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A New Formulation of System Reliability for Consecutive *k* out of *n* Structure with Sink-Source Pole

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ABSTRACT

The derived expressions and computations of the system reliability in the consecutive k out of n failure structure with sink-source pole are discussed and a new simplified formulation to compute the reliability of consecutive k out of n^* failure system is presented, and the relations with system reliability when the number of first consecutive falied components from source is greater than k are presented in this paper. The new simplified formulation is illustrated in the numerical experiments.

1. Introduction

A consecutive k out of n structure is a system of components in sequence such that the system fails if some k consecutive components are all fail. The often quoted examples for

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such systems are telecommunication, Oil and water pipeline systems. An example is an oil pipeline system with n pump stations. Source is Inchon, station 4(Bupeung), 3(Buchon), 2(Orudong), 1(Youngdeungpo), and sink is Yangjaedong. The sink can be supplied by both stations 1 and 2, station 1 can be received by both stations 2 and 3, and station 2 can be received by both stations 3 and 4, Also Source(Inchon) can send oil to the station 4 and 3. etc. Thus, when station 2 is failed, the oil pipeline system still is able to flow the oil from station 1 to station n. However, if both stations 2 and 3 are failed, that is, when two consecutive pump stations are down, then the oil flow is stopped and the system fails.

This example is what we call the consecutive 2 out of n failure structure, because any two consecutive components are a cut set [2], [5], [6]. Generally, a system with n components in sequence is called consecutive k out of n failure system if the system fails whenever k consecutive components fail [1], [3]. In this example some object, be it a flow, is to be relayed from a source to a sink through a sequence of intermediate stations. Now care should be taken as to if the source and the sink are also considered components of the systems, i.e., whether they serve the same function as the intermediate stations. In the oil or water pipeline system example and the telecommunication system example, source, sink and the intermediate stations are all the same kind of relayed stations. Thus both source and sink are considered components of the systems.

This paper studies several system reliability problems about the consecutive k out of n failure structure related with the state of source and sink pole through the derivation of the rotative formula based on the idea of experimenting the first sequence of consecutive k out of n failure structure.

2. Notation

n^*	: number of stations including sink-source pole
n	: number of stations excluding sink-source pole
k	: minimum number of consecutive failed stations excluding sink-source pole
I_i	: station i ($i = 1, 2, \cdots, n.$)
P, Pr	: Station reliability ($P_i = P$), Just probability
q	: Unreliability of station ($q_i = q$)
$L(p, k, n)$	: a consecutive k out of n F structure of n $i.i.d.$ stations with reliability p
$L^*(p, k, n^*)$	: a consecutive k out of n F structure with sink & source pole of n $i.i.d.$stations with reliability p
$\Phi(x)$	: structure function of the system.
X_i	: state of station i ($i = 1, 2, \cdots, n.$)

X	: Vector of station states
U	: random variable indicating the index of first 0 in vector of station states(X)
V	: random variable indicating the index of first 1 after U position in X.
C_i	: Cutset i (minimal cutset).
I_0, I_{n+1}	: Source-Pole, Sink-Pole
X_0, X_{n+1}	: State of Source-Pole, State of Sink-Pole
Re	: System Reliability
CN	: The number of first consecutive failed components from source

3. Generalized Formulation Using the Minimal Cutset.

Consider a system consisting of n components(stations), and suppose that each component (station) is either operating or has failed. To indicate whether or not the ith component (station) is operating, we define the indicator variable X_i by

$$X_i = \begin{cases} 1, & \text{if the ith component(station) is operating} \\ 0, & \text{if the ith component(station) has failed} \end{cases}$$

The vector X indicates which of the components(stations) are operating and which have failed.

We further suppose that whether or not the system as a whole is operating is completely determined by the state vector X.

Specifically, it is suppose that there exists a function $\Phi(x)$ such that

$$\Phi(x) = \begin{cases} 1, & \text{if the system is operating when the state vector is } X \\ 0, & \text{if the system is failed when the state vector is } X \end{cases}$$

(1) For k = 1.

