Journal of the KSQC
Vol.21, No.1, June 1993

1993년 6월 韓國品質管理學會誌 제21권 제1호

On the Validity of SN Ratio in Parameter Design

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ABSTRACT

In parameter design Taguchi analyzed a statistic which he called signal-to-noise(SN) ratio by using the experimental design technique. However he gave no justification for using SN ratios in the optimization procedure of parameter design. In this paper we discuss the validity of such SN ratios as proper statistics to be analyzed in parameter design. Moreover, using the real empirical data we examine the appropriateness of SN ratios, and we explain how transformation technique can be applied in parameter design as an alternative method of analysis.

1. Introduction

In manufacturing process, there are two types of factors which affect the product's quality characteristic. These are called control and noise fators. The control factors are those inputs to the production process which can be adjusted by the operator. Noise factors are those variables which are difficult, if not possible, to control. These noise factors are the

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main sources of variation in the product's quality characteristic. In the late 1970's, Taguchi (1986) introduced a new statistical quality control method, so called parameter design, to reduce such product variation cost-effectively.

As an illustration, let $\underline{\theta}$ denote the setting of the control factors and let $\underline{W}$ represent the set of noise factors in a manufacturing process of the product. The output (quality characteristic) Y is then determined by $\underline{\theta}$ and $\underline{W}$ through transfer function f, that is, $Y = f(\underline{W}; \underline{\theta})$. Then the setting of the control factors $\underline{\theta}$ is a set of parameters of the distribution of Y, and for a given $\underline{\theta}$ the noise factors generate the distribution of Y. The noise fators are assumed to be random variables so that Y is also a random variable that is assumed here to be continuous. The ideal value of Y is called a target value and we denote it by τ. Then the goal of parameter design is to achieve the minimum dispersion of Y with the mean of Y close to target value τ.

Statistically, Taguchi employed a loss function in explaining the aim of parameter design. A loss is incurred if Y differs from a target τ. The objective of parameter design is then to identify the optimal setting of control factors $\underline{\theta}$ that minimizes average loss caused by noise factors $\underline{W}$. In evaluating average loss, Taguchi used a qudratic loss function

$$L(y, \tau) = k(y-\tau)^2 \qquad (1.1)$$

, where k is some constant. The average loss $R(\underline{\theta})$ at the setting $\underline{\theta}$ is thus proportional to mean squared error,

$$R(\underline{\theta}) = E[L(y, \tau)] = E[k(y-\tau)^2] = k[\sigma^2(\underline{\theta}) + (\mu(\underline{\theta})-\tau)^2]. \qquad (1.2)$$

Since the average loss (1.2) consists of two parts of the bias and the variance, we have to control the quality characteristic Y through both the mean and the variance in order to minimize average loss. Therefore, in parameter design we need to devise an objective measure which takes these two parts into account simultaneously. Taguchi called such objective measure as signal-to-noise(SN) ratio, and he suggested several kinds of SN ratios for the different cases of target value. However, Taguchi demonstrated no connection between SN ratios and the goal of parameter design of minimizing average loss. And some authors, for example, León, *et al.*(1987), Box(1988) criticized such SN ratios and suggested different methods of analysis.

The aim of this paper is to investigate the validity of SN ratios, and to explain how data transformation can be used as an alternative method of analysis of parameter design. In following section, we introduce SN ratios and optimization procedure suggested by Taguchi. In section 3 we review some criticism about SN ratios. Section 4 deals with the transformation technique suggested by Box(1988) as an alternative method of analyzing parameter design. Finally in section 5, we examine the appropriateness of SN ratios by analyzing real empirical experiments conducted in industries, and explain how data transformation can be applied to the analysis of parameter design.

2. SN Ratios and Optimization Procedure

According to the property of the target value of Y, Taguchi(1986) suggested the following SN ratios.

2-1. Nominal is the best

Here the quality characteristic Y is continuous and the nominal target value, say $\tau = \tau_0$, is specified, and the average loss (1.2) increases as Y deviates from τ_0 in either direction. For this case, Taguchi recommended the following SN ratio, denoted here by $(SN)_1$

$$(SN)_1 = 10\, log\, (\frac{\bar{y}}{s})^2 \qquad (2.1)$$

, where $\bar{y}$ and s^2 are the sample mean and the sample variance, respectively.

