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# An Optimal Design of Man−On−Board Storage and Retrieval Warehousing System[†]

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## Abstract

This paper deals with the design problem of a man-on-board(MOB) storage and retrieval warehousing system which is suitable for storing items of small size and light weight. It is assumed that the operator carries out a sequence of retrieving(or storing) operations traveling on a specially designed truck. Considering the operating characteristics of the man-on-board system, an optimal design model is developed in which the investment and maintenance costs of the system are minimized over a time horizon satisfying a set of constructional restrictions. The model is formulated as a nonlinear integer program and a search algorithm is proposed to find an optimum solution.

## Introduction

The man-on-board(MOB) system is a storage and retrieval(S/R) warehousing system which has been used predominantly for storing and retrieving the items of small size and light weight. In the MOB system an operator works on the specially designed S/R truck, such as the order-picking truck[1] and the narrow-aisle order picker truck [5]. The operator usually carries out multi-command which consists of a number of storage or retrieval requests between successive returns to the input and output(I/O) point. Compared to conventional warehousing systems, the MOB system offers a number of benefits such as better space utilization, precise inventory control, shorter lead time, and efficient order picking. Presently, it is not difficult to find domestic manufacturing firms which are utilizing the MOB system, especially among the aero, electric and electronic parts manufacturing companies. These firms adopt the MOB system either for their main storage facility or as a supple-

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mentary warehousing system to unit-load automated storage/retrieval systems(AS/RS). The MOB system adopts "Chebyshev travel" for the movement of the S/R truck below a predetermined rack height, but it adopts "rectilinear travel" above the height to ensure the operator's safety. In Chebyshev travel, the S/R truck travels simultaneously in the horizontal and vertical directions, whereas in rectilinear travel, the S/R truck can travel in only one direction at a time. We will refer to the above travel method of the S/R truck as "combined travel". Hwang and Song[2] developed the expected travel time models of the system and proposed an order sequencing heuristic, considering the operating characteristics of the MOB system.

A firm's successful performance in the warehouse is dependent upon the appropriate design, layout and operation of the warehouse storage and handling system. A number of articles have been published in the area of unit-load AS/RS design. As far as we know, there has been no literature regarding the design problem for the MOB system up to the present time.

This paper presents an analytical design model of the MOB system with the objective of minimizing the installation cost and operating cost over a time period. Through the solution algorithm proposed, the number of S/R truck required and system parameters, such as width, length and height of the system are determined. All these decision variables are subject to the storage volume requirement, system throughput requirement and dimension restrictions due to the building site.

# Assumptions and Notation

A design model of the system is developed based on the following assumptions.

1) All rack openings are of the same size.

2) The I/O point is located at the lower left-hand corner.

3) Randomized storage assignment policy is adopted.

4) The demand of each storage item is known and constant during some time period(e.g. year). That is, the required storage volume and system throughput are known and constant.

5) The S/R truck adopts the combined travel for its movement.

To facilitate the model development, the following notation is introduced.

$w$ = width of a rack opening containing allowance,
$h$ = height of a rack opening containing allowance,
$l$ = length of a rack opening containing allowance,
$w_a$ = width of an aisle between adjacent two racks,
$w_m$ = width of the main aisle between rack and conveyor system,
$w_c$ = width of conveyor system,
$wt$ = average weight of items stored in a rack opening,
$d_w$ = allowance for width of the MOB system,
$d_h$ = allowance for length of the MOB system,
$d_l$ = allowance for length of the MOB system,
$s_x$ = average horizontal S/R truck speed,
$s_y$ = average vertical S/R truck speed,
$t_p$ = operation time for pick-up or deposit,

$e_w$=system width allowed,
$e_h$=system height allowed,
$e_l$=system height allowed,
A=predetermined height for the operator's safety,
i=annual discount rate,
T=planning horizon(years) to be investigated,
V=storage volume required, i.e., the minimum number of rack openings needed for the storage of the items in the MOB system,
TR=system throughput, i.e., the required number of retrieval(or storage) requests handled by the operator per time unit.