This case is like as series system, generally and clearly, any permutation is invariant optimal. The number of minimal cutset is n, and the size of cutset is just $k = 1$.

The minimal cutsets are $C_0 = \{ I_0 \}$, $C_1 = \{ I_1 \}$, $C_2 = \{ I_2 \}$, $\cdots$, $C_n = \{ I_n \}$, $C_{n+1} = \{ I_{n+1} \}$. We have

$$\begin{aligned} \Phi(x) &= \max(X_0) \cdot \max(X_1) \cdots \max(X_n) \cdot \max(X_{n+1}) \\ &= \min(X_0, X_1, \cdots, X_n, X_{n+1}) \\ &= \prod_{i=0}^{n+1} X_i \end{aligned} \qquad (1)$$

Hence, $ReL^*(p, k, n^*) = Pr\{\Phi(x) = 1\} E[\Phi(x)] = \prod_{i=0}^{n+1} P_i = P^{n+2}$ (2)

(2) Case for $k = 2$.

(2-1) For $k = 2$, the number of minimal cutsets are $n - k + 1$.

$$ReL^*(p, k = 2, n^*) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k = 2, n)$$
$$= E[\max(X_0 \cdot X_{n+1}) \cdot \max(1, 2) \cdot \max(2, 3) \cdots \max(n-1, n)] \quad (3)$$

(2-2) For $k = 3$

$$ReL^*(p, k = 3, n^*) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k = 3, n)$$
$$= E[\max(X_0 \cdot X_{n+1}) \cdot \max(1, 2, 3) \cdot \max(2, 3, 4) \cdots \max(n-2, n-1, n)] \quad (4)$$

(2-3) For $k = 4$

$$ReL^*(p, k = 4, n^*) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k = 4, n)$$
$$= E[\max(X_0 \cdot X_{n+1}) \cdot \max(1, 2, 3, 4) \cdot \max(2, 3, 4, 5) \cdots \max(n-3, n-2, n-1, n)]$$
$$= E[\{1-(1-X_0 \cdot X_{n+1})\}\{1-(1-X_1)(1-X_2)(1-X_3)(1-X_4)\}\{1-(1-X_2)(1-X_3)(1-X_4)(1-X_5)\}\cdots \{1-(1-X_{n-3})(1-X_{n-2})(1-X_{n-1})(1-X_n)\}] \quad (5)$$

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(2-4) For $k = k$

$$ReL^*(p, k = k, n^*) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k, n)$$
$$= E[\max(X_0 \cdot X_{n+1}) \cdot \max(1, 2, \cdots, k) \cdot \max(2, 3, \cdots, k+1) \cdot \max(3, \cdots, k+1, k+2) \cdots \cdot \max(n-k, n-k+1, \cdots, n-1) \cdot \max(n-k+1, n-k+2, \cdots, n)]$$

$$= E[\{1-(1-X_0 \cdot X_{n+1})\}\{1-(1-X_1)\cdots(1-X_k)\}$$
$$\{1-(1-X_2)\cdots(1-X_k)(1-X_{k+1})\}\cdot\cdots\cdot\{1-(1-X_{n-k})$$
$$(1-X_{n-k+1})\cdot\cdots\cdot(1-X_{n-1})\}\{1-(1-X_{n-k+1})$$
$$(1-X_{n-k+2})\cdot\cdots\cdot(1-X_n)\}]. \qquad (6)$$

Now let's consider about the number of first consecutive failed-components from source : $CN \geq k+1$, and the others are k.
if $CN = k+1$

$$ReL^*(p, k = n^*{}_{CN}) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k, n-1)$$
$$= E[\max(X_0 \cdot X_{n+1} \cdot \max(1, 2, \cdots, k)$$
$$\cdot \max(2, 3, \cdots, k+1) \cdot \max(3, \cdots, k+1, k+2)$$
$$\cdots \cdot \max(n-k, n-k+1, \cdots, n-1)] \qquad (7)$$

and hence $CN > k+i$ where $i = 1, 2, \cdots, n$

$$ReL^*(p, k = n^*{}_{CN}) = E[\max(X_0 \cdot X_{n+1})] \cdot ReL(p, k, n-i)$$
$$= E[\max(X_0 \cdot X_{n+1}) \cdot \max(1, 2, \cdots, k)$$
$$\cdots \cdot \max(n-k+1-i, n-k+2-i, \cdots, n-i)]. \qquad (8)$$

