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A Study on IFGP Model for Solving Multiobjective Quality Management under Fuzzy Condition

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ABSTRACT

This paper purports to study on interactive fuzzy goal programming model which leads to the compromise solution which the decision maker satisfies through the interactive approach.

We also attempted to calculate local proxy preference function from utility function of sum-of-logarithms in connection with marginal rate of substitution and interactive approach for the purpose of applying weight of multiobjective function.

In an attempt to grasp compromise solution from fuzzy efficient solution, we decided to take the interactive method and presented stopping rule for this.

I. Introduction

A lot of multiobjective decision making problems with application to quality management take place in an environment in which the goals, constraints, and consequences of possible actions are not kown precisely. The tools of fuzzy set theory can be utilized to deal with such imprecision in a quantitative manner.

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In particular, it is demonstrated how fuzzy or imprecise aspirations of the decision maker can be quantified throught the use of piecewise linear and continuous functions.

Typical fuzzy goal programming could measure the aspiration level and also find out the decision maker's satisfaction level through utilizing the membership function. The algorithm of traditional fuzzy goal programing depicts the satisfaction level by decision maker's subjective judgesment. In this paper, howbeit, we demonstrated the new algorithm indication the decision maker's degree of satisfaction with little more systematic methodology.

This paper purports to study on interactive fuzzy goal programming (IFGP) which leads to the compromise solution which the decision maker satisfies through the interactive approach.

II. Study of IFGP Algorithm

The following study is divided into fuzzy efficient solution and compromise solution, the logical background of IFGP algorithm is shown as follows.

1. Fuzzy Efficient Solution

1.1 Conception of Fuzzy Efficient Solution

To find fuzzy goal programming, first of all, fuzzy efficient solution which is most suitable thing to compromise solution should be researched by the possible collection of fuzzy efficient solution.

The conception of efficient solution means non-dominated solution and is called pareto optimality. [S 15]

The definition of such fuzzy efficient solution is as follows. [Z 19]

(Definition 2-1) Fuzzy Efficient Solution : On the assumption that $\mu_i(X)$, $j=1, \cdots, m$ is membership function of (formula 2-1) fuzzy constraints, X found by the interactive fuzzy goal programming algorithm becomes fuzzy efficient solution in case of the following $X'(\in X)$ is not appeared.

$$
\begin{aligned}
& Z_i(X') \leq Z_i(\mathrm{X}),\ i=1, \cdots, k, \qquad (2\text{-}1) \\
\wedge\ & \mu_j(X') > \mu_j(\mathrm{X}),\ j=1, \cdots, m \\
\wedge\ & Z_{i0}(\mathrm{X}) < Z_{i0}(\mathrm{X}),\ \text{for at least one } io \in (1, \cdots, k\} \\
\vee\ & \mu_{j0}(X') > \mu_{j0}(\mathrm{X}),\ \text{for at least one } jo \in (1, \cdots, m\}.
\end{aligned}
$$

1.2 Range of Objective Function

Because the range of each objective function has been decided by subjective and optional decision maker in the existing FGP, the range of real objective function is decided too widely, and the iteration should be increased to find the compromise solution. In case of beyond the range of objective function, the satisfaction can't be done.

Therefore, this study decided positive ideal solution(PIS) and negative ideal solution(NIS) and within its range the minimum limit Z_i^L and the maximum limit Z_i^U was made under the limited condition by cutting down repeating times for solution and compromise solution. [SGI 16]

Positive ideal solution is the best solution under the limited condition given in the problem of multiobjective decision as each objective function looks for solution by itself while negative ideal solution is the worst solution under the limited condition given as each objective function looks for solution by itself.

Namely, on the assumption that multiobjective decision was given as follows.

$$\begin{aligned} \text{Max } & Z_i(x) = [Z_1(x),\ Z_2(x),\ \cdots,\ Z_p(x)] \\ & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2-2}$$

To find this problem as follows, it becomes PIS of Z_1 by selecting objective function Z_1.

$$\begin{aligned} \text{Max } & Z_1(x) \\ & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2-3}$$

If we find this problem as follows, it becomes NIS of Z_1.

$$\begin{aligned} \text{Min } & Z_1(x) \\ & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2-4}$$

1.3 Structure and Induction of Membership Function

The decision maker indicates membership function as a satisfaction degree for numerous fuzzy goals. This can be indicated $\mu(x)$ as the real number at interval of (0, 1).

Such membership functions are linear membership function, exponential membership fucntion, hyperbolic membership function, hyperbolic inverse membership function, piecewise linear membership function [S 15], and its structure and solution induction is decided by membership function structure.

Piecewise linear membership function suggested by Charnes and Cooper is applied in this study, and membership function and objective function within the range of goal value of objective function applied by algorithm are researched as follows.

1.3.1 Membership Function within Goal Value of Each Objective Function

(HYPOTHESIS 1) Membership function of each objective function is assumed as piecewise linear membership, and piecewise linear membership function connected with goal value $\mu_i(Z_i)$ of free objective function is piecewise linear concave membership function.