The decision variables for the MOB system design are

N=number of aisles,
$N_h$=number of loads high(Load high is one row of the rack which is composed of the connected rack openings),
$N_b$=number of bays(Bay is one column of the rack which is composed of the connected rack openings).

Then the building width, height and length become

$$X_w = \text{building width} = (2\times w + w_a)\times N + d_w \quad (1)$$

$$X_h = \text{building height} = h\times N_h + d_h, \quad (2)$$

$$X_l = \text{building length} = l\times N_b + d_l \quad (3)$$

## Throughput Estimation

To evaluate the throughput of the MOB system, we need to know the average number of stops(n) the operator has to make during a tour and its expected round trip time(ETT). It is reasonable to expect that as the number of rack openings in an aisle becomes larger, n may increase approaching to a certain upper bound. Power or logarithm function is known to represent the above characteristics well. In our study, we adopt the following power function.

$$n = [k_1(1 - k_2^{-2N_h N_b})],$$

where [x] is the nearest integer of x, $k_1$ the maximum number of stops during a tour and $k_2$ the parameter related to the slope of the curve. $k_1$ could be determined considering the carrying capacity of the S/R truck and volume(or weight) of each item. Figure 1 shows curves, each corresponding to a given value of $k_2$ with $k_1$ set to 20.

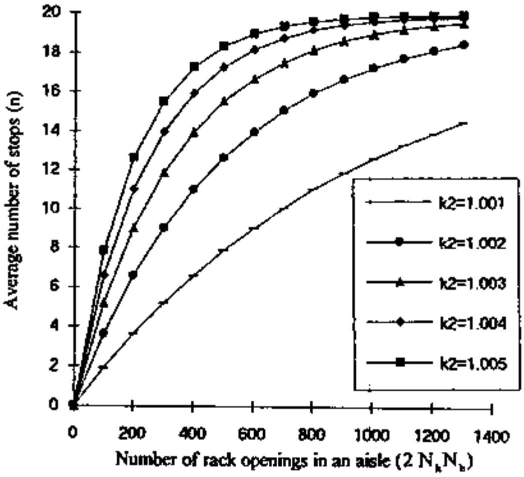

Figure 1. Average number of stops($k_1$=20)

Suppose the operator has a task of picking n different items(or visiting n different rack locations) before returning to the I/O point. The ETT of the task depends on how the traveling sequence is formulated. Any algorithm for the Traveling Salesman Problem (TSP) can be utilized to obtain the minimum round trip time of the operator. At the design stage, the information on each task is not known and so the ETT based on TSP is impossible to find in closed form.

We present a sequencing heuristic from

which the ETT can be readily determined. Even though the heuristic is not competitive in its efficiency compared to the combined hull heuristic [2], tests with various data have shown that it generates a fairly good sequence. Also, it has an advantage of being simple to understand by the manager in charge of the warehouse operations. This makes implementation considerably easier than in the case of the sophisticated TSP algorithm. Consider a set of retrieval requests whose locations are given on the rack surface which is partitioned into two regions. These regions are separated by horizontal line corresponding to the predetermined height with the lower part being Chebyshev travel region and the upper part rectilinear travel region. The points in Chebyshev region are sorted in the increasing x-coordinate direction, while the points in rectilinear region are sorted in the opposite direction. The heuristic starts with the I/O point as the beginning of a tour. Then these points are connected sequentially whose last point is connected to the I/O point to form a tour. Let nc and nr be the expected number of stops in Chebyshev region and rectilinear region, respectively. For randomized assignment policy on a continuous rack face, each retrieval location can be assumed to be uniformly distributed, Then, with n given, nc and nr are