2-2. The smaller the better

In this case, Y is continuous, positive and the target value is zero. Then the average loss $R(\underline{\theta})$ in (1.2) is proportional to $E(Y^2)$. For this case Taguchi suggested the SN ratio, denoted here by $(SN)_2$, as a function of the moment estimate of $E(Y^2)$.

$$(SN)_2 = -10\, log\, (\frac{1}{n} \sum_{i=1}^{n} y_i^2) \qquad (2.2)$$

2-3. The larger the better

This is the case of a continuous quality characteristic which we want to be as large as possible. If we take the reciprocal $1/Y$, then we have the same situation as in the case of SN ratio (2.2). The SN ratio, denoted by $(SN)_3$, suggested by Taguchi is

$$(SN)_3 = -10\, log\, [\frac{1}{n} \sum_{i=1}^{n} (\frac{1}{y_i})^2] \qquad (2.3)$$

2-4. Optimization Procedure in Parameter Design.

In parameter design, we carry out the experiments and calculate a SN ratio for each setting $\underline{\theta}$ of control factors. Then we perform the standard analysis of variance(ANOVA) procedure using calculated SN ratios as data, and decompose the control factors into two groups $\underline{\theta}_1$ and $\underline{\theta}_2$, where $\underline{\theta}_1$ is a subset of control factors having significant effects on the SN ratio. Taguchi identifies $\underline{\theta}_1$ as dispersion effects which affect the variability of Y as measured by the SN ratio. Moreover, $\underline{\theta}_2$ can also be decomposed into $\underline{\theta}_2 = (\underline{\theta}_3, \underline{\theta}_4)$, where $\underline{\theta}_3$ is a set of influential factors on the mean, and $\underline{\theta}_4$ of non-influential factors on the SN ratio and the mean.

To minimize the average loss, Taguchi recommended the two-step procedure such that (a) first find the setting $\underline{\theta}_1 = \underline{\theta}_1{}^*$ that maximize the SN ratio, (b) and then set $\underline{\theta}_2 = \underline{\theta}_2{}^*$ at which $E(Y)$ is close to τ as possible. For more details about construction and analysis of parameter design, refer to Taguchi(1986), Taguchi and Wu(1980), Phadke(1982), or Kackar(1985).

3. Discussion of Taguchi SN Ratios

In two-step optimization procedure, we manipulate the mean $\mu(\underline{\theta})$ to be close to target value τ in the second step. Thus, for this two-step procedure to be successful the SN ratio analyzed in the first step should be functionally independent of $\mu(\underline{\theta})$. Therefore, we can easily see that the suitable SN ratio should be a statistic that is independent of $\mu(\underline{\theta})$.

Consider the first SN ratio (2.1) which is a function of sample coefficient of variation $s/\bar{y}$. Here note that only when $\bar{y}$ and s are linked in a linear function, the coefficient variation $s/\bar{y}$ is constant, and thus independent of $\bar{y}$. Consequently we can see that the SN ratio (2.1) is relevant only when the mean and the variance of Y have a linear relationship. For this case, we can minimize the average loss (1.2) by applying the two-step procedure : (a) maximize the SN ratio by controlling $\underline{\theta}_1$ which affect the SN ratio (b) then use $\underline{\theta}_2$ (or $\underline{\theta}_3$) in adjusting $\mu(\underline{\theta})$ close to the value at which (1.2) is minimized.

However, if the mean and the variance of Y are functionlly independent, then the average loss can be minimized by minimizing the variance first and then minimizing the bias in the second step. Therefore, for the case of independency, the suitable SN ratio which is maximized in the first step is some funtion of the variance. For example,

$$SN = -10\, log\,(s^2). \qquad (2.4)$$

At this point, it is clear that the information about data structure is very important in developing a proper SN ratio. And we should make all efforts to get concrete informations about the relationship between the mean and the variance.

Generally assume that the $\sigma(\theta)$ and the $\mu(\underline{\theta})$ are linked in some function g such that $T(\underline{\theta}) = [\sigma(\theta)/g(\mu(\underline{\theta}))]^2$ is independent of μ in a sence that only a subset $\underline{\theta}_1$ of $\underline{\theta}$ affects $T(\underline{\theta})$. That is, $T(\underline{\theta})$ is a measure of dispersion which is functionally independent of μ. Then the average loss (1.2), apart from constant, can be expressed as

$$R(\underline{\theta}) = [g(\mu(\underline{\theta}))]^2 T(\underline{\theta}_1) + (\mu(\underline{\theta}) - \tau)^2. \tag{2.5}$$

Then $R(\underline{\theta})$ can be minimized by minimizing $T(\underline{\theta})$ by using $\underline{\theta}_1$ first, and next adjusting $\mu(\underline{\theta})$ by contolling $\underline{\theta}_2$ of $\underline{\theta} = (\underline{\theta}_1, \underline{\theta}_2)$. Note that the exact minimum can be achieved by adjusting $\mu(\underline{\theta})$ to the point μ^* satisfing $\partial R(\underline{\theta})/\partial\underline{\theta}_2 = 0$, that is

$$\mu^* = \tau - g(\mu^*)g'(\mu^*)T^* \tag{2.6}$$

, where T^* is the minimum value of $T(\underline{\theta})$ attained in the first step. But if we ignore the second part of the right hand side of (2.6), then the adjustment point is $\mu^* = \tau$.

