Journal of the KSQC
Vol.21, No.1, June 1993

1993년 6월 韓國品質管理學會誌 제21권 제1호

An Economic Life Test Sampling Plan for Repairable Products with Exponential Interfailure Time Distribution *

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ABSTRACT

In this article an economic life test sampling plan is considered for repairable products when the products in each lot have the same interfailure time distribution, but the mean time between failure (MTBF) of a lot varies from lot to lot according to a known prior distribution. A cost model is constructed which consists of test cost, accept cost, and reject cost. Determination of the optimal plan which minimizes the expected average cost per lot is discussed. Numerical examples are presented to illustrate the use of the proposed sampling plans and sensitivity analyses for parameters of the prior distribution are performed.

* This paper was supported in part by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1992.
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1. Introduction

When lifetime of a product is an important quality characteristic, sampling plans to determine acceptability of a product with respect to lifetime are called life test sampling plans. In life testing, a fixed number of items are often tested simultaneously and test continues until some fixed number of items on test fail (Type II censoring) or for some fixed period of time(Type I censoring).

Traditionally, life test sampling plans have largely been based upon statistical criteria. For exponential distribution, a test based on Type II censored data and one based only on the r-th order statistic were proposed by Epstein and Sobel(1953). Epstein(1954) proposed a test with Type II censoring and one with hybrid censoring which combines both Type I and Type II censoring. A two stage test with Type II censoring at each stage was proposed by Bulgren and Hewette(1973) and one with hybrid censoring by Fairbanks(1988). Sequential life test in the exponential case was considered by Epstein and Sobel(1955) and truncated sequential test was proposed by Aroian(1964).

Conducting life tests are relatively expensive because the tests are time consumming and destructive in that some or all of the products on tests fail during the tests. When prior information on quality of a product is available, sample size and/or test duration may be reduced by utilizing the prior information in designing life test sampling plans. Therefore, it is worthwhile to consider Bayesian life test sampling plans based on cost model. Costs related to life test sampling plans are test cost, accept cost, and reject cost. Test cost consists of the costs associated with the length of test duration and the number of test items. Accept cost means overall cost associated with the products in the accepted lot. Reject cost consists of scrap or reprocess costs for products in the rejected lot. Soland(1968) considered a Bayesian life test sampling plan based on cost model for the Weibull distribution with unknown scale parameter and Thyregod(1975) proposed a Bayesian sampling plan for one parameter exponential distribution with Type II censoring. Nigm and Ismail (1985) extended Thyregod's works to two parameter exponential distribution. Dunsmore and Wright(1985) proposed a Bayesian sequential sampling plan based on costs for exponential distribution. Sandoh and Fujii(1991) considered an economic design of life test plan with Type I censoring in a situation where only one testing machine is available and the machine tests items sequentially.

Lieberman(1985) noted that the test duration of the acceptance sampling plans in MIL-STD-781 tends to be long as reliability of products increases and suggested that a Bayesian alternative to MIL-STD-781 be constructed. Ascher(1987) pointed out that MIL-STD-781 is intended for the reliability testing of repairable products and in practice, it is usually, if not always, used solely for that purpose, rather than for the testing of nonrepairable products.

In MIL-STD-781, two types of acceptance sampling plans based on statistical criteria are included : fixed length test(FLT) and probability ratio sequential test(PRST).

In this study, Bayesian sampling plans for repairable products with hybrid censoring which is similar to the FLT in MIL-STD-781 is considered. It is assumed that after an accept/reject decision, products in the accepted lot are sold under the warranty policy guaranteeing a minimal lifetime for the products. For repairable products, the warranty policy commonly used is the failure-free policy. In this policy, the manufacturer pays the repair costs for all failures occurring during the warranty period. Karmarkar(1978) developed a model for estimating the wattanty costs assumming that the repair costs are the same for all failures. Nguyen and Murthy(1984) considered a model with the assumption that the repair cost depends on the number of repairs carried out. Failure-free policy proposed by Karmarkar (1978) is considered in the design of acceptance sampling plans for repairable products.