Namely,

$$\{\mu_i(Z_i^{j+1})-\mu_i(Z_i^j)\}/(Z_i^{j+1}-Z_i^j)\leq\{\mu_i(Z_i^j)-\mu_i(Z_i^{j-1})\}/(Z_i^j-Z_i^{j-1}) \quad (2\cdot5)$$

is a goal value that objective function i exists between maximum value Z_i^U and the minimum value Z_i^L. [RN 11]

(HYPOTHESIS 2) If the value of Z_i^j is divided in detail and find $\mu_i(Z_i^j)$ and connect it by line, the assumed objective function $\mu_i(Z_i)$ becomes negative quadratic function of which the peak is 1

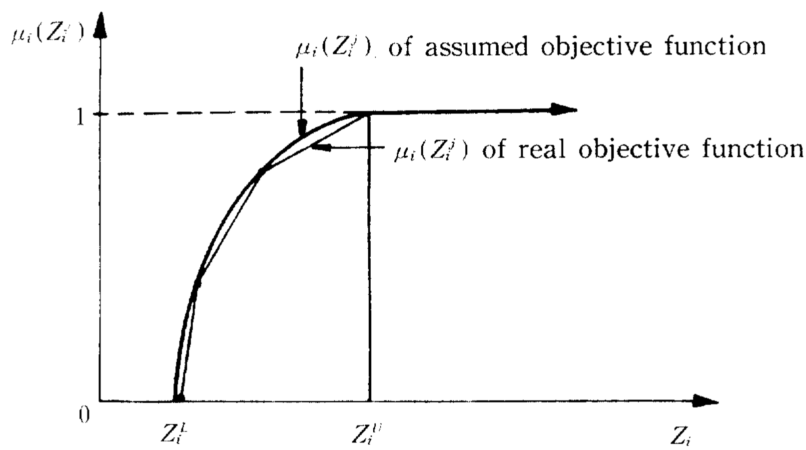


Figure 2-1 The maximum problem $\mu_i(Z_i^j)$

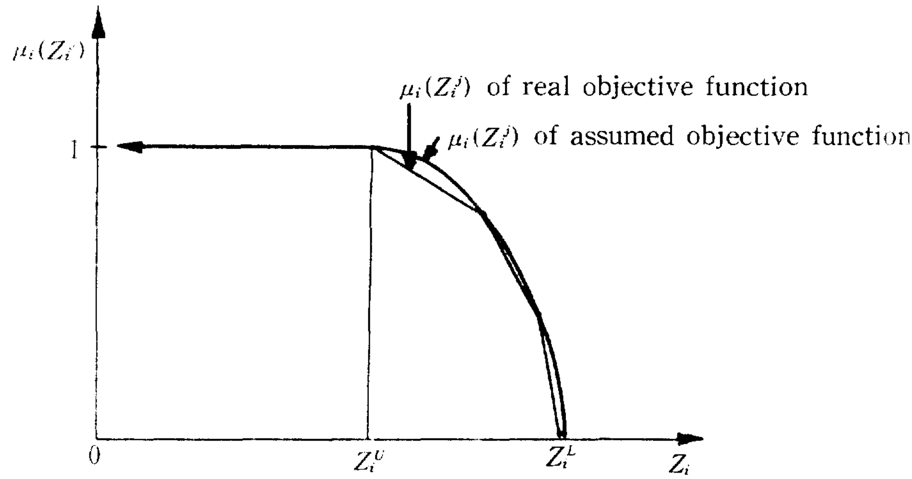


Figure 2-2 The minimum problem $\mu_i(Z_i^j)$

$$\therefore\ \mu_i(Z_i^j)=1-\left[\frac{Z_i^U-Z_i^j}{Z_i^U-Z_i^L}\right]^2 \tag{2-6}$$

1.3.2 Membership Function of Objective Function

In general, a decision maker expects that each objective function becomes larger than aspiration level. And as shown an figure 2-3, when a membership function of possible goal function is lined to piecewise linear Z_i exists between g_i and Z_i^U [H 4]

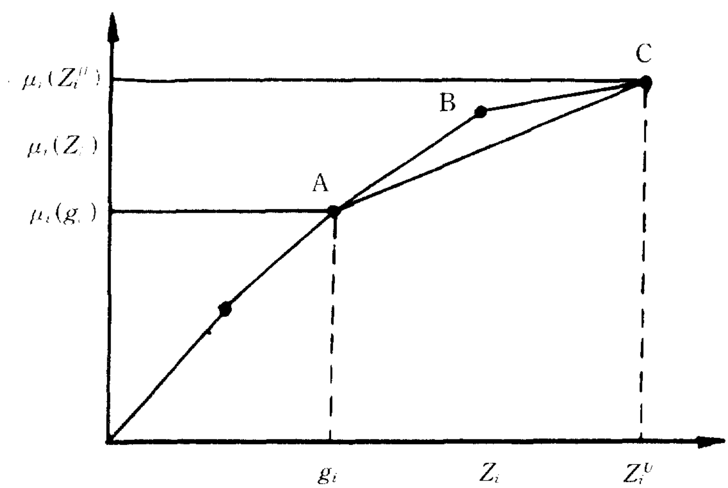


Figure 2-3 Membership Function of New Objective Function

In the model of fuzzy objective programming of Hannan, Rubin and Narasimhan [N 7], membership function $\mu_i(Z_i)$ of Z_i from $\overline{AB}$ and $\overline{BC}$ was found.

However, in the piecewise linear function, The membership function was induced for the a lineal distance between two points of A and C. This membership function induction formula is as follows.

$$\mu_i(Z_i)=\frac{\mu_i(Z_i^U)-\mu_i(g_i)}{Z_i^U-g_i}(Z_i-g_i)+\mu_i(g_i) \qquad (2\text{-}7$$

Here, Z_i is $g_i \leq Z_i \leq Z_i^U$ and this formula was found by interpolation as Z_i is between g_i and Z_i^U.