Moreover, León, *et al.*(1987) also discussed the appropriateness of (2.1) from the point of view of transfer function f. They showed that if the Y is generated by a particular multiplicative transfer function

$$Y = f(\underline{\theta})\varepsilon(w, \underline{\theta}) \tag{2.7}$$

, where $E[\varepsilon(w, \underline{\theta})] = 1$ and $E(Y) = f(\underline{\theta})$ is a strictiy monotone function of $\underline{\theta}$, then the SN ratio (2.1) is appropriate in the two-step optimization procedure. However, it may not be possible for most actual manufacturing processes to identify the specific mechanistic model f of manufacturing system.

Now consider the second SN ratio (2.2) suggested Taguchi. Since (2.2) can be written as

$$(\mathrm{SN})_2 = -10\,log\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) = -10\,log\left(\bar{y}^2 + \frac{n-1}{n}s^2\right) \tag{2.8}$$

, we can see that (2.8) is the function of the mean and the variance, and thus it confounds the location and dispersion effects. For this reason, Box(1988) argues that it is very doubtful whether we can achieve the minimum variance as well as minimum bias by applying the two-step procedure. It can be argued in favor of (2.8) that it combines information about changes in mean and changes in variance. However, if $\bar{y}$ is dominating part over s^2 in (2.8), then the maximization of (2.8) is essentially same as minimization of $\bar{y}$, instead of minimization of s^2. And the second step of adjustment is meaningless, and furthermore misleading. Therefore, it is desirable to analyze the dispersion effect and the location effect separately.

The same arguments can be justified for the third SN ratio (2.3), since (2.3) is also made of the mean and the variance of the transformed data $1/Y$. Moreover, Box(1988) argues that (2.3) is likely to be exceptionally sensitive to outliers since it depends on the square of reciprocals of the data. For this case, we should exploit another form of transformation to develop a suitable SN ratio.

4. Data Transformation

In the field of design of experiment, the transformation of data has been widely used, especially when we want to achieve the stabilization in variance, normality, and additivity of the model. In parameter design, we should decompose the control factors into two groups which affect the location and the dispersion, respectively, and analyze these two groups separately in order for the two-step procedure to be successful. For decomposing the control factors, Box(1988) suggested to use transformation technique, especially variance stabilization.

Assume that $X = h(Y)$ is a variance stabilization transformation such that $\mathrm{Var}(X) = \sigma_X^2$ is functionally independent of μ, and suppose further that σ_X^2 depends only on the subset $\underline{\theta}_1$ of $\underline{\theta}$. Then the average loss (1.2) can be expressed approximately as

$$R(\underline{\theta}) \approx [\, h'(\mu)^{-2}\, \sigma_X^2(\underline{\theta}_1) \,] + (\mu(\underline{\theta}) - \tau)^2 \qquad (3.1)$$

, since $\sigma^2(\underline{\theta}) \approx (h'(\mu))^{-2}\, \sigma_X^2$. Therefore, by analyzing the transformed data $X = h(Y)$, we can achive the objective of parameter design with applying the two-step procedure such that we first find the setting of $\underline{\theta}_1$ at which σ_X^2 is minimized and then adjust the mean of Y to some point at which $R(\underline{\theta})$ is minimized.

In practice, it is convenient to perfrom the second step of adjustment in the transformed data. For this case, Box(1988) showed that we should adjust $E(X)$ to the point $\mu_X{}^*$,

$$\mu_X{}^* = h\,[\, \tau + \frac{3}{2}(h'(\tau))^{-3}\, h''(\tau)\,(\sigma_X{}^2)^* \,] \qquad (3.2)$$

, where $(\sigma_X{}^2)^*$ is the minimum value of σ_X^2 obtained in the first step.

