A SPATIAL PREDICTION THEORY FOR LONG-TERM FADING IN MOBILE RADIO COMMUNICATIONS

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CONTENTS

ABSTRACT

There have been traditional approaches to model radio propagation path loss mechanism both theoretically and empirically. Theoretical approach is simple to explain and effective in certain cases. Empirical approach accommodates the terrain configuration and distance between base station and mobile unit along the propagation path only. In other words, it does not accommodate natural terrain configuration over a specific area. In this paper, we propose a spatial prediction technique for the mobile radio propagation path loss accommodating complete natural terrain configuration over a specific area. Statistical uncertainty analysis is also considered.

I. INTRODUCTION

Demands for mobile radio communication services are increasing very rapidly. To accommodate demands and to satisfy the quality of mobile radio communication, it is required to study mobile radio propagation mechanisms for both wanted and unwanted signals. One of them is to study point-to-point prediction of propagation path loss in a specific area since it can provide information to ensure coverage of a designated signal and avoidance of co-channel interference.

The prediction of propagation loss is very complicated and difficult because of the irregular natural terrain configuration, the various shapes of architecture structures, changes of weather, and changes of foliage conditions throughout the year. There have been traditional approaches to model the mechanism of the radio propagation path loss both theoretically and empirically. Theoretical approach is trying to model propagation mechanism with physical theory under some specific conditions. Thus it is simple to explain and effective in certain cases. Empirical approach is trying to accommodate the natural terrain configuration and artificial architecture structures. Empirical approach is composed of area-to-area prediction and point-to-point prediction. An area-to-area prediction is useful to predict path loss over a flat terrain and a point-to-point prediction uses information about natural terrain configuration of a specific type. Further detail is explained by Lee [1]. This approach accommodates the terrain configuration and distance between base station and mobile unit along the propagation path only. In other words it does not accommodate natural configuration over a specific area.

II. SPATIAL PREDICTION TECHNIQUE

Two well-known methods for spatial prediction are Matheron' s Kriging method [2] and Hardy' s Multiquadric-Biharmonic (MQ-B) method [3]. Kriging is a stochastic approach and MQ-B is a deterministic approach. As has become recognized[4], many spatial prediction methods are formally equivalent. In Kriging, the prediction is dependent on semivariogram which is a function of distance and the estimation of semivariogram is heavily dependent on experience of researcher and the processing time is long. Hardy's MQ-B is not dependent on experience of the researcher and the kernel function is dependent on distance and it can be easily extended as a function of some important parameters characterizing the response variable. Indeed, with regard to the afore-mentioned two methods, the MQ-B method may be seen as substituting distance itself for the function of distance that is the semivariogram of Kriging. Kriging could provide statistical uncertainty interval or prediction interval for the prediction since it is based on the stochastic model. A way to get statistical uncertainty interval based on deterministic prediction technique, MQ-B, is proposed by David and Yoo [5].

Suppose that

$$F(s) = m(s) + \delta(s), \; s \in D \subset R^d \qquad (1)$$

where, $F(s)$ is the field strength at a spatial site s which varies continuously over D, a subset of d-dimensional space R^d, and $m(s)$ is a deterministic component which characterizes the field strength, and $\delta(s)$ is a random component of the field strength which is not characterized by the deterministic component. In deterministic model, the random component $\delta(s)$ is zero.

Actually, the deterministic component $m(s)$ should depend on some deterministic parameters such as frequency of the wanted signal, antenna gain and height of the transmitter and itself, etc. In stochastic model, we assume that the expected value of the field strength is equal to deterministic component, i.e., $E\{F(s)\}=m(s)$ and we call $F(s)$ is unbiased. The deterministic component $m(s)$ is constant in ordinary Kriging and, in universal Kriging, it is a linear combination of variables that could include trend-surface terms or other explanatory variables thought to influence the behavior of the large-scale variation. Clearly it is reasonable to predict field strength $F(s)$ by the parameters which characterize deterministic component. In this section, we explain ordinary Kriging as a stochastic approach and MQ-B as a deterministic approach for the prediction of strength of long-term fading. For the simplicity of explanation we consider 2-dimensional case, i.e., $d=2$.

1. ORDINARY KRIGING METHOD

It might be true that the measured values of field strength from mutually adjacent sites are strongly correlated. The Kriging method is useful in this situation.