2. Model Formulation

We consider a situation where the products in each lot have the same interfailure time distribution, but the mean time between failure(MTBF) of a lot varies according to a known distribution. Suppose that lots of size N are submitted for testing and the interfailure times of the individual products have an exponential distribution with probability density function (p.d.f.)

$$f(t\mid\theta) = \begin{cases} \theta^{-1}e^{-t/\theta}, & t>0, \\ 0, & \text{otherwise.} \end{cases} \qquad (1)$$

The parameter θ is itself a random variable, varying from lot to lot according to the known distribution function $G(\theta)$. The conjugate prior for θ is an inverted gamma distribution with p.d.f.

$$g(\theta) = \begin{cases} b^{a}\Gamma(a)^{-1}\theta^{-(a+1)}e^{-b/\theta}, & \theta>0, \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

where a and b are known constants and $\Gamma(.)$ denotes the gamma function.

The test procedure with hybrid censoring is considered here and it can be summarized as follows :

A sample of size n is drawn at random from the production lot and placed on test simultaneously. Failures which occur during the test are repaired and test continues until either the r-th failure occurs or a fixed test time t_0 is reached. If the r-th failure occurs first, the lot is rejected and otherwise, the lot is accepted.

It is assumed that all repair actions take only negligible time and that if a lot is accepted, all products in that lot are accepted with the refurbishment of the sample products used on test as in the case of acceptance sampling plans in MIL-STD-781. It is also assumed that, after an accept/reject decision, products in the accepted lot are sold under the failure-free warranty policy in which the manufacturer pays the repair costs for all external failures occurring during the warranty period and those in the rejected lot are sold at a discounted price with no warranty or scrapped or reprocessed.

The following notations are used for the model formulation.

N : lot size
n : sample size
r : a fixed integer
t_0 : a fixed length of test time
t_r : r-th failure time
T^* : Min $\{t_0, t_r\}$, test duration
N_f : number of failures during the test
N_c : number of repairs during the test
w : warranty period
c_s : cost of sampling and putting a product on test
c_t : cost per unit of test time
c_d : repair cost per failure during the test
c_p : cost of rejecting a product
c_a : cost associated with an external failure
c_h : refurbish cost per product

For given n, r, and t_0, the probability of accepting a lot with MTBF θ is given by

$$\begin{aligned}\lambda(n, r, t_0 \mid \theta) &= Pr\{T^* = t_0\} \\ &= Pr\{N_f < r\},\end{aligned} \qquad (3)$$

where the probability density of N_f is

$$Pr(N_f = k) = \begin{cases} p(k : m) & \text{for} \quad k \le r-1, \\ 1 - \sum_{k=0}^{r-1} p(k : m) & \text{for} \quad k = r, \end{cases} \qquad (4)$$

where $m = nt_0/\theta$ and $p(k : m)$ denotes a Poisson p.d.f. with parameter m.

Since

$$T^* = \begin{cases} t_0 & \text{if} \quad N_f < r, \\ t_r & \text{if} \quad N_f = r, \end{cases}$$

the expected test time is obtained by

$$\begin{aligned} \rho(n, r, t_0 \mid \theta) &= E(T^* \mid \theta) \\ &= \sum_{k=0}^{r-1} E(t_0 \mid N_f = k) Pr(N_f = k) + E(t_r \mid t_r < t_0) Pr(t_r < t_0) \\ &= t_0 \sum_{k=0}^{r-1} p(k : m) + (\theta r/n)\{1 - \sum_{k=0}^{r} p(k : m)\} \end{aligned} \quad (5)$$

Number of repairs during the test is

$$N_e = \begin{cases} N_f, & \text{if} \quad N_f < r, \\ r-1, & \text{if} \quad N_f = r, \end{cases}$$

since the r-th failure will not be repaired in the case of rejection. Hence the expected number of repairs during the test is

$$\begin{aligned} \eta(n, r, t_0 \mid \theta) &= E(N_e \mid \theta) \\ &= \sum_{k=0}^{r-1} kp(k : m) + (r-1)\{1 - \sum_{k=0}^{r-1} p(k : m)\} \end{aligned} \quad (6)$$

The number of external failures per product during the warranty period w has a Poisson distribution with parameter w/θ. According to the failure-free warranty policy, the expected accept cost per product is $c_a w/\theta$.