4. Simplified Rotative Formulation for System Reliability

In order to compute the reliability of consecutive k out of n^* Failure system with sink-source pole, all possible combinations of sequences of failed components with size less than k must be specified. This procedure of computing for system reliability becomes too complicate and dissy. So we show a comparatively simple rotative formulation to compute the system reliability of consecutive k out of n failure structure.

Let X be a n-vector such that the component X_i is 1 if the ith component is operating and 0 if the ith component has failed. Thus the vector $X = (X_1, \cdots, X_n)$ is called the state vector of all components in the system.

We further suppose that if or not the system as a whole is operating is completely determined by the state vector X. The derivation of the rotative formulation is subject to the experimenting the first sequence of consecutive $X_i = 0$ in the X vector. If the number of consecutive $X_i = 0$ in the first sequence is greater than or equal k, then the system is failed. If the number of consecutive $X_i=0$ in the first sequence is less than k, then the system reliability is a consecutive k out of n^* Failure system. For all $n < k$, the system reliability of a consecutive k out of n^* Failure system is P^2, that is, the system reliability of a consecutive k out of n Failure system is just 1 by definition. Now we may rotatively compute the reliability of consecutive k out of n^* Failure system for $n^*-2 \geq k$.

We can get the new rotative formula as following (with Sink-Source Pole) :

$$ReL^*(p, k, n^*) = \sum_{u=1}^{n-k+1} \sum_{u=r+1}^{n+k-1} ReL(p, k, n-v)\, P^{u+2}(1-P)^{v-u} + P^{n-k+3} \qquad (9)$$

if and if only $\quad ReL(p, k, n-v) = 1, \quad \text{if } k > n-v \geq 0$

$\quad ReL(p, k, n-v) = 0, \quad \text{if } n-v < 0$

⟨ Proof ⟩

$$\begin{aligned} ReL^*(p, k, n^*) &= \text{System is operating } (E(X) = Pr(\Phi(X)) = 1) \\ &= E[E(\Phi(x) \mid U=u, V=v] \\ &= \sum_u \sum_v E[\Phi(x) \mid U=u, V=v] \cdot Pr[U=u, V=v] \end{aligned}$$

Now, the right side of the above can be written

$$\begin{aligned} &\sum_u \sum_v E[\Phi(x) \mid U=u, V=v] \cdot Pr[U=u, V=v] \\ &= \sum_u \sum_v \sum_1 1 \cdot Pr[\Phi(x)=1 \mid U=u, V=v] \cdot Pr[U=u, V=v] \\ &= \sum_u \sum_v Pr[\Phi(x)=1 \mid U=u, V=v] \cdot Pr[U=u, V=v] \qquad (10) \\ &= \sum_u \sum_v \sum_1 1 \cdot \frac{Pr\{[\Phi(x)=1, U=u, V=v] \cdot Pr[U=u, V=v]\}}{Pr\{U=u, V=v\}} \\ &= \sum_u \sum_v \sum_1 1 \cdot Pr\{\Phi(x)=1, U=u, V=v\} \\ &= \sum_1 1 \sum_u \sum_v Pr\{\Phi(x)=1, U=u, V=v\} \\ &= \sum_1 1 \cdot Pr\{\Phi(x)=1\} \\ &= E[X] \end{aligned}$$

and the result is obtained. From equation (10), that is,

$$\sum_u \sum_v Pr[\ \Phi(x)=1 \mid U=u, V=v\ ] \cdot Pr[U=u, V=v\ ]$$

$$= \sum_{u=1}^{n-k+1} \sum_{v=u+1}^{n+k-1} Pr[\ \Phi(x)=1 \mid U=u, V=v\ ] \cdot Pr[U=u, V=v\ ] + P^{n-k+3} \qquad (11)$$

$$= \sum_{u=1}^{n-k+1} \sum_{v=u+1}^{n+k-1} Pr[\ \Phi(x)=1 \mid U=u, V=v\ ] \cdot (1-P)^{v-u} P^{u+2} + P^{n-k+3} \qquad (12)$$

As the explanation of notation, U is the random variable indicating the index of first 0 invector of station(component) states (X) and V is the random variable indicating the index of first 1 after U position in X in the cut set of the for the consecutive k out of n^* Failure System.