Suppose

$$Var\{F(s+h)-F(s)\}=2\gamma(h), \forall s,\ s+h \in R^2,$$

where $F(s)$ and $F(s+h)$ are the measured values of field strength at site s and $s+h$ respectively, h is the lag. The quantity $2\gamma(\cdot)$, named by Matheron [2], is a model-based parameter which is generally a function only of the difference between the spatial locations s and $s+h$. Further assume that

$$Cov\{F(s+h),\ F(s)\}=C(h),\ \forall s,\ s+h \in R^2 \qquad (2)$$

where $C(\cdot)$ is the covariance function. The condition (2) and unbiasedness of $F(\cdot)$ defines intrinsic stationarity. It could be noted that $C(0)$is the variance of the measured values of field strength at a given location and $\gamma(h)=C(0)-C(h)$. Practically we have to estimate semivariogram based on measured values of field strength from some mathematical models. This estimation step requires trial and error in the specific situation and it takes time to do so. Further details are explained by Cressie [6] and McBratney and Webster [7]. We could assume that two measured values of field strength between two far apart sites are independent then it is clear that

$$\lim_{h\to\infty} \gamma(h)=C(0).$$

It is further assumed that the value of field strength at site s_p is a linear combination of m measured values F_i at sites s_i, $i=1, \cdots, m$. The Kriging coefficients are calculated by minimizing the mean squared prediction error $E\left\{F(s_p)-\sum_{i=1}^{m} a_i F_i\right\}^2$ with unbiasedness condition $\sum_{i=1}^{m} a_i = 1$. In Kriging, the model fitting process for semivariogram implicitly creates an error during the fitting and it depends on the justification and an experience of the researcher as to which mathematical model he should choose.

2. MQ-B METHOD

Let (x_i, y_i), $i=1, \cdots, m$,be the coordinates of the sampled test site s_i, and F_i the field strength at s_i. Let s_p, with coordinates (x_p, y_p), denote an arbitrary site, and $F(s)$ denote the field strength at s. In deterministic model there is no random component and it is assumed that the deterministic component is related linearly to measured F_i at various sites in the area of consideration, i.e., the predicted value is a linear combination of measured ones. MQ-B is based on the physical theory of equilibrium: the final state is in a state of equilibrium after a past expenditure of energy. This state leads to the result that the sum of coefficients for the linear combination is zero.

Now define

$$Q=\begin{pmatrix} Q_{11} & \cdots & Q_{1m} & 1 \\ \cdot & \cdots & \cdot & \cdot \\ Q_{m1} & \cdots & Q_{mm} & 1 \\ 1 & \cdots & 1 & 0 \end{pmatrix},$$

$$Q_p' = (Q_{p1}, \cdots, Q_{m1}, 1),$$

$$R' = (\rho_1, \cdots, \rho_m, \rho_0),$$

and $F' = (F_1, \cdots, F_m, 0)$,

where $Q_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2}$. The system equations in MQ-B with the equilibrium constraint $\left(i.e., \sum_{i=1}^{m}\rho_i = 0\right)$ are written as QR=F and thus the field strength $F(s_p)$ at site s_p is given by $Q_p'R=F(s_p)$. Thus it is easily noted that

$$F(s_p) = Q_p' Q^{-1} F = \sum_{i=1}^{m} \beta_i(s_p) F_i,$$

with $(\beta_1(s_p), \cdots, \beta_m(s_p))$ equal to the first m components of $Q_p'Q^{-1}$. In other words, the estimated field strength at site s_p is a linear combination of the observed field strength from m sampled test sites. It is noted that $Q_{ij} = Var(F_i - F_j)/2$ in Kriging and $Q_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$ in MQ-B. Hardy observed that the structure of system equations of MQ-B is very similar to that of Kriging if we minimize $\left\{F(s_p) - \sum_{i=1}^{m} \alpha_i F_i\right\}^2$ instead of minimizing the mean squared prediction error $E\left\{F(s_p) - \sum_{i=1}^{m} \alpha_i F_i\right\}^2$. Generally speaking, MQ-B is simple and more flexible since it can easily accommodate deterministic component as a function of parameters in addition to distance and it is not necessary to worry about modeling of semivariogram. It is others' experience, as documented in Hardy [8] and David and Yoo [5], that it is not disadvantageous, for purposes of accurate spatial prediction, to forego using an estimated spatial covariance structure. Note, however, that this is not to say that stochastic considerations might not play a role in the evaluation of prediction method.

III. APPLICATION TO MOBILE RADIO COMMUNICATIONS

A radio propagation mechanism involves not only distance but also natural terrain configuration, propagation frequency, natural environment such as foliage effect, and human-made environment such as rural, suburban, and urban area. Thus it is reasonable to predict field strength by the appropriate spatial prediction method accommodating characterizing variables other than distance.

In this section we consider spatial prediction method accommodating not only natural terrain configuration in 3-dimensional spatial domain over a specific area but also the other variables characterizing field strength such as propagation frequency and foliage effect in a specific area. Additionally, we propose prediction interval for the statistical uncertainty analysis in the prediction of long-term fading following log-normal distribution.