From the above results, the average cost per lot is given by

$$\begin{aligned} \kappa(n, r, t_0 \mid \theta) &= c_s n + c_t \rho(n, r, t_0 \mid \theta) + c_d \eta(n, r, t_0 \mid \theta) \\ &\quad + N\{c_p + (c_h n/N + c_a w/\theta - c_p)\lambda(n, r, t_0 \mid \theta)\} \end{aligned} \quad (7)$$

Therefore the expected average cost per lot for a given (n, r, t_0) plan is

$$\kappa(n, r, t_0) = \int_0^\infty \kappa(n, r, t_0 \mid \theta) g(\theta) d\theta. \tag{8}$$

Let $z = nt_0$, then formula (8) can be rewritten as

$$\begin{aligned} K(n, r, z) &= \kappa(n, r, z/n) \\ &= c_s n + c_t R(r, z)/n + c_d U(r, z) + N\{c_p + H(n, r, z)\}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} R(r, z) &= \int_0^\infty n p(n, r, z/n \mid \theta) g(\theta) d\theta \\ &= \sum_{k=0}^{r-1} \{z - r(b+z)/(a+k-1)\}\beta(k : a, b, z) + rb/(a-1) - z\beta(r-1 : a, b, z), \end{aligned}$$

$$\begin{aligned} U(r, z) &= \int_0^\infty \eta(n, r, z/n \mid \theta) g(\theta) d\theta \\ &= r - 1 - \sum_{k=0}^{r-1} (r-k-1)\beta(k : a, b, z), \end{aligned}$$

$$\begin{aligned} H(n, r, z) &= \int_0^\infty [c_h n/N + c_a w/\theta - c_p]\lambda(n, r, z/n \mid \theta) g(\theta) d\theta \\ &= \sum_{k=0}^{r-1} \{c_a w(a+k)/(b+z) + c_h n/N - c_p\}\beta(k : a, b, z), \end{aligned}$$

and

$$\beta(k : a, b, z) = \frac{z^k \Gamma(a+k) b^a}{k!\,\Gamma(a)(b+z)^{a+k}}.$$

To find the sampling plan that minimizes $\kappa(n, r, t_0)$ is equivalent to finding the one that minimizes $K(n, r, z)$. For a given (n, r, z) plan, the expectation of the probability of acceptance is given by

$$\begin{aligned} L(r, z) &= \int_0^\infty \lambda(n, r, z/n \mid \theta) g(\theta) d\theta \\ &= \sum_{k=0}^{r-1} \beta(k : a, b, z). \end{aligned}$$

Let K_a and K_r be the expected cost pet lot for the special cases of acceptance and rejection without test ($n=0$), respectively. Then

$$K_a = c_a waN/b,$$

$$K_r = c_p N.$$

We note that, if the MTBF θ of a lot is known precisely, then the optimal decision is to accept the lot if $\theta \geq \theta^*$, and reject it otherwise, where $\theta^* = c_a w / c_p$.

3. The Optimal Sampling Plan

The optimal sampling plan can be obtained by minimizing formula(9) with respect to (n, r, z). We first determine the optimal number $n^*(r, z)$ for given r and z and then determine $z^*(r)$ for given r and finally determine r^* minimizing the expected average cost.

LEMMA 1 : For given r, $L(r, z)$ is decreasing in z and $R(r, z)$ and $U(r, z)$ are increasing in z.

PROOF : The lemma follows from the fact that

$$\partial L(r, z)/\partial z = -r\beta(r : a, b, z)/z < 0,$$

$$\partial R(r, z)/\partial z = \sum_{k=0}^{r-1} \beta(k : a, b, z)/z > 0,$$

and

$$\partial U(r, z)/\partial z = \sum_{k=0}^{r-2} (a+k)\beta(k : a, b, z)/(b+z) > 0, \qquad \text{Q.E.D.}$$

Lemma 1 implies that, for fixed r, the expectation of the probability of acceptance is decreasing in z and that the expected test time and the expected number of repairs during the test are increasing in z.