Hwan-Oh's table for the Indexing of U and V shows in Appendix, and by this Indexing table we may have the $Pr\{U=u, V=v\}$ is equal to the $(P^u(1-P)^{v-u})\ P^2$, and $Pr\{R > n-k+1\} = P^{n-k-1}$, that is, $Pr\{U > n^*-k+1\} = P^{n-k+3}$.

For example, in the consecutive $k=3$ out $n=5$ F System excluding sink-source pole,

$$\text{if} \left[\begin{array}{l} U=4 > n-k+1=3,\ Pr\{U=4\} = P^3(1-P)(1-P) + P^3(1-P)P \\ U=5 > n-k+1=3,\ Pr\{U=5\} = P^4(1-P) \\ U=6 > n-k+1=3,\ Pr\{U=6\} = P^5 \end{array} \right.$$

hence the $Pr\{U=3\} = Pr\{U=4\} + Pr\{U=5\} = Pr\{U=6\} = P^{n-k+1} = P^3$.

In case of including Sink-Source Pole, $Pr\{U > 3\} = P^{n-k+3} = P^5$.

Also we have $Pr\{\Phi(x)=1 \mid U > n-k+1\} = 1$. When $v \geq u+k$, the system is failed since the system abready has k failed components.

Hence $u+1 \leq v \leq r+k-1$, since $X_u = 0$, $X_v = 1$, the event of the consecutive k out of n^* : Failure System is operating now is equivalent to the event of a consecutive k out of (n^*-2-v) Failure System is operating, that is,

$$Pr\{\Phi(X) = 1 \mid U=u, V=v\} = ReL(p, k, n-v).$$

Therefore, the new rotative formulation of the system reliability is

$$ReL^*(p, k, n^*) = \sum_{u=1}^{n-k+1} \sum_{v=u+1}^{n+k-1} ReL(p, k, n-v) \cdot (1-P)^{v-u} P^{u+2} + P^{n-k+3}$$

if and if only $\quad ReL(p, k, n-v) = 1$, if $k > n-v \geq 0$

$\qquad\qquad\qquad ReL(p, k, n-v) = 0$, if $n-v < 0$

Hence the result is proven

5. Numerical Experiment and Result

For the computation procedure of rotative formula, Let's continuously compute such that

Experiment 1.

When $k = 2, n^* > 3$

$$\begin{aligned} ReL^*(p, 2, 4) &= ReL(p, 2, 2) \cdot p^2 \\ &= ReL(p, 2, 0) \cdot p(1-p) + p^3 \\ &= 2p^3 - p^4 \end{aligned}$$

$$\begin{aligned} ReL^*(p, 2, 5) &= ReL(p, 2, 3) \cdot p^2 \\ &= ReL(p, 2, 1) \cdot p(1-p) \\ &\quad + ReL(p, 2, 0) \cdot p^4(1-p) + p^4 \\ &= p^3 + p^4 - p^5 \end{aligned}$$

$$\begin{aligned} ReL^*(p, 2, 6) &= ReL(p, 2, 4) \cdot p^2 \\ &\quad + ReL(p, 2, 2) \cdot p^3(1-p) \\ &\quad + ReL(p, 2, 1) \cdot p^4(1-p) \\ &\quad + ReL(p, 2, 0) \cdot p^5(1-p) + p^5 \\ &= 3P^4 - 2P^5 \end{aligned}$$