$$\mathrm{nc} = [\mathrm{An}/(\mathrm{hN}_h)], \tag{4}$$

$$\mathrm{nr} = \mathrm{n} - \mathrm{nc}. \tag{5}$$

Also the normalized time, Tb, of the S/R truck to reach the row most distance from the I/O point and the time, Ta, to reach the predetermined height become

$$\mathrm{Tb} = \mathrm{hs}_x \mathrm{N}_h/(\mathrm{ls}_y \mathrm{N}_b), \tag{6}$$

$$\mathrm{Ta} = \mathrm{As}_X/(\mathrm{ls}_y \mathrm{N}_b). \tag{7}$$

It will be assumed, without loss of generality, that Tb is less than or equal to one. Let PCI and PCA be the points in Chebyshev region with the minimum and maximum x coordinate, respectively. Similarly, we denote PRI and PRA as the points in rectilinear region with the minimum and maximum x coordinate, respectively. Let PO be the I/O point and $t(P_i, P_j)$ be the travel time to reach $P_j$ from $P_i$. With the application of the probability theory to tours determined by the heuristic, its expected round trip time, ETT, can be found[3] and

$$\mathrm{ETT} = t(\mathrm{PO},\mathrm{PCI}) + t(\mathrm{PCI},\mathrm{PCA}) + t(\mathrm{PCA},\mathrm{PO}), \text{ if } \mathrm{nr}=0 \tag{8}$$

$$t(\mathrm{PO},\mathrm{PRA}) + t(\mathrm{PRA},\mathrm{PRI}) + t(\mathrm{PRI},\mathrm{PO}), \text{ if } \mathrm{nc}=0 \tag{9}$$

$$t(\mathrm{PO},\mathrm{PCI}) + t(\mathrm{PCI},\mathrm{PCA}) + t(\mathrm{PCA},\mathrm{PRA}) + t(\mathrm{PRA},\mathrm{PRI}) + t(\mathrm{PRI},\mathrm{PO}), \text{ otherwise} \tag{10}$$

,where

$$t(\mathrm{PO}, \mathrm{PCI}) = \frac{1-(1-\mathrm{Ta})^{\mathrm{nc}+2}}{(\mathrm{nc}+1)(\mathrm{nc}+2)\,\mathrm{Ta}} + \frac{\mathrm{Ta}}{2} \tag{11}$$

$$t(\mathrm{PCI},\mathrm{PCA}) = (\mathrm{nc}-1) \left\{ \frac{2}{(\mathrm{nc}+1)(\mathrm{nc}+2)\mathrm{Ta}} + \frac{\mathrm{Ta}}{3} - \frac{2-2(1-\mathrm{Ta})^{\mathrm{nc}+3}}{(\mathrm{nc}+1)(\mathrm{nc}+2)(\mathrm{nc}+3)\mathrm{Ta}^2} \right\} \tag{12}$$

$$t(\mathrm{PCA},\mathrm{PO}) = \frac{\mathrm{nc}}{\mathrm{nc}+1} + \frac{\mathrm{Ta}^{\mathrm{nc}+1}}{(\mathrm{nc}+1)(\mathrm{nc}+2)}, \tag{13}$$

$$t(\mathrm{PO},\mathrm{PRA}) = \frac{\mathrm{Ta}+\mathrm{Tb}}{2} + \frac{(1-\mathrm{Ta})^{\mathrm{nr}}}{\mathrm{nr}}, \tag{14}$$

$$t(\mathrm{PRA},\mathrm{PRI}) = (\mathrm{nr}-1) \left\{ \frac{1}{(\mathrm{nr}+1)} + \frac{\mathrm{Tb}-\mathrm{Ta}}{3} \right\}, \tag{15}$$

$$t(\mathrm{PRI},\mathrm{PO}) = \frac{\mathrm{nr}+\mathrm{Ta}^{\mathrm{nr}+1}}{\mathrm{nr}+1} + \frac{\mathrm{Tb}-\mathrm{Ta}}{2} \tag{16}$$