THEOREM 1 : Define $k(r, z) = c_t R(r, z) / \{c_s + c_h L(r, z)\}$ and n_0 = integer part of $\sqrt{k(r, z)}$. Then, for given r and z, the optimal sample size is

$$n^*(r, z) = \begin{cases} n_g & \text{if } n_g \leq N, \\ N & \text{if } n_g > N, \end{cases}$$

where

$$n_g = \begin{cases} n_0, & \text{if} \quad n_0(n_0+1) \ge k(r, z), \\ n_0+1, & \text{otherwise.} \end{cases}$$

PROOF : The expected cost (9) can be rewritten as

$$K(n, r, z) = \{c_s + c_h L(r, z)\}n + c_t R(r, z)/n + c_d U(r, z) + N\{c_p + H'(r, z)\} \qquad (10)$$

where $H'(r, z) = \int_0^\infty (c_a w/\theta - c_p) L(r, z \mid \theta) g(\theta) d\theta$.

For given r and z, the optimal sample size can be obtained by minimizing the first two terms in formula (10). It is clear that, for given r and z, $K(n, r, z)$ is unimodal with respect to n and has its minimum at $n = \sqrt{k(r, z)}$. Since $n^*(r, z)$ is integer-valued and

$$K(n_0, r, z) \le (>) K(n_0+1, r, z) \quad \text{if} \quad n_0(n_0+1) \ge (<) k(r, z),$$

the optimal sample size is given by $n^*(r, z) = n_g$ with the obvious restriction that $n \le N$. Q.E.D.

The following results follow directly from lemma 1.

LEMMA 2 : For given r, $n^*(r, z)$ is increasing in z and $n^*(r) \le \bar{n}(r)$ where $\bar{n}(r)$ is the integer part of $\sqrt{c_t rb/c_s(a-1)}+1$.

THEOREM 2 : Define $d(n, r, z) = c_p - c_h n/N - c_a w(a+r)/(b+z)$ and $\bar{z}(r) = c_a w(a+r)/\{c_p - c_h \bar{n}(r)/N\} - b$. For given r, (i) optimal $z^*(r) = 0$ when $d(\bar{n}(r), r, 0) \ge 0$ and (ii) $z^*(r) \le \bar{z}(r)$ when $d(\bar{n}(r), r, 0) \le 0$.

PROOF : We obtain

$$\partial H(n, r, z)/\partial z = -\frac{\Gamma(a+r) z^{r-1} b^a}{\Gamma(a)\Gamma(r)(b+z)^{a+r}} d(n, r, z).$$

(i) When $d(\bar{n}(r), r, 0) \geq 0$, $d(n, r, z) \geq 0$ for all $n \leq \bar{n}(r)$ and $z \geq 0$. Hence for given r, $H(n, r, z)$ is increasing in z. From this and the results of lemma 1, it follows that for given r, $K(n, r, z)$ is increasing in z for $z \geq 0$.

(ii) When $d(\bar{n}(r), r, 0) < 0$, $d(\bar{n}(r), r, z) \leq (>)\ 0$ for $z \leq (>)\ \bar{z}(r)$ and it follows that for $n \leq \bar{n}(r)$, $d(n, r, z) > 0$ for $z > \bar{z}(r)$. Hence, for given r, $K(n, r, z)$ is increasing in z for $z > \bar{z}(r)$ and this implies that $z^*(r) \leq \bar{z}(r)$ when $d(\bar{n}(r), r, 0) < 0$. Q.E.D

Theorem 2 implies that, when $d(\bar{n}(r), r, 0) \geq 0$, optimal decision is to accept the lot regardless of the sample outcome and when $d(\bar{n}(r), r, 0) < 0$, an upper bound for $z^*(r)$ is $\bar{z}(r)$. Hence, when $d(\bar{n}(r), r, 0) < 0$, a unidimensional search over $0 \leq z \leq \bar{z}(r)$ can be utilized to find the optimal value $z^*(r)$ for given r.