Z_i^U is the most optimistic vlaue of Z_i and Z_i^L is the worst pessimistic price of Z_i Therefore, $\mu_i(Z_i^U)=1$, $\mu_i(Z_i^L)=0$

1.4 Weighting Algorithm for Objective Function

Generally, the preferred solution of multiobjective decision making (MODM) can be obtained depending on the preference structure of desision maker (DM). Accordingly, we can the apply the preemptive priority when DM distinguishes his preferences in the multiobjective functions and the weights when DM doesn't distinguish his preferences.

This paper is based on the weighting method which is one of multiobjective decision making techniques. The overview of this algorithm is represented as follows. The methods used make a partial revision of proxy approach method which was proposed by Oppenheimer [0 8], and Sequential Proxy Optimization Technique(STOP) which was suggested by Sakawa [S 15].

1.4.1 Initialization

The new model V is represented as follows.

Model V : (2-8)

Max (or Min) $\alpha\ Z_i(x)$

s. t. $x \in X$

where α_i is the weighting vector

$\sum_{i=1}^{n} \alpha_i=1,\ \alpha_i>0,\ i=1, \cdots\cdots, n$

The initial weighting vector α_i may be selected arbitrarily (ex, $\alpha_1=\alpha_2=\alpha_3=1/3$).

The model V with the initial weighting vector α^k, $k=1$ generates a nondominated solution using the weighting method and we obtain the vector solution Z^1 and its α^k of the model V, where the index of k represents the number of iterations.

[THEOREM] Let Z be convex and let x^* be a nondominated solution. Then there exist nonnegative weights α_i such that at least one $\alpha_i > 0$ and x^* is an optimal solution of the model V

1.4.2 Assess Marginal Rate of Substitution

This paper relates directly to the global procedure of Barrager[B 1] and Keelin and the local procedure of Boyd [B 2] and Geoffrion, Dyer, and Feinberg. Both the global and local procedures are built upon one fundamental preference measure : The marginal rate of substitution(MRS). MRS is the trade off. [0 9]

MRS is a well-known first order approximation of an indifference curve. Mathematically, MRS is the negative of the slop of an indifference curve at a point defined as follows. [S 13]

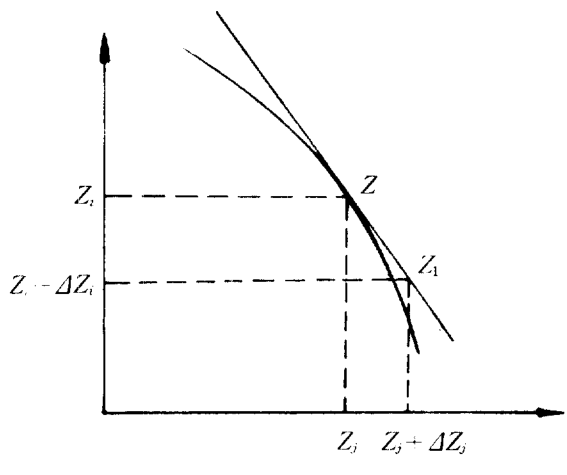


Figure 2-4 The negative slope of an indifference curve.

(Definition 2-2) At any Z, the amount of Z_i that the DM is willing to sacrifice to acquire an additional unit of Z_j is called the marginal rate of substitution [S 13] ;

$$m_{ij}(Z) = \frac{\delta p(Z)/\delta Z_j}{\delta p(Z)/\delta Z_i} = -\frac{dZ_i}{dZ_j}\Bigg|_{\substack{dP=0 \; dZ_r=0 \\ r \neq i,\; j}} \qquad (2\text{-}9)$$

The MRSs are assessed by representing the following prospects to DM

$Z = (Z_1, \cdots, Z_i, \cdots, Z_j, \cdots, Z_n)$
$Z^1 = (Z_1, \cdots, Z_i - \Delta Z_i, \cdots, Z_j + \Delta Z_j, \cdots, Z_n)$

for a small fixed ΔZ_j, ΔZ_i is varied until they are indifferent between Z and Z^1, respectively.

1.4.3 Local Proxy Preference Function

Concerning the convergency of the algorithm, it is necessary to assume some kind of preference (utility) functions, to this end, local proxy preference functions mentioned in [O 19] are used.

(Definition 2-3) Attribute Z_1, Z_2, $\cdots$, Z_n are the additive independence if preference over lotteries on Z_1, Z_2, $\cdots$, Z_n depend only on their marginal probability distributions and not on their joint probability distribution [K 6]

Using the deterministic additive independence condition $P(Z)=\sum P_i(Z_i)$ of Keeney et al., together with assumptions about a marginal rate substitution variation, Keelin [K 5] derives the following three local utility functions :

(1) sum-of-exponentials.

$$\text{If } \frac{-\delta m_{ij}(Z)/\delta Z_j}{m_{ij}(Z)}=W_j, \text{ then } P(Z)=-\sum a_i\cdot\exp(-W_iZ_i)$$

The constant a_1 can arbitrarily be set equal to one and remaining parameters a_2, $\cdots$, a_n, W_i, must be calculated from tradeoff assessment.

(2) sum-of-power ($\zeta_j\neq 0$)

$$\text{If } \frac{-\delta m_{ij}(Z)/\delta Z_j}{m_{ij}(Z)}=\frac{1+\zeta_j}{M_j+Z_j}, \text{ then } P(Z)=-\sum a_i\cdot(M_i+Z_i)^{\zeta_i}$$

Where M_i is a constant such that $M_i+Z_i>0$, $i=1$, $\cdots$, n. The parameters a_i must be calculated from tradeoff assessment.

(3) sum-of-logarithm.

$$\text{If } \frac{-\delta m_{ij}(Z)/\delta Z_j}{m_{ij}(Z)}=\frac{1}{M_j-Z_j}, \text{ then } P(Z)=\sum a_i\cdot \ell n(M_i-Z_i)$$

where M_i is a constants such that $M_i-Z_i>0$, $i=1$, $\cdots$, n, must be calculated from tradeoff assessment.