$$t(\mathrm{PCA},\mathrm{PRA}) = \frac{\mathrm{Tb}}{2} + \frac{1}{2\mathrm{Ta}} \left\{ \frac{\mathrm{nr}}{\mathrm{nr}+1} - \frac{\mathrm{nc}}{\mathrm{nc}+1} \right.$$

$$-\frac{2ncnr}{(nc+nr+1)(nr+1)} + \frac{2nc}{nc+nr+1}\Big\}^2 \qquad (17)$$

## Development of the Design Model

In this section, we present a mathematical model from which an economic configuration of the MOB system is obtained. We try to incorporate as many cost components and constraints involved in the design process as possible. The cost components of the objective function consist of initial investment cost and discounted operating and maintenance cost as follows :

1) S/R truck cost(CSR) : $C_1N$ *where* $C_1$ represents the unit cost of the S/R truck.

2) Rack structure cost(CRK) : $C_2(2NN_hN_b)$ *where* $C_2$ is the rack structure cost estimate per rack opening. Adopting Zollinger's strdy [6] on $C_2$ is the rack structure cost can be expressed as

$$CRK = C_{21}NN_bN_n + C_{22}NN_bN_h^2 + C_{23}NN_bN_h^3,$$

where

$$C_1 = 46.242 + 1.25whl + 0.02212wt - wt^2/1650000,$$

$$C_{22} = 11.664 \text{ and } C_{23} = -0.238.$$

3) Land cost(CLN) : $C_3(x_l + w_m + w_c)x_w$ where $C_3$ is the land price per square feet. The total space contains the rack building, main aisle and conveyor system space.

4) Conveyor system cost(CCV) : $2C_4x_w$ where $C_4$ is the conveyor cost per feet.

5) Discounted operating and maintenance cost(COP) :

i ) Annual truck maintenance cost (CMN) : $C_5N$ where $C_5$ is the annual S/R truck maintenance cost per unit.

ii ) Annual labor cost(CLB) : $C_6N$ where $C_6$ is the annual labor cost per operator

Assuming a lifetime of T years and a discount rate i, COP becomes

$$COP = \sum_{t=1}^{T} \frac{CMN + CLB}{(1+i)^t}$$

The MOB system under consideration must satisfy the storage volume and system throughput required. Also, it has to meet restrictions coming from the constructional site condition. These constraints are introduced as follows :

1) Storage volume constraint

The total number of rack openings should be greater than or equal to the required storage volume V. Thus the storage volume constraint becomes

$$2NN_hN_b \geq V \qquad (18)$$

2) System throughput constraint

With the expected round trip time, ETT, the required throughput, TR, and the average number of stops per trip, n, the system throughput constraint becomes

$$\frac{nN}{ETT + 2nt_p} \geq TR. \qquad (19)$$

Note that both n and ETT are the functions of $N_h$ and $N_b$

3) Site restrictions on the system

The system configuration should be within the available site capacity, which can be expressed as

system width : $x_W \leq e_W$ (20)

system height : $x_h \leq eh$ (21)

system length : $x_l + w_m + w_c \leq e_l$ (22)

Now, the design model can be stated as follows :

(P) Minimize

$$TC = C_1N + 2C_2NN_hN_b + C_3(x_l + x_m + w_c)x_w + 2C_4x_w + \sum_{t=1}^{T} \frac{(C_5+C_6)N}{(1+i)^t} \qquad \text{㉓}$$

subject to

$$x_w \le e_w \qquad \text{㉔}$$

$$x_h \le e_h \qquad \text{㉕}$$

$$x_l + w_m + w_c \le e_l \qquad \text{㉖}$$

$$2NN_hN_b \ge V \qquad \text{㉗}$$

$$\frac{nN}{ETT + 2nt_p} \ge TR \qquad \text{㉘}$$

$$x_w,\ x_h,\ x_l\ > 0 \qquad \text{㉙}$$

$$N, N_h, N_b > 0 \text{ and integer.} \qquad \text{㉚}$$

The problem(P) is reformulated into(P1) by rewriting after substituting the rack cost, $C_2$, with those of Zollinger [6] and utilizing equations (1), (2) and (3)