It is difficult to show analytically whether the expected cost is unimodal or not with respected to r. However numerical studies indicate that the expected cost is unimodal with respect to r. Therefore a unidimensional search over r is utilized for finding the optimal sampling plan. We can see that $\bar{z}(r)$ is increasing in r. Hence, from Theorem 2, it follows that a lower bound for the optimal value r^* is

$$\underline{r} = \text{Min}\{r \mid \bar{z}(r) > 0\}.$$

4. Numerical Examples

Life test sampling plans are being used for an electrical assembly used in a personal computer. The assemblies are manufactured in the batches of $N = 500$. For the products, failure-free warranty with $w = 120$ unit hours is offered considering warranty policies of other competitive companies which produce the same kind of products. It is known that the interfailure times of the products have an exponential distribution. Life test history shows that the MTBF, θ, of a lot can be regarded as having inverted gamma distribution with $a = 6$ and $b = 350^{-1}$. The estimated test costs, refurbish cost, external failure cost, and reject cost in dollars are $c_s = 1.5$, $c_t = 5.0$, $c_d = 3.5$, $c_k = 2.5$, $c_a = 18.0$, and $c_p = 42.0$. The optimal sampling plan is $(n^*, r^*, z^*) = (48, 31, 1545.4)$. Hence, for this case, the maximum test duration is $t_0 = 1545.4/48 = 32.2$ and the expected cost per lot is 17156.5. Optimal sampling plans for various lot sizes are given in Table 1.

For some selected combinations of the cost components and the parameters a and b, the optimal sampling plans and their expected cost per lot are given in Table 2. We can see that the differences in optimal expected costs for the same values of $E(\theta)$ are relatively small and that optimal sampling plans (n^*, r^*, z^*) are not influenced so much for small changs in cost components.

Table 1. Optimal Sampling plans for $(a, b) = (6, 1/350)$, $w_1 = 80$, $w_2 = 160$, $c_s = 1.5$, $c_t = 5.0$, $c_d = 3.5$, $c_h = 2.5$, $c_a = 18.0$, and $c_p = 42.0$,

Lot size N	n^*	r^*	t_0^*	Expected Cost per Item	$L(r^*, t_0^*$
-60	0	0	--	37.03	1.0
70	20	6	12.78	36.96	.703
80	22	7	13.10	36.70	.696
90	22	7	14.06	36.48	.693
100	24	8	15.05	36.29	.689
120	27	10	17.18	35.98	.686
140	30	12	18.92	35.74	.682
160	31	13	19.94	35.54	.682
180	32	14	21.00	35.38	.679
200	34	16	22.76	35.24	.679
250	38	19	24.45	34.97	.677
300	39	21	26.44	34.77	.677
350	42	24	28.19	34.62	.677
400	45	27	29.79	34.50	.676
450	47	29	30.70	34.40	.676
500	48	31	32.19	34.31	.676
600	51	35	34.35	34.18	.675
700	54	39	36.27	34.08	.675
800	57	43	37.96	33.99	.675
900	59	46	39.28	33.93	.675
1000	61	50	41.35	33.87	.675
1200	64	55	43.42	33.78	.675
1400	68	61	45.44	33.71	.674
1600	71	67	47.85	33.66	.674
1800	73	71	49.35	33.61	.674
2000	76	76	50.78	33.57	.674
2500	81	88	55.25	33.50	.674
3000	86	98	58.07	33.45	.674

Table 2. Optimal sampling plans for selected combinations of the cost components and the parameters a and b.