For these three types of proxy functions, the constant a_1 can arbitrarily be set equal to one in $P(Z)$. The remaining parameters can be calculated from the MRS assessment. For the sum-of-exponentials or the sum-of-powers proxy, $n-1$ MRS at each of two points plus a single MRS at third point are required to fit the $2n-1$ parameters ; whereas the sum-of-logarithms proxy requires only $n-1$ MRS at any point to fit the $n-1$ parameters.

The numerical MRS actually assessed relates to the sum-of-logarithm parameters a_i and a_j in the following way ;

$$m_{ij}=\frac{\delta P(Z)/\delta Z_i}{\delta P(Z)/\delta Z_j}=\frac{a_i(M_j-Z_j)}{a_j(M_i-Z_i)}$$

$j=1, \cdots, n$

1.4.4 Stopping Rule

Deciding when to stop this algorithm, we obtain the discrepancy index d^k between the real disposable gradient a^k and the desirable gradient W_D^k. The discrepancy index d^k means difference for the direction gradients between previous iteration and current iteration. This can be achieved by their normalized scalar product, i.e.,

$$d^k=\frac{a^k \cdot W_D^k}{\|a^k\| \cdot \|W_D^k\|} \qquad (2\text{-}10)$$

If the discrepancy vanishes, i.e., if a^k is colinear with W_D^k, this algorithm stops.

That is, if $1-d^k \leq \varepsilon$(error bound) stop, as the solution is the preferred on which satisfied by the DM. Otherwise, the next step is accomplished.

There is a question which necessitates a skill of decision maker. It is a question of selection for ε value. If the ε value decreases, then the solution of this algorithm is more exact. On the other hand, the computational efforts increase. That is, the tradeoffs exist between the computational efforts and the quality of solutions. Accordingly, decision maker selects a proper ε value case by case.

Moreover, if $\|P(Z)$ generated by the n iteration$-P(Z)$ by the $(n-1)$ iteration$\|$ is less than δ_1, this algorithm stops. Otherwise, the decision maker selects the new weights.

1.4.5 Generate the New Weighting Vector

If the iteration of this algorithm does not stop in the stopping rule, it is necessary to generate the new weighting vector. The new weighting vector is generated by a^k, W_D^k and reasonable β^k.

$$a^{k+1}=(1-\beta^k)\cdot a^k+\beta^k \cdot W_D^k, \qquad 0\leq\beta^k\leq 1$$

The β^k is determined by the utility function of DM.

The curve labeled proxy is the slice of the proxy function along the line determined by a^k and W_D^k. This quantity $[P(W_D^k)-P(a^k)]$, can be negative or positive.

In this case, W_D^k overshoots the true maximum for enough to $[P(W_D^k)<P(a^k)]$. Since movement in the W_D^k direction quarantees improvement if small enough a step is taken [0 8], there must be a point a^{k+1} along the line determined by a^k and W_D^k, but

somewhere in between, such that $P(W_D^k) > P(\alpha^k)$. An optimization procedure called relaxation technique is used to find such a point α^{k+1}. For some β^k, $0 < \beta^k < 1$. [0 10]

The parameter β^k is increased until improvement is achieved. Figure (2-5) shows the actual change A', measured at α^{k+1}, has the same direction as predicted improvement $P(Z)$.

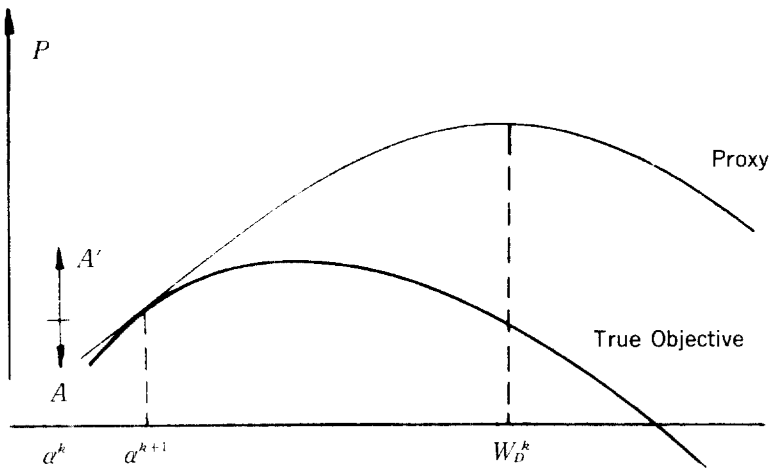


Figure 2-5 Relaxation Technique

Hence, decision maker selects the β^k to maximize $P(Z)$. Hence, $P(Z)$ is represented as follows :

$$P(Z) = \sum a_i \cdot l_n(M_i - Z_i)$$

$$\text{then,}\quad m_{ij} = \frac{\delta P(Z)/\delta Z_i}{\delta P(Z)/\delta Z_j} = \frac{a_i(M_j - Z_j)}{a_j(M_i - Z_i)}$$

The fixed small change $\Delta\beta^k$ is suggested to decision maker and the decision maker determines the β^k(i. e., $0 \le \Delta\beta^k \le \Delta 2\beta^k \le \cdots \le 1$).

If the decision maker selects the β^k as $\beta^k = 0$, the algorithm is stopped, because the utility of decision maker is not more improved.

The overview of this algorithm is represented in Figure(2-6) and each step of this algorithm is presented in a step-by-step manner.