$$\begin{aligned} ReL^*(p, 2, 7) &= ReL(p, 2, 5) \cdot p^2 \\ &= ReL(p, 2, 3) \cdot p^3(1-p) + ReL(p, 2, 2) \cdot p^4(1-p) \\ &\quad + ReL(p, 2, 1) \cdot p^5(1-p) + ReL(p, 2, 0) \cdot p^6(1-p) + p^6 \\ &= p^4 + 3p^5 - 4p^6 + p^7 \end{aligned}$$

$$\begin{aligned} ReL^*(p, 2, 8) &= ReL(p, 2, 6) \cdot p^2 \\ &= ReL(p, 2, 4) \cdot p^3(1-p) + ReL(p, 2, 3) \cdot p^4(1-p) \\ &\quad + ReL(p, 2, 2) \cdot p^5(1-p) + ReL(p, 2, 1) \cdot p^6(1-p) \\ &\quad + ReL(p, 2, 0) \cdot p^7(1-p) + p^7 \\ &= 4p^5 + 2p^6 - 2p^7 + p^8 \end{aligned}$$

Experiment 2.

When $k = 3, n^* > 4$

$$ReL^*(p,3,5) = ReL(p,3,3)\cdot p^2$$
$$= [ReL(p,3,1)\cdot p(1-p) + ReL(p,3,0)\cdot p(1-p)^2] + p^3$$
$$= 3p^3 - 3p^4 + p^5$$

$$ReL^*(p,3,6) = ReL(p,3,4)\cdot p^2$$
$$= [ReL(p,3,2)\cdot p^3(1-p) + ReL(p,3,1)\cdot p^3(1-p)^2]$$
$$+ [ReL(p,3,1)\cdot p^4(1-p) + ReL(p,3,0)\cdot p^4(1-p)^2] + p^4$$
$$= 2p^3 - 2p^5 + p^6$$

$$ReL^*(p,3,7) = ReL(p,3,5)\cdot p^2$$
$$= [ReL(p,3,3)\cdot p^3(1-p) + ReL(p,3,2)\cdot p^3(1-p)^2]$$
$$+ [ReL(p,3,2)\cdot p^4(1-p) + ReL(p,3,1)\cdot p^4(1-p)^2]$$
$$+ [ReL(p,3,1)\cdot p^5(1-p) + ReL(p,3,0)\cdot p^3(1-p)^2] + p^5$$
$$= p^3 + 3p^4 - 5p^5 + 2p^6$$

$$ReL^*(p,3,8) = ReL(p,3,6)\cdot p^2$$
$$= [ReL(p,3,4)\cdot p^3(1-p) + ReL(p,3,3)\cdot p^3(1-p)^2]$$
$$+ [ReL(p,3,3)\cdot p^4(1-p) + ReL(p,3,2)\cdot p^4(1-p)^2]$$
$$+ [ReL(p,3,2)\cdot p^5(1-p) + ReL(p,3,1)\cdot p^5(1-p)^2]$$
$$+ [ReL(p,3,1)\cdot p^6(1-p) + ReL(p,3,0)\cdot p^6(1-p)^2] + p^6$$
$$= 6p^4 - 8p^5 + 3p^6$$

Result when $K = k, n^* > k+1$

Defining the rule of the $K = k$, and $n^* > k+1$

$$ReL^*(p,k,n^* > k+1) = ReL(p,k,n^*-2)\,p^2$$
$$= [ReL(p,k,n^*-4)\cdot p(1-p)\cdot p^2$$
$$+ ReL(p,k,n^*-5)\cdot p(1-p)^2\cdot p^2 + \cdots$$
$$+ \text{To } k-1 \text{ Term of } ReL\cdot p^1(1-p)^{k-1}\cdot p^2]$$
$$+ [ReL(p,k,n^*-5)\cdot p^2(1-p)^2\cdot p^2$$
$$+ ReL(p,k,n^*-6)\cdot p^2(1-p)^2\cdot p^2 + \cdots$$
$$+ \text{To } k-1 \text{ Term of } ReL\cdot p^2(1-p)^{k-1}\cdot p^2]$$

$$\left.\begin{matrix}\cdot\\\cdot\\\cdot\\\cdot\\\cdot\end{matrix}\right] \text{To terms of } n^* - k - 1$$