(P1) Minimize

$$TC = \{C_1 + C_3(d_l + w_m + w_c)(2w + w_a) + 2C_4\ (2w + w_a)\}N + C_3l\ d_w\ N_b + C_3\ l\ (2w + w_a)NN_b + C_{21}N\ \ N_hN_b + C_{22}\ NN_h\ N_b^2 + C_{23}\ NN_hN_b^3 + C_3(d_w\ d_l + d_w\ w_c) + 2C_4\ d_w + \sum_{t=1}^{T}\frac{(C_5+C_6)N}{(1+i)^t} \qquad \text{㉛}$$

subject to

$$N \le (e_w - d_w)/(2w + w_a) \qquad \text{㉜}$$

$$N_h \le (e_h - d_h)/h \qquad \text{㉝}$$

$$N_b \le (e_l - d_l - w_m - w_c)/l \qquad \text{㉞}$$

$$N \ge V/(2N_h\,N_b) \qquad \text{㉟}$$

$$N \ge TR(ETT + 2nt_p)/n \qquad \text{㊱}$$

$$N, N_h, N_b > 0 \text{ and integer} \qquad \text{㊲}$$

## Algorithm for the Model

The problem(P1) is a nonlinear integer program. To find an optimal solution with most well known optimization techniques must be very tedious and time consuming. We present a simple search procedure which is stated in an algorithmic form.

**Phase 1** : Determine upper bounds.

Let UBN, $UBN_h$ and $UBN_b$ denote the upper bounds for N,$N_h$ and $N_b$, respectively. Then each bound can be determined with equations ㉜,㉝ and ㉞ as follows :

Step 1. $UBN = \left\lfloor (e_w + d_w)/(2 \times w + w_a) \right\rfloor$,
where $\lfloor x \rfloor$ is the largest integer less than or equal to x.
$UBN_h = \left\lfloor (e_h - d_h)/h \right\rfloor$
$UBN_b = \left\lfloor (e_l - d_l - w_c)/l \right\rfloor$.

**Phase 2** : Find lower bounds and feasible solutions.

Let LBN, $LBN_h$ and $LBN_b$ denote the lower bounds for N, $N_h$ and $N_b$,respectively. Then the feasible solutions of N, $N_h$ and $N_b$ are found using equations㉟ and ㊱ by the following steps :

Step 2. $LBN = \left\lceil (V/(2 \times UBN_h \times UBN_b) \right\rceil$,
where$\lceil x \rceil$ is the smallest integer greater than or equal to x.
N=LBN.

Step 3. $LBN_h = \left\lceil V/(2 \times N \times UBN_b) \right\rceil$.
$N_h = LBN_h$.

Step 4. $LBN_b = \left\lceil V/(2 \times N \times N_h) \right\rceil$.
$N_b = LBN_b$.

Step 5. If the throughput constraint ㊱ is satisfied, then keep the values of N, $N_h$ and $N_b$ as a feasible solution. Otherwise, increase $N_b$ by one and repeat this step until $N_b = UBN_h$

Step 6. $N_h = N_h + 1$ and go to Step 4 until $N_h = UBN_h$

Step 7. N=N+1 and go to Step 3 until N=UBN.

**Phase 3** : Find an optimal solution.

Step 8. Find an optimal value of N, $N_h$ and $N_b$ and the minimum cost among a set of the feasible solution.

To illustrate the solution algorithm, a case problem is solved. The data for the problem are quoted from the case study of Park and Webster[4] and listed in Table 1.

