bar{q} V$. Then $f^{-1}(V)$ is fuzzy pre-q-nbd of y_β such that $x_\alpha \bar{q} f^{-1}(V)$. Hence X is fuzzy pre-T_1.

(b): The proof is similar to (a).

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Department of Natural Sciences,
Pusan National University of Technology,
Pusan, 608–739, Korea

Department of Mathematics Education,
Korea National University of
Education, Choongpook, 363–791, Korea.