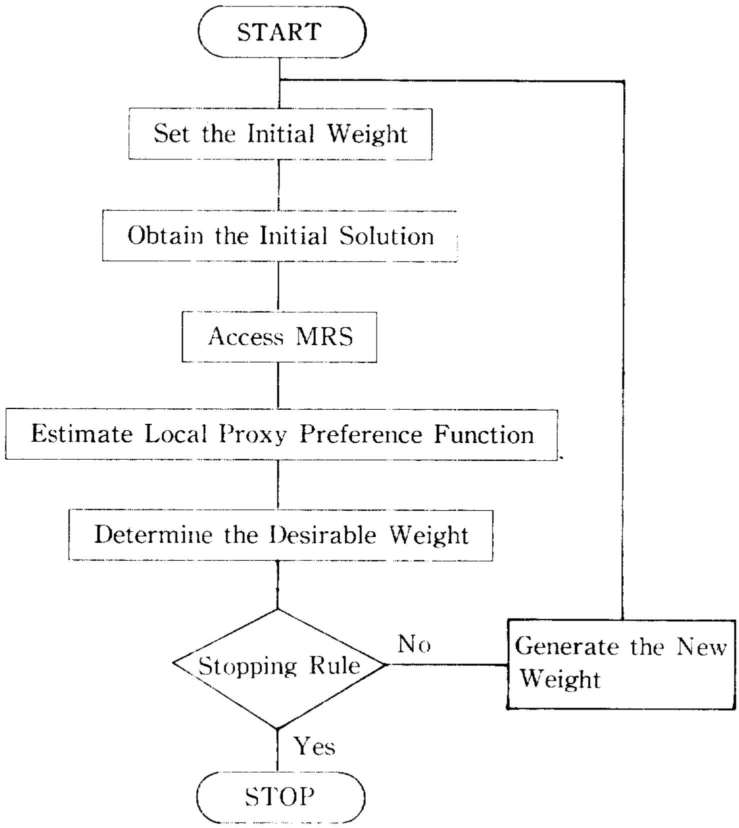


Figure 2-6 Flow Chart of Weighting Algorithm

2. Compromise Solution

2.1 Conception of Compromise Solution

This study accepted the interactive approach, and the interactive approach [S 15] (SGI16) of Sakawa and Sasaki was compromised and supplemented, and the compromise solution was found for the satisfaction of the decision maker.

(Definition 2-3) [Compromise Solution]:Compromise solution means satisfied membership function $\mu_i(g_i)$ of sapiration level g_i or the closest efficient solution among fuzzy efficient solution by the decision maker in the matters of mulitobjective decision making.

The closest efficient solution means the efficient solution that satisfies condition of Stopping Rule.

2.2 Stopping Rule

If membership function of new aspiration level is satisfied instead of membership function $\mu_i(g_i)$ of aspiration level presented in the first stage, the obtained solution will be ended as a compromise solution.

If all is not satisfied, aspiration level g_i may not be satisfied even though g_i is adjusted many times because of its impossibility of g_i's convertibility.

Therefore, if the efficient solution is suitable to Stopping Rule, the efficient solution will be stopped as the compromise solution. If not, new g_i should be made.

(1) Sum of Membership Function

As membership function assumed a negative quadratic function, it increases largely to increase g_i near Z_i^l than to reduce near Z_i^u.

Therefore, if not accomplished g_i of Z_i is increased a little, membership function becomes increased largely, and the total membership function becomes large comparing with the first membership function in general. And a decision maker will be satisfied with the solution if sum of membership function found by new g_i is bigger than the sum of membership function presented at first to a certain extent.

Namely, if the sum of all the membership function is bigger than the sum of membership function found by the first efficient solution and allowance level (T_α), it will be stopped as the compromise solution.

If T_α becomes small, it comes to be stopped generally as finite iteration.

(2) Sum of Deviation Rate

In case there is no solution to satisfy all $\mu_i(g_i)$, the decision maker will be satisfied with the closest efficient solution which is the minimum solution of the sum of deviation rate from new g_i.

And if the sum of deviation rate of each objective function about aspiration level is smaller than allowance level(T_β), the gained solution will be stopped as the compromise soultion.

(3) Sum of Efficient Solution Numbers

Probably there is no satisfied solution according to T_α, T_β from membership function sum and the sum of deviation rate. Therefore, to cut down unnecessary iteration, it should be stopped at a certain point of iteration. Consequently more than ten efficient solutions gained by iteration are considered as quite a bit comparatively, and the closest efficient solution among them will be stopped as the compromise solution.

If stopping rule is applied, the solution will be stopped as finite iteration even though the accurate iteration is not known.

3. Development of Interactive Fuzzy Goal Pragramming Algorithm

Step 1. Based on Weighting Algorithm for objective function in previous section, weights for multiobjective function are decided.

Step 2. Positive ideal solution(PIS) Z_i^+ and negative ideal solution(NIS) Z_i^- for each objective function (Z_i), and the efficient solution Z_i of multiobjective linear programming(MOLP) are obtained.
These will give solution for more matching objective range of decision maker's range of each goal values. In here, PIS is the best solution that each objective function(Z_i) could obtain under given constraints in MOLP. In contrast, NIS is the worst solution [SGI 17].

Step 3. Decision maker decides the range of upper and lower limit of Z_i^j after examining PIS, NIS, and the solution of MOLP. Upper limit(Z_i^L) will not be exceed PIS, and lower limit (Z_i^L) will not be decided less than NIS.

Step 4. Decision maker determines the first aspiration level g_i discussing with expert analyst that the object function should have a certain minmum value. g_i should be decided after examine Z_i^* and Z_i^-, and it is always less than Z_i^+.