$$\begin{aligned}
&+ [ReL(p, k, j = k-2) \cdot p^{n-k-1}(1-p) \cdot p^2 \\
&+ [ReL(p, k, j-1 = k-3) \cdot p^{n-k-1}(1-p)^2 \cdot p^2 + \\
&\cdots + ReL(p, k, 2) \cdot p^{n-k-1}(1-P)^{k-3} \cdot p^2 \\
&+ ReL(p, k, 1) \cdot p^{n-k-1}(1-P)^{k-2} \cdot p^2 \\
&+ ReL(p, k, 0) \cdot p^{n-k-1}(1-P)^{k-1} \cdot p^2] \\
&+ p^{n-k+1}
\end{aligned}$$

Now, let's take an example of case, Sink is Seoul and Source is Ulsan. There are 7 stations between Ulsan and Seoul, and the system fails whenever 3 consecutive stations fail. This is the system of consecutive 3 out of 7 with sink and source pole.

The reliabilities of each station is p, respectively.

That is $P_0 = P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = 0.9$. From the experiment(2) using the equation(9) we can get :

$$\begin{aligned}
ReL^*(p, 3, 9^*) &= [ReL(p, 3, 5) \cdot p^3(1-p) + ReL(p, 3, 4) \cdot p^3(1-p)^2] \\
&\quad + [ReL(p, 3, 4) \cdot p^4(1-p) + ReL(p, 3, 3) \cdot p^4(1-p)^2] \\
&\quad + [ReL(p, 3, 3) \cdot p^5(1-p) + ReL(p, 3, 2) \cdot p^5(1-p)^2] \\
&\quad + [ReL(p, 3, 2) \cdot p^6(1-p) + ReL(p, 3, 1) \cdot p^6(1-p)^2] \\
&\quad + [ReL(p, 3, 1) \cdot p^7(1-p) + ReL(p, 3, 0) \cdot p^7(1-p)^2] \\
&\quad + p^7 \\
&= 3P^4 + 4P^5 - 16P^6 + 14P^7 - 4P^8 \\
&= 0.80149176
\end{aligned}$$

If pumping power of the only source pole has the number of $k+1$(that is, 4), $k+2$(5), $k+3$(6), $k+4$(7), $k+5$(8)

$$\begin{aligned}
ReL^*(p, 3, 9^*) = ReL^*(p, 3, 9^*)_{c4} &= 6p^4 - 8p^5 + 3p^6 = 0.807003 \\
ReL^*(p, 3, 9^*)_{c5} &= p^3 + 3p^4 - 5p^5 + 2p^6 = 0.807732 \\
ReL^*(p, 3, 9^*)_{c6} &= 2p^3 - 2p^5 + 2p^6 = 0.80843 \\
ReL^*(p, 3, 9^*)_{c7} &= 3p^3 - 3p^4 + p^5 = 0.80919
\end{aligned}$$

$$ReL^*(p, 3, 9^*)_{c8} = ReL(p, 3, 7)c_6 \cdot P^2$$

$$= 1 \cdot P^2 = 0.81$$

6. Conclusion

The simplified formulation for system reliability represented in this paper has showed so scientific technique as comparing with traditional computation like as appling the minimal cut set or combinations between operating and failed component (station).

In order to use this traditional generalized method for computing the system reliability, all possible combinations of sequences of failed components (stations) with size less than k have to be specified. Since this computation procedure is too much dissy and messy, this paper has presented a simplified formulation to compute the system reliability of consecutive k out of n failure structure with sink-source pole, and also presented the relations with system reliability in the case that the number of first consecutive failed components number from source is greater than k.

Furthermore, Hwan-Oh table of appendix appeared the relations among the number of k and n, the value of u and v from the experimenting the first sequence of minimal cut set for consecutive k out of n failure structure.

Finally, we hope that the simplified formulation in this paper might be useful for the real field and further research is being pursued to consider stochastic processes in the system reliability view point.