1. SPATIAL PREDICTION

First, we consider spatial prediction method accommodating natural terrain configuration in 3-dimensional spatial domain over a specific area. One way to get the spatial prediction accommodating natural terrain configuration including distance is to simply define the kernel function as $Q_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2\right]^{1/2}$. The other way is as follows: Now define

$$Q = \begin{pmatrix} Q_{11} & \cdot & \cdot & \cdot & Q_{1m} & 1 & h_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Q_{m1} & \cdot & \cdot & \cdot & Q_{mm} & 1 & h_m \\ 1 & \cdot & \cdot & \cdot & 1 & 0 & 0 \\ h_1 & \cdot & \cdot & \cdot & h_m & 0 & 0 \end{pmatrix},$$

$$Q_p' = (Q_{p1}, \cdot\cdot\cdot, Q_{pm}, 1, h_p)$$

$$R' = (\rho_1, \cdot\cdot\cdot, \rho_m, \rho_n, \eta),$$

and $F' = (F_1, \cdot\cdot\cdot, F_m, 0, 0)$,
where $Q_{ij} = [(x_i-x_j)^2+(y_i-y_j)^2]^{1/2}$ and h_i is the altitude of the site s_i above the sea level. The system equations in MQ-B with the uniform equilibrium constraint $\left(i.e., \sum_{i=1}^{m}\rho_i = 0 \text{ and } \sum_{i=1}^{m}\rho_i h_i = h_p\right)$ are written as $QR=F$ and thus the field strength $F(s_p)$ at site s_p is given by $Q_p'R=F(s_p)$. Thus it is easily noted that

$$F(s_p) = Q_p'Q^{-1}F = \sum_{i=1}^{m}\beta_i(s_p)F_i, \tag{3}$$

with $(\beta_1(s_p),\cdots,\beta_m(s_p))$ equal to the first m components of $Q_p'Q^{-1}$. In other words, the estimated field strength at site s_p is a linear combination of the observed field strengths from m sampled test sites.

Next, we consider spatial prediction method accommodating characterizing variable other than natural terrain configuration in 3-dimensional spatial domain over a specific area. For simplicity we assume that transmitting frequency from the base station is a characterizing variable. It is well known [1] that the empirical received signal power is proportional to inverse power a of the transmitted frequency f_i from the base station with a between 2 and 3. Then we could assume that

$$F_f(s_p) = \alpha + \beta f + \varepsilon, \tag{4}$$

where $F_f(s_p)$ is the estimated field strength at site s_p with the transmitted frequency $f \equiv f_i^a$ from the base station, α and β are unknown parameters, and ε follows Gaussian distribution with mean zero and unknown variance. Now it is possible to estimate $F_{f_p}(s_p)$ from equations (3) and (4) for possible channel with frequency f_p over a specific area such as a cell with a base station if we have some measured values of field strength from several sites over a specific area for several different channels with different frequencies. Thus we could get some information to ensure coverage of a designated signal and avoidance of co-channel interference between two adjacent areas such as cells.

More frequently the mobile radio signal between the base station and mobile unit may be propagated over more than one type of environment. In this case we could divide the specific area with several subareas according to the type of environment, and apply the proposed spatial prediction method for each subarea.

2. STATISTICAL EVALUATION OF THE PREDICTION INTERVAL

To formulate statistical uncertainty interval of the empirical field strength $F_f^{(m)}(s_p)$, based on m measured sites, as predictor of the actual field strength map $F_f(s_p)$, we assume that all covariances required below, in other words the semivariogram $\gamma(\cdot)$, are known. In mobile radio communications, it is well known that the signal strength of a mobile radio signal received at each site is ergodic, in other words, it remains unchanged in different time intervals and long-term fading follows log-normal distribution [1]. Define $y \equiv y_f(s_p) = \log\{F_f(s_p)\}$ to be the log-transformed field strength of long-term fading and $\hat{y} \equiv \hat{y}_f(s_p) = \log\{F_f^{(m)}(s_p)\}$ is the log-transformed empirical strength at site s_p. Two approaches now suggest themselves.