Step 5. Membership function about goal value(Z_i) of which each objective function could elicit is as follows.

$$\mu_i(Z_i^j) = 1 - \left[\frac{Z_i^U - Z_i^j}{Z_i^U - Z_i^L}\right]^2$$

Here, $i = 1, \cdots, k$
$j = 1, \cdots, r$

Z_i^L means the worst pessimistic lower limit of decision maker's point of view among obtainable goal value(Z_i^j) of each objective function, and Z_i^l means the most optimistic upper limit among Z_i^j of earch objective function.

Here, $\mu_i(Z_i^U) = 1$, $\mu_i(Z_i^L) = 0$.

Step 6. Link to the piecewise linear continuous function of membership function μ_i (Z_i^j) for Z_i^j given in Step 5.

Step 7. In order to present the membership function between two points of the line, the membership function of each objective function which have a piecewise linear concave function is generated as follows.

$$\mu_i(Z_i) = \frac{\mu_i(Z_i^U) - \mu_i(g_i)}{Z_i^U - g_i}(Z_i - g_i) + \mu_i(g_i)$$
$$g_i \le Z_i \le Z_i^U$$

Step 8. The Interactive Fuzzy Goal Programming (IFGP) Model will be established as follows.

$$\text{Min} \sum_{i=1}^{k} W_i d_i^-$$
$$\text{s.t. } Ax \le b$$
$$\mu_i(Z_i) + d_i^- - d_i^+ = \mu_i(g_i)$$
$$d_i^-,\ d_i^+ \ge 0$$
$$x \ge 0$$

In $\sum_{i=1}^{k} w_i d_i$, w_i is the weight value for the membership function of each objective function, and will be given the weight value of each objective function which decided is Step 1. And the reason of minimizing the deviation variables d_i is that $\mu_i(Z_i)$ will be hoped to have greater value than $\mu_i(g_i)$ because Z_i was limited $g_i \le Z_i \le Z_i^U$ in Step 7.

About $\mu_i(g_i)$, Hannan treated it as assumed membership function, but in this article, the membership function $\mu_i(g_i)$ of aspiration level g_i is built the RHS of constraint.

Step 9. The fuzzy efficient solution is decided.

By the established model in Step 8, the fuzzy efficient solution is decided. If it is the first efficient solution of repeatedless, then delves the compromise solution in next Step.

If it is efficient solution delved by more than 2 iteration, then compromise solution is obtained by the Stopping Rule.

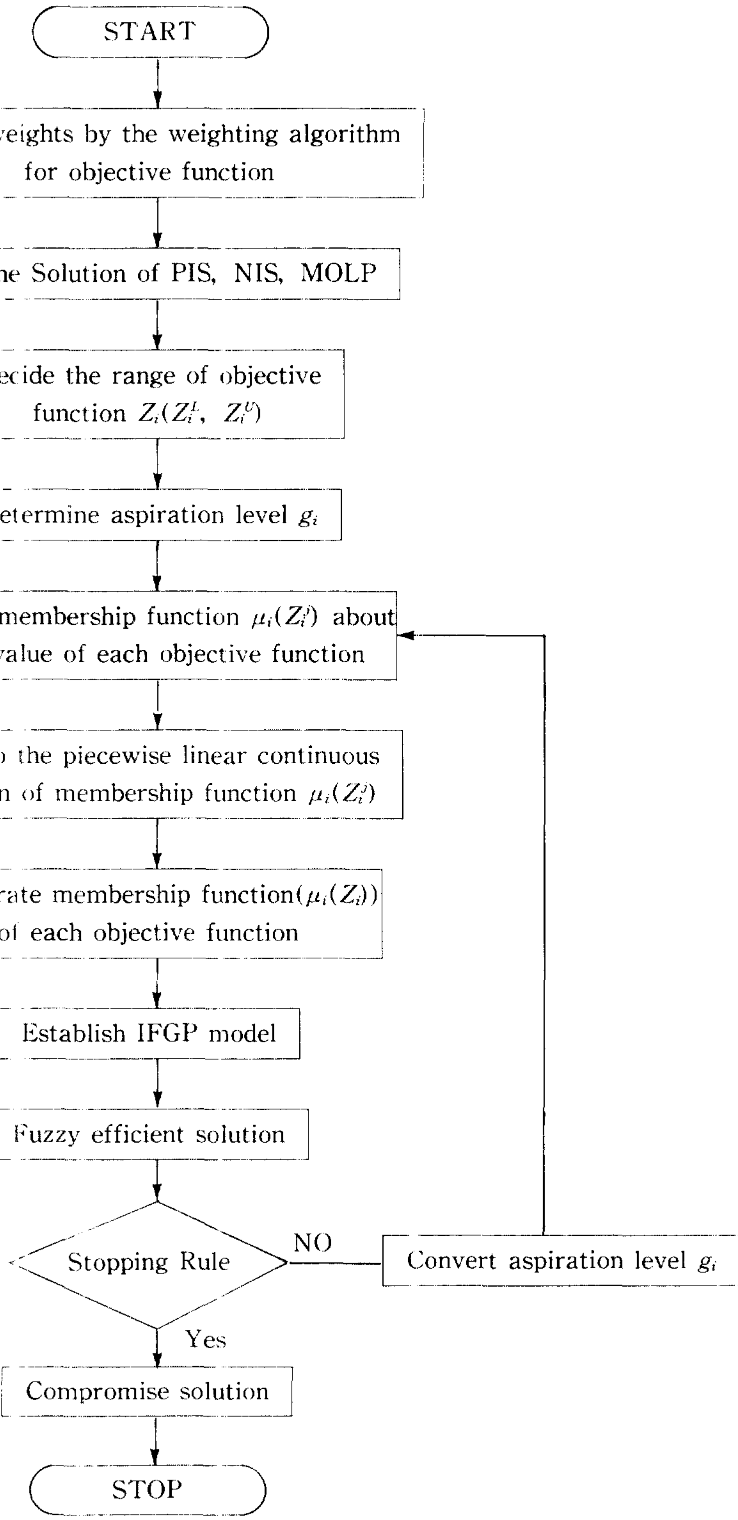
Step10. The compromise solution is obtained.