As an easy way of finding a suitable transformation, Box(1988) recommended using the power transformation suggested by Box and Cox(1964) such that $X = h(Y) = Y^{\alpha}$. For selecting appropriate value α in the power transformation, some quantitity such as t-value, F-value or relative proportion of sum of squares of each effect in the ANOVA procedures both for the mean and the variance can be plotted with changing value of α. In such plottings, if we can identify the value of α at which we can decompose the control factors $\underline{\theta}$ into $\underline{\theta}_1$ and $\underline{\theta}_2$ successfully, then we use such value in the power transformation. In such optimal transformation, the number of influential factors could be small, and we may also achieve the additivity of the model. Moreover factors which in the optimal transformation have influence on variance but not on the mean will have influence on both in another transformation. This procedure of selecting a suitable α is explained in the examples in section 5.

And note here that for a power transformation $X = Y^{\alpha}$, the adjustment point (3.2) is reduced to

$$\mu_X^* = \left\{ \tau \left[1 - \frac{3}{2} \frac{(1-\alpha)}{\alpha^2} \frac{(\sigma_X^2)^*}{(\tau)^{2\alpha}} \right] \right\}^{\alpha} \qquad (3.3)$$

Moreover, if we ignore the second term in the parenthesis of (3.3) then the adjustment point is approximately

$$\mu_X^* = \tau^{\alpha}. \qquad (3.4)$$

5. Examples of Empirical Data

Here, we discuss the validity of Taguchi SN ratios by analyzing real data of experiments conducted in manufacturing industries.

〈 **Example 1** 〉 The experiment in this example was conducted by M.G. White at Military Aerospace Division of ITT Cannon Company in America. The quality characteristic Y in this experiment is the thickness of gold plating and the target value is 50 micro-inch.

In the experiment, there are 9 control factors denoted by A, B, C, D, E, F, G, H, I, each having two levels. The objective was to reduce the variation of Y with it's mean close to target value. White used an $L_{16}(2^{15})$ orthogonal array and the actual arrangement of factors is illustrated in Table 1 in which the simbol e denotes the error. The thickness values for 20 samples taken from product manufactured at each setting of control factors are shown in Table 2 with values of SN ratio of (2.1). The details about the whole experiment are illustrated in Yum(1991, pp. 221－237). White performed the usual ANOVA using SN ratio values as data, and he also did ANOVA using the mean values of 20 samples. According to these two ANOVA procedures, it was found that two factors D and C and one interaction $A \times B$ had significant effect on the SN ratio, and only one factor C affected the mean value. Following these results, White obtained the optimal setting of control factors by identifying the levels A, B, D at which the SN ratio is maximized, and identifying the level of C at which the mean value is close to target value.

〈 Table 1 : Arrangement in $L_{16}(2^{15})$ orthogonal array 〉

Run	A	I	$A \times I$	$E \times F$	D	B	$A \times B$	G	H	$B \times C$	E	C	$A \times C$	e	F
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

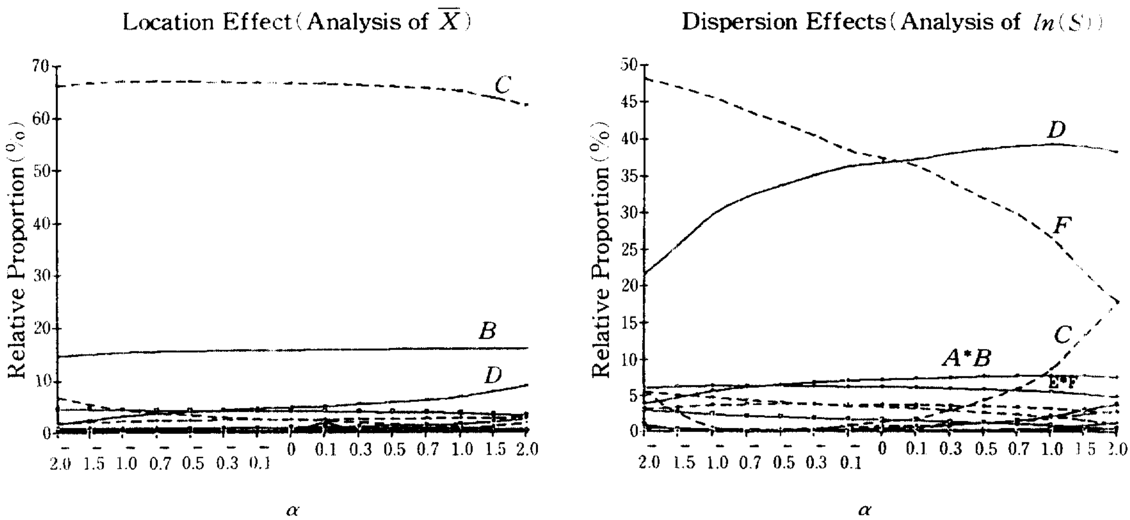
〈 Table 2 : Data and SN ratio of gold plating experiment 〉