The first is to base the statistical uncertainty statement on the conditional distribution of $y-\hat{y}$ given the vector $Y=(y_1-\bar{y},\cdots,y_{m-1}-\bar{y})$ where $\bar{y}=(y_1+\cdots+y_m)/m$. This conditional distribution of $y-\hat{y}$ given Y is free of the unknown expected field strength level. In detail, the resulting conditional uncertainty interval for true field strength $y_f(s_p)$ at an arbitrary site s_p with transmitted frequency f from the base station would be obtained as follows: The joint density of $y-\hat{y}$ and y is m-variate normal with zero means and covariance matrix V:

$$V=\begin{pmatrix} V_{ss} & V_{s1} & \cdot & \cdot & \cdot & V_{sb} \\ V_{sb} & V_{11} & \cdot & \cdot & \cdot & V_{1b} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ V_{sb} & V_{b1} & \cdot & \cdot & \cdot & V_{bb} \end{pmatrix}$$

where $b=m-1$, Define $\sum_{s1}'=(V_{s1},\cdots,V_{sb})'$ and

$$V=\begin{pmatrix} V_{11} & \cdot & \cdot & \cdot & V_{1b} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ V_{b1} & \cdot & \cdot & \cdot & V_{bb} \end{pmatrix}.$$

Then the conditional mean and variance of $y-\hat{y}$ given Y are given by

$$\sum_{s1}' \sum_{11}^{-1} Y' \tag{5}$$

and

$$V_{ss}-\sum_{s1}' \sum_{11}^{-1} \sum_{s1}, \tag{6}$$

respectively. We may construct a 100(1-α)% uncertainty interval of $y-\hat{y}$ given Y by

$$\left((5)-z_{\alpha/2}\sqrt{(6)},\ (5)+z_{\alpha/2}\sqrt{(6)}\right),$$

and an uncertainty interval for y is therefore given by

$$\left(\hat{y}+(5)-z_{\alpha/2}\sqrt{(6)},\ \hat{y}+(5)+z_{\alpha/2}\sqrt{(6)}\right). \tag{7}$$

This conditional approach might be only valid when the semivariogram has a limited value since it strongly depends on the intrinsic stationarity assumption.

If the intrinsic stationarity assumption seems to be violated however there is an apparent limited value of the semivariogram, the following marginal approach based on the marginal distribution of $y-\hat{y}$, with only the variance of $y-\hat{y}$ estimated from the measured data, may be more plausible. The resulting marginal 100(1-α)% uncertainty interval of y is constructed by centering on $\hat{y}$ an interval of half-length equal to $z_{\alpha/2}$ times the standard deviation of $y-\hat{y}$:

$$\left(\hat{y}-z_{\alpha/2}\sqrt{V_{ss}},\ \hat{y}+z_{\alpha/2}\sqrt{V_{ss}}\right). \tag{8}$$

If the intrinsic stationarity assumption seems to be strongly violated, we could not assume that the structure of semivariogram are known any more. However if we have a history of log-transformed values of measured signal strength $y_1,\cdots,y_n$ at a site for n different channels using n different frequencies $f_1,\cdots,f_n$, respectively, we could apply the following classical approach from regression analysis as long as the assumption of model (4) is valid: The 100(1-α)% confidence interval or uncertainty interval of the mean of $y_i \equiv y_{f_i}(s_p)$ is given by

$$\bar{y}+\hat{\beta}(f_i-\bar{f})\pm t_{n-2,1-\alpha/2}S_y\sqrt{S^2_{\mu(y)}}, \tag{9}$$

where

$$\bar{y}=\sum_{i=1}^{n}y_i/n,\ \hat{\beta}=\sum_{i=1}^{n}(f_i-\bar{f})(y_i-\bar{y})\Big/\sum_{i=1}^{n}(f_i-\bar{f})^2,\ \bar{f}=\sum_{i=1}^{n}f_i\Big/n,$$

$t_{n-2,1-\alpha/2}$ is the 100(1-α)% point of the t distribution with n-2 degrees of freedom, S_y^2 and S_f^2 are the sample variances of y_i and f_i, respectively, and $S^2_{\mu(y)}=1/n+(f_i-\bar{f})^2/\{(n-1)S_f^2\}$. If we are interested in the 100(1-α)% prediction interval of y at unmeasured frequency f it is given by (9) by replacing $S^2_{\mu(y)}$ with

$$1+1/n+(f_i-\bar{f})^2/\{(n-1)S_f^2\}.$$

Since log-transformation is applied into the strength of long-term fading the uncertainty interval of the field strength of long-term fading should be obtained by taking exponential.

IV. CONCLUDING REMARKS

Prediction of field strength of long-term fading is a necessary step to achieve both high capacity and high quality. The prediction technique should be based on theoretical model to get precise prediction of field strength if the underlying physical assumption under a specific condition is valid. In most cases the specific conditions for the theoretical approach are rather ideal than practical. Empirical approach accommodates the terrain configuration and distance between base station and mobile unit along the propagation path only. In other words it does not accommodate natural terrain configuration over a specific area. We proposed some spatial prediction techniques for the mobile radio propagation path loss accommodating complete natural terrain configuration over a specific area. Moreover statistical uncertainty analysis is given for those spatial predictions. Practical applications with real data should be considered for future research.

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