If the obtained solution satisfies all membership function $\mu_i(g_i)$ that was presented to decision maker's first aspiration level g_i, then it becomes the compromise solution and stop.

Otherwise, DM makes new aspiration level g_i and go to step 5, where g_i is to be reduced as follows

$$g_j = \sum \frac{Z_i - g_i'}{M_i} + Z_i$$

However ; $i \neq j$


START
Decide weights by the weighting algorithm for objective function
Find the Solution of PIS, NIS, MOLP
Decide the range of objective function $Z_i(Z_i^L, Z_i^U)$
Determine aspiration level g_i
Elicit membership function $\mu_i(Z_i^j)$ about goal value of each objective function
Link to the piecewise linear continuous function of membership function $\mu_i(Z_i^j)$
Generate membership function($\mu_i(Z_i)$) of each objective function
Establish IFGP model
Fuzzy efficient solution
Stopping Rule
NO
Convert aspiration level g_i
Yes
Compromise solution
STOP

III. An Illustrative Example

The following data show that each objective function in fuzzy.
Find compromise solution through interactive method to be close to aspiration level by using important degree of each objective function as weight and its presented aspiration level $g_1=5$, $g_2=4$, $g_3=3$

$$\text{Max } Z_1 = 3X_1 + X_2 + X_3$$
$$\text{Max } Z_2 = X_1 - X_2 + 2X_3$$
$$\text{Max } Z_3 = X_1 + 2X_2$$

$$\text{s.t. } 4X_1 + 2X_2 + 3X_3 \le 10$$
$$X_1 + 3X_2 + 2X_3 \le 8$$
$$X_3 \le 5$$
$$X_1,\ X_2,\ X_3 \ge 0$$

(1) Find the fuzzy efficient solution
The result found by the first iteration solution of IFGP is as follows.

Table 3-1 The first iteration solution of IFGP

decision variable	(+) deviation variable	(−) deviation variable	objective function	$\mu_i(Z_i)$
$X_1=2.1981$	0.2474	0	$Z_1=6.767$	$\mu_1(Z_1)=0.909$
$X_2=0.4010$	0	0	$Z_2=1.9771$	$\mu_2(Z_2)=0$
$X_3=0.0000$	0	0	$Z_3=3.0001$	$\mu_3(Z_3)=0.48$

As apiration level $\mu_2(g_2)=0.69$ is not satisfied, the value of g_i should be reduced and adjusted and it returns to step 5.
The result found by the second iteration solution of IFGP is as follows. (Table 3-2)

Table 3-2 The second iteration solution of IFGP

decision variable	(+) deviation variable	(−) deviation variable	objective function	$\mu_i(Z_i)$
$X_1=1.3333$	0.1033	0	$Z_1=5.8332$	$\mu_1(Z_1)=0.79$
$X_2=0.8333$	0	0	$Z_2=2.5$	$\mu_2(Z_2)=0.21$
$X_3=1.0000$	0	0	$Z_3=3$	$\mu_3(Z_3)=0.48$

Find the compromise solution at stage of Stopping Rule because it is efficient solution found by the second iteration.

(2) Find the compromise solution

It satisfies membership function $\mu_i(g_i) = (0.69,\ 0.21,\ 0.48)$ of aspiration level g_i presented newly by the result of (Table 3-2). And this solution is stopped as compromise solution by Stopping Rule.

FGP applied important degree of each objective function as preemptive priority while IFGP applied important degree of objective function as weights.

It is difficult to have an objective comparison because the range of each objective function is FGP and that of IFGP are applied differently.

Therefore, for the objective comparison, IFGP algorithm is applied preemptive priority and objective function range of FGP. The result of applied algorithm is as (Table 3-3).

As results shown on (Table 3-3), the time required and calculated of CPU was shorten in the model of algorithm by the use of brief membership function induction of interpolation while complicated piecewise linear membership function of Charnes and Cooper required more time in FGP.

Table 3-3 Comparison of FGP with IFGP applied preemptive and range of objective function of FGP, and IFGP