**Table 1. System specifications and cost elements**

| parameter | value | | parameter | value | |
|---|---|---|---|---|---|
| w | 4.5 | feet | $e_w$ | 110 | feet |
| h | 4.5 | feet | $e_h$ | 70 | feet |
| l | 4.5 | feet | $e_l$ | 215 | feet |
| $w_a$ | 6.5 | feet | A | 13.5 | feet |
| $w_m$ | 20 | feet | i | 0.1 | |
| $w_c$ | 15 | feet | T | 8 | years |
| wt | 500 | lb | V | 3600 | pallets |
| $d_w$ | 0.0 | feet | TR | 7 | /min |
| $d_h$ | 0.0 | feet | $k_1$ | 20 | |
| $d_l$ | 0.0 | feet | $k_2$ | 1.003 | |
| $s_x$ | 240 | ft/min | $C_1$ | $40000 | |
| $s_y$ | 80 | ft/min | $C_3$ | $22 | |
| $t_p$ | 0.2 | min | $C_4$ | &744 | |
| | | | $C_5$ | $1000 | |
| | | | $C_6$ | $30000 | |

A computer program is compiled in Turbo PASCAL and executed on personal computer 486/AT. The results are summarized in Table2.

**Table 2. Solution of the case problem**

| Feasible solutions | | | | Optimal solution | |
|---|---|---|---|---|---|
| N | $N_h$ | $N_b$ | TC($) | parameter | value |
| 5 | 8 | 45 | 1995617.2 | N | 5 |
| 5 | 9 | 45 | 1970967.1 | $N_h$ | 10 |
| 5 | 10 | 36 | 1953132.7 | $N_b$ | 36 |
| 5 | 11 | 36 | 2013832.8 | | |
| 5 | 12 | 39 | 2148737.5 | $x_w$ | 77.5 feet |
| 5 | 13 | 45 | 2373386.3 | $x_h$ | 45.0 feet |
| 6 | 7 | 43 | 2273514.5 | $x_l$ | 162.0 feet |
| 6 | 8 | 38 | 2246581.7 | | |
| 6 | 9 | 34 | 2226730.0 | CSR | $200000.0 |
| 6 | 10 | 30 | 2193514.4 | CRK | $475014.1 |
| 6 | 11 | 33 | 2335406.9 | CLN | $335885.0 |
| 6 | 12 | 36 | 2491085.4 | CCV | $115320.0 |
| 6 | 13 | 39 | 2660627.1 | COP | $826913.6 |
| 6 | 14 | 42 | 2844006.6 | TC | $1953132.7 |
| 6 | 15 | 45 | 3041095.5 | | |
| 7 | 6 | 43 | 2564402.2 | system throughput | |
| 7 | 7 | 37 | 2517118.0 | | 7.2899 |
| 7 | 8 | 33 | 2497546.2 | storage volume | |
| 7 | 9 | 29 | 2463266.4 | | 3600 |
| 7 | 10 | 30 | 2559100.1 | computation time | |
| 7 | 11 | 33 | 2724641.4 | | 0.54 seconds |
| 7 | 12 | 36 | 2906266.3 | | |
| 7 | 13 | 39 | 3104064.9 | | |
| 7 | 14 | 42 | 3318007.7 | | |
| 7 | 15 | 45 | 3547944.8 | | |

## Conclusions

Due to its low initial investment cost and flexibility in its operations, the MOB system becomes popular in domestic industries for storing small and light weight items. Also, the system has been frequently adopted as a supplementary warehousing facility to unit-load AS/RS. We formulated a nonlinear integer program to solve the design problem whose objective is to minimize the initial investment and operating costs over a given time horizon. A solution algorithm was proposed which determines the system parameters, such as length and height of the system and number of aisles. An example problem was solved to demonstrate the validity and efficiency of the design model and its algorithm. In formulating the design model, a major difficulty was how to evaluate the system throughput. In this regard, we proposed a sequencing heuristic and a way to represent the average number of stops in a tour. The methodology proposed in this paper to estimate the system throughput is by no means complete and needs further study.

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