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〈Appendix〉

Hwan-Oh Table

j = Component Number

Xu = ⓪

Xv = ①

$\underline{k = 2}$

$u \setminus j$	1	2	3	4	5	6	7	8	…	$n-k+1$	$n-k+2$
1	⓪	①									
2	1	⓪	①								
3	1	1	⓪	①							
4	1	1	1	⓪	①						
5	1	1	1	1	⓪	①					
6	1	1	1	1	1	⓪	①				
7	1	1	1	1	1	1	⓪	①			
⋮									⋮		
$n-k+1$	1	1	1	1	1	1	1	1	…1	⓪	①

$\underline{k = 3}$

$u \backslash j$	1	2	3	4	5	6	7	8	…	$n-k+1$	$n-k+2$	$n-k+3$
1	⓪	①							…			
	⓪	0	①									
2	1	⓪	①						…			
	1	⓪	0	①								
3	1	1	⓪	①					…			
	1	1	⓪	0	①							
4	1	1	1	⓪	①				…			
	1	1	1	⓪	0	①						
5	1	1	1	1	⓪	①			…			
	1	1	1	1	⓪	0	①					
6	1	1	1	1	1	⓪	①		…			
	1	1	1	1	1	⓪	0	①				
·									…			
·									…			
·									…			
$n-k+1$	1	1	1	1	1	1	1	1	…1	⓪	①	
	1	1	1	1	1	1	1	1	…1	⓪	0	①

$\underline{k = 4}$

$u \backslash j$	1	2	3	4	5	6	7	8	9	…	$n-k+1$	$n-k+2$	$n-k+3$	$n-k+4$
1	⓪	①								…				
	⓪	0	⓪							…				
	⓪	0	0	①						…				
2	1	⓪	①							…				
	1	⓪	0	①						…				
	1	⓪	0	0	①					…				
3	1	1	⓪	①						…				
	1	1	⓪	0	①					…				
	1	1	⓪	0	0	①				…				
4	1	1	1	⓪	①					…				
	1	1	1	⓪	0	①				…				
	1	1	1	⓪	0	0	①			…				
5	1	1	1	1	⓪	①				…				
	1	1	1	1	⓪	0	①			…				
	1	1	1	1	⓪	0	0	①		…				
6	1	1	1	1	1	⓪	①			…				
	1	1	1	1	1	⓪	0	①		…				
	1	1	1	1	1	⓪	0	0	①	…				
										…				
										…				
										…				
$n-k+1$	1	1	1	1	1	1	1	1	1	…1	⓪	①		
	1	1	1	1	1	1	1	1	1	…1	⓪	0	①	
	1	1	1	1	1	1	1	1	1	…1	⓪	0	0	①

$\underline{K = k}$

$u \setminus j$	1	2	3	4	5	…	k	$k+1$	$k+2$	…	$n-k$	$n-k+1$	$n-k+2$	$n-k+3$	…	n
1	⓪	①														
	⓪	0	①													
	⓪	0	0	①												
	⓪	0	0	0	①											
	·	·	·	·	0											
	·	·	·	·	·											
	·	·	·	·	·											
	⓪	0	0	0	0	…	①									
2	1	⓪	①													
	1	⓪	0	①												
	1	⓪	0	0	①											
	1	⓪	0	0	0	①										
	·	·	·	·	·	0										
	·	·	·	·	·	·										
	·	·	·	·	·	·										
	1	⓪	0	0	0	0	…	①								
3	1	1	⓪	①												
	1	1	⓪	0	①											
	1	1	⓪	0	0	①										
	1	1	⓪	0	0	0	①									
	·	·	·	·	·	·	0									
	·	·	·	·	·	·	·									
	·	·	·	·	·	·	·									
	1	1	⓪	0	0	0	0	…	①							
			·													
			·													
			·													
$n-k+1$	1	1									1	⓪	①			
	1	1									1	⓪	0	①		
	1	1									1	⓪	0	0		
	1	1									1	⓪	0	0		
	·	·									·	·	·	·		
	·	·	·	·	·	·	·	·	·	·	·	·	·	·		
	·	·									·	·	·	·		
	1	1									1	⓪	0	0	…	①