a	b	$E(\theta)$	$V(\theta)$	c_s	c_t	c_d	c_k	c_a	c_p	n^*	r^*	t_0^*	Expected Cost
4	180	60	1800	1.7	6.0	4.0	2.8	20.0	45.0	51	28	29.94	20041.0
4	180	60	1800	1.5	5.0	3.5	2.5	18.0	42.0	50	30	31.38	18448.8
5	240	60	1200	1.7	6.0	4.0	2.8	20.0	45.0	53	30	30.70	20099.4
5	240	60	1200	1.5	5.0	3.5	2.5	18.0	42.0	52	32	31.98	18475.0
6	300	60	900	1.7	6.0	4.0	2.8	20.0	45.0	54	31	30.99	20164.3
6	300	60	900	1.5	5.0	3.5	2.5	18.0	42.0	53	33	32.19	18512.5
5	280	70	1633	1.7	6.0	4.0	2.8	20.0	45.0	51	30	30.99	18846.6
5	280	70	1633	1.5	5.0	3.5	2.5	18.0	42.0	49	31	31.95	17236.4
6	350	70	1255	1.7	6.0	4.0	2.8	20.0	45.0	50	30	31.28	18794.9
6	350	70	1255	1.5	5.0	3.5	2.5	18.0	42.0	48	31	32.19	17155.0
7	420	70	980	1.7	6.0	4.0	2.8	20.0	45.0	50	30	30.95	18756.1
7	420	70	980	1.5	5.0	3.5	2.5	18.0	42.0	47	31	32.45	17093.0
6	400	80	1600	1.7	6.0	4.0	2.8	20.0	45.0	44	26	29.40	17342.3
6	400	80	1600	1.5	5.0	3.5	2.5	18.0	42.0	42	26	29.37	15749.8
7	480	80	1280	1.7	6.0	4.0	2.8	20.0	45.0	42	24	27.61	17191.0
7	480	80	1280	1.5	5.0	3.5	2.5	18.0	42.0	39	24	28.18	15580.5
8	560	80	1067	1.7	6.0	4.0	2.8	20.0	45.0	39	22	26.25	17056.6
8	560	80	1067	1.5	5.0	3.5	2.5	18.0	42.0	34	20	25.37	15430.4

The sensitivity of the expected cost to the parameters a and b, in the above example, is given in Figure 1. Instead of parameters (a, b), we use $(E(\theta), a)$ in Figure 1 because $E(\theta)$ gives the average quality level and $b=(a-1)E(\theta)$. The percentage errors (PE) are computed for selected combinations of a and b with other cost components fixed. PE is computed as

$$PE = \frac{\{EC_1(a, b) - EC_0(a, b)\}}{EC_0(a, b)} \times 100(\%) \qquad (11)$$

where $EC_0(a, b)$ is the expected cost of the optimal sampling plan using correct a and b (in the example, $a=6$, $b=350$ and $E(\theta)=70$) and $EC_1(a, b)$ is the expected cost of the optimal sampling plan using incorrect a and b. Figure 1 show that, in all cases, the percentage errors of the expected cost do not exceed 5% when $50 \le E(\theta) \le 90$. Hence, the sampling plan based on the expected cost per lot using inverted gamma prior distribution seems to be reasonably robust to small changes in the parameters of the prior distibution.