Fuzzy Goal Programming	IFGP applied preemptive priority and range of objective function of FGP	Interactive Fuzzy Goal Programming
Min $M_1e_1^- + M_2e_2^- + M_3e_3^-$	Min $M_1d_1^- + M_2d_2^- + M_3d_3^-$	Min $0.3d_1^- + 0.3d_2^- + 0.3d_3^-$
s. t.	s. t.	s. t.
$\mu_1(Z_1) + e_1^- - e_1^+ = 0.4$	$\mu_1(Z_1) + d_1^- - d_1^+ = 0.4$	$\mu_1(Z_1) + d_1^- - d_1^+ = 0.69$
$\mu_2(Z_2) + e_2^- - e_2^+ = 0.3$	$\mu_2(Z_2) + d_2^- - d_2^+ = 0.3$	$\mu_2(Z_2) + d_2^- - d_2^+ = 0.21$
$\mu_3(Z_3) + e_3^- - e_3^+ = 0.4$	$\mu_3(Z_3) + d_3^- - d_3^+ = 0.4$	$\mu_3(Z_3) + d_3^- - d_3^+ = 0.48$
$4X_1 + 2X_2 + 3X_3 \leq 10$	$4X_1 + 2X_2 + 3X_3 \leq 10$	$4X_1 + 2X_2 + 3X_3 \leq 10$
$X_1 + 3X_2 + 2X_3 \leq 8$	$X_1 + 3X_2 + 2X_3 \leq 8$	$X_1 + 3X_2 + 2X_3 \leq 8$
$X_3 \leq 5$	$X_3 \leq 5$	$X_3 \leq 5$
$3X_1 + X_2 + X_3 + d_1^- - d_1^+ = 5$	$x_j \geq 0,\ j = 1,\ 2,\ 3$	$x_j \geq 0,\ j = 1,\ 2,\ 3$
$3X_1 + X_2 + X_3 + d_2^- - d_2^+ = 6$	$d_i^+,\ d_i^- \geq 0,\ i = 1,\ 2,\ 3$	$d_i^+,\ d_i^- \geq 0,\ i = 1,\ 2,\ 3$
$X_1 - X_2 + 2X_3 + d_3^- - d_3^+ = 4$		
$X_1 + 2X_2 + d_4^- - d_4^+ = 4$		
$x_j \geq 0,\ j = 1,\ 2,\ 3$		
$d_i^+,\ d_i^- \geq 0,\ i = 1,\ 2,\ 3,\ 4$		
$e_i^+,\ e_i^- \geq 0,\ i = 1,\ 2,\ 3$		

[Table 3-4] shows the result that decision maker's intention is completely not reflected in MOLP and aspiration level for decision maker's satisfaction is $\mu_1(Z_1)=0.4$, $\mu_2(Z_2)=0.3$, $\mu_3(Z_3)=0.4$, with exception of $\mu_3(Z_3)=0.164$, not-aspiration level of membership function in FGP.

In this algorithm, the preemptive priority of FGP and range of each objective function should be same and the solution under not-interactive condition and that of FGP come to be same.

Therefore, this algorithm was demonstrated to find solution in simple method.

Table 3-4 Comparison of results with other algorithm

MOLP	Fuzzy Goal Programming	IFGP applied preemptive priority order range of objective function of FGP	Interactive Fuzzy Goal Programming
$X_1=2.5$	$X_1=0.2$	$X_1=0.71$	$X_1=1.3333$
$X_2=0.0$	$X_2=0.85$	$X_2=0.85$	$X_2=0.8333$
$X_3=0.0$	$X_3=1.82$	$X_3=1.82$	$X_3=1.0000$
$Z_1=7.5$	$\mu_1(Z_1)=0.4$	$\mu_1(Z_1)=0.46$	$\mu_1(Z_1)=0.79$
$Z_2=2.5$	$\mu_2(Z_2)=0.3$	$\mu_2(Z_2)=0.44$	$\mu_2(Z_2)=0.21$
$Z_3=2.5$	$\mu_3(Z_3)=0.164$	$\mu_3(Z_3)=0.3$	$\mu_3(Z_3)=0.48$
	$Z_1=4.8$	$Z_1=4.8$	$Z_1=5.8332$
	$Z_2=3.5$	$Z_2=3.5$	$Z_2=2.5$
	$Z_3=2.41$	$Z_3=2.41$	$Z_3=3$

However, membership function $\mu_3(Z_3)=0.3$ of objective function Z_3 is beyond aspiration level $\mu_3(Z_3)=0.4$ because interactive method was not accepted. The solution of FGP and IFGP algorithm are same but membership functions are different because the induction formula is different.

Compromise solution of IFGP algorithm which used the important degree of objective function is not satisfied with aspiration level $\mu_2(Z_2)$ as a result of the first iteration, and it was the second iteration to make $g_2=2.5$ which satisfies aspiration level $\mu_1(g_1)=0.69$, $\mu_2(g_2)=0.21$, $\mu_3(g_3)=0.48$.

Therefore, the characteristics of this algorithm shortens the required time of CPU because there is few the number of constraints comparing with the existing FGP, and it seems that it reflects in each objective function of decision maker's satisfaction degree completely.

Ⅳ. Conclusion

The major findings in this research as characteristics of interactive fuzzy goal programming could be capsuled as follows :

(1) In the traditional algorithm, the weight of objective function is decided by the decision maker's experience. Howbeit in this research, we decided it by employing the MRS and local proxy preference in the hope of expressing the importance of objective function.

(2) In the established algorithm membership function of objective function was determined by decision maker's ambiguous experiences, while we presented the systematic methodology including membership function, because we postulated nonlinear in this algorithm.

(3) It was simplified to elicit membership function which introduces the assumption of piecewise linear membership function in which elicits membership function for objective function.

(4) The algorithm in this study proposed NIS, PIS and the solution of MOLP for deciding the range of goals which corresponds to more practical in decision making under fuzzy environment.

(5) We delved deeper into the decision maker through the interactive approach in order to grasp compromise solution and suggested stopping rule.

(6) On behalf of calculating modus, we employed LP program to grasp weight value of objective function. And we utilized the simplex method of goal programming for the interactive fuzzy goal programming.

For the further study we contemplate the following caveats.

(i) To maker easier decision on mass with decision support system.

(ii) To present the objective mathematic model by which we can employ the diagram of membership function pertaining to nonlinear programming.

(iii) To find out membership function and aggregate operator which might be applicable to characteristics of fuzzy programming.

(iv) To make it easy to understand the result of solution by using graphics.

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