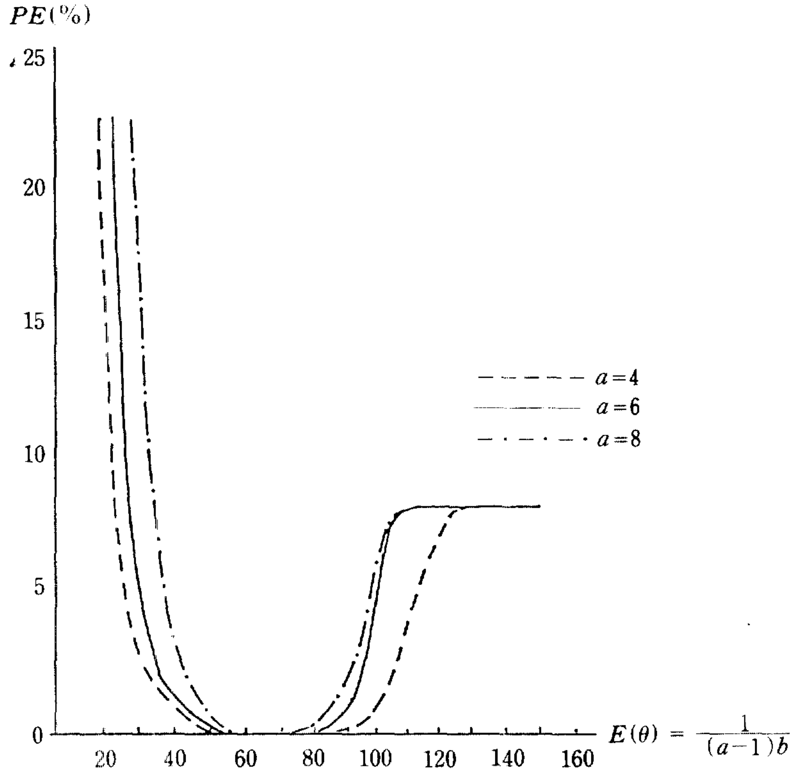

Figure 1. Percentage errors of expected cost for different values of a and b.

5. Conclusion

In this article an economic life test sampling plan for repairable products which are sold under the failure free warranty policy is proposed. A cost model is constructed which consist of test cost, accept cost, and reject cost and the method of determining optimal sampling plans is discussed. Inverted gamma distribution is considered as a prior distribution for MTBF of a lot. Results of numerical examples show that, the sampling plans based on the expected cost using inverted gamma prior distribution is reasomably robust to small changes in the parameters of the prior distribution. Economic design of life test sampling plans for products whose interfailure time distribution is other than exponential distribution may be considered to widen the applicability of the proposed sampling plans.

REFERENCES

1. Aroian, L. A. (1964), "*Some Comments on Truncated Sequential Tests for the Exponential Distribution*," Industrial Quality Control, 21, 309 – 312.
2. Ascher, H. (1987), "*MIL-STD-781C : A Vicious Circle*," IEEE Trans. on Reliability, R-36, 397 – 402.
3. Bulgren, W., and Hewette, J. (1973), "*Double Sample Test for Hypotheses about the Mean of an Exponential Distribution*," Technometrics, 15, 187 – 190.
4. Dunsmore, I. R., and Wright, D. E. (1985), "*A Decisive Predictive Approach to the Construction of Sequential Acceptance Sampling Plans for Life times*," Applied Statistics, 34, 1 – 13.
5. Epstein, B. (1954), "*Trunceted Life Tests in the Exponential Case*," Annals of Mathematical Statistics, 25, 555 – 564.
6. Epstein, B., and Sobel, M. (1953), "*Life Testing*." Journal of the American Statistical Association, 48, 485-502.
7. Epstein, B., and Sobel, M. (1955), "*Sequential Life Test in the Exponential Case*," Annals of Mathematical Statistics, 26, 82 – 93.
8. Fairbanks, K. (1988), "*A Two-Stage Life Test for the Exponential Parameter*." Technometrics, 30, 175 – 180.
9. Karmarkar, U. S. (1978), "*Future Cost of Service Contracts for Consumer Durable Goods*," AIIE Trans., 10, 380 – 387.
10. Lieberman, G. J. (1985), "*Highlights and Review of the Quality Control Workshop : New Developments and Practice for Sampling Inspection*. March 7 – 8, 1983," Naval Research Logistics Quarterly, 32, 113 – 117.
11. MIL-HDBK-781 (1987), Reliability Test Methods, Plans and Environments for Engineering Development, Qualification and Production, Dept. of Defense, Washington DC.
12. Nguyen, D. G., and Murthy, D.N.P. (1984), "*A General Model for Estimating Warranty Costs for Repairable Products*," IIE Trans., 16, 379 – 386.
13. Nigm, A. M., and Ismail, M. A. (1985), "*Bayesian Life Test Sampling plans for the Two Parameter Exponential Distribution*," Common. in Statist-Simula. Computa., 14, 691 – 707.
14. Sandoh, H., and Fujii, S. (1991), "*Designing an Optimal Life Test with Type I censoring*." Naval Research Logistics, 38, 23 – 31.
15. Soland, R. M. (1968), "*Bayesian Analysis of the Weibull Process with Unknown Scale Parameter and Its Application to Acceptance Sampling*," IEEE Trans. on Reliability, R – 17, 84 – 90.
16. Thyregod, P. (1977), "*Bayesian Single Sampling Plans for Life Testing with Truncation of the Number of Failures*," Scandinavian Journal of Statistics, 2, 61 – 70.