Run	Data	SN ratio
1	63 52 57 60 51 64 57 56 61 57 58 51 54 60 54 64 50 54 55 49	21.76
2	59 60 63 61 58 66 53 63 57 61 61 60 73 81 59 62 68 53 60 72	19.32
3	60 66 78 83 64 76 89 106 68 78 52 86 91 69 71 57 73 96 83 60	14.61
4	67 77 66 51 53 55 59 58 62 62 81 76 60 58 53 59 55 54 70 60	17.18
5	71 74 78 63 62 67 47 69 49 58 54 80 71 67 62 47 51 66 57 49	15.60
6	70 58 69 65 65 74 71 75 75 65 70 64 65 66 55 70 71 74 65 75	21.79
7	61 66 65 74 66 73 73 65 83 81 60 75 77 62 60 69 65 60 89 76	18.39
8	68 62 54 51 43 59 57 64 53 52 48 58 46 46 47 42 51 48 50 44	17.16
9	66 47 67 56 55 56 49 53 39 54 42 46 66 89 42 68 61 46 92 58	12.14
10	48 63 69 60 89 81 63 53 68 76 53 67 66 68 69 65 89 67 74 70	16.13
11	75 76 75 70 71 70 84 68 75 73 80 76 69 71 70 75 67 68 76 66	24.03
12	58 55 47 49 59 45 53 56 41 53 61 52 55 55 54 50 53 52 56 55	20.92
13	64 65 57 76 54 54 65 60 64 67 62 62 67 57 67 58 55 61 64 56	20.94
14	45 79 77 72 71 99 50 74 77 74 72 96 75 89 98 77 41 77 96 75	13.45
15	84 53 56 64 61 74 57 56 69 65 72 57 48 64 64 67 55 68 56 55	17.23
16	61 60 55 50 54 51 50 56 57 55 53 52 57 52 53 52 54 55 55 49	24.63

As an alternative way of analyzing this data, we transform the data by $X = Y^{\alpha}$, where α is real number. For various values of α, we perform the standard ANOVA procedures by using $\bar{x}$ and logarithm of standard deviation $ln(s)$ of the transformed data $X = Y^{\alpha}$, and we calculate the relative proportion of sum of squares of each factor. The relative proportion is obtained by dividing each sum of squares by total sum of squares. The plottings of these values of relative proportions against α are shown in Figure 1.

An examination of Figure 1 reveals that the value $\alpha = 0$, which correspond to the transformation of $X = ln(Y)$, is appropriate in decomposing the control factors into two sets $\underline{\theta}_1$ and $\underline{\theta}_2$. For such transformation, we can see that two effects D and F are influential on the dispersion, and C has influence on the mean. Furthermore, no interaction is influential so that we can also achieve the additivity.

Moreover, Figure 1 indicates that the analysis of the untransformed original data may be also appropriate since we can also decompose successfully the control factors at $\alpha = 1$. For this case, factor F has influence less significantly than D on the dispersion. Therefore both for the transformed and the untransformed data, the optimal setting $\underline{\theta}^*$ can be obtained approximately by identifying the levels of factors D and F that give the minimum variance, and identifying the levels of C at which the mean value is close to target value. If we compare these results with those of White, we can see that the blind use of SN ratio produces wrong conclusion.



⟨ Figure 1 : Plots of Relative Proportions in Gold Plating Experiment ⟩

〈 **Example 2** 〉 For further illustration, consider the experiment conducted by Schild and Pishko at Texas Instrument, Inc. The quality characteristic of the experiment was tensil strength of the product and the target value was infinity. In this experiment, there are five control factors (A, B, C, D, E) each having two levels, and four noise factors. For construction of parameter design, they used an orthogonal arrays $L_{16}(2^{15})$ and $L_8(2^3\times 4)$ for the control factors and the noise factors, respectively. For each setting of control factors, they replicate $L_8(2^3\times 4)$ array twice so that 16 observations of tensil strenth are available. The $L_{16}(2^{15})$ array used in this experiment is the same array of the Example 1. The arrangement of the control factors in $L_{16}(2^{15})$ array and the data are shown in Table 3 and Table 4. For more details about this experiment, refer to Yum(1991, pp. 271−281).

〈 Table 3 : Arrangement in $L_{16}(2^{15})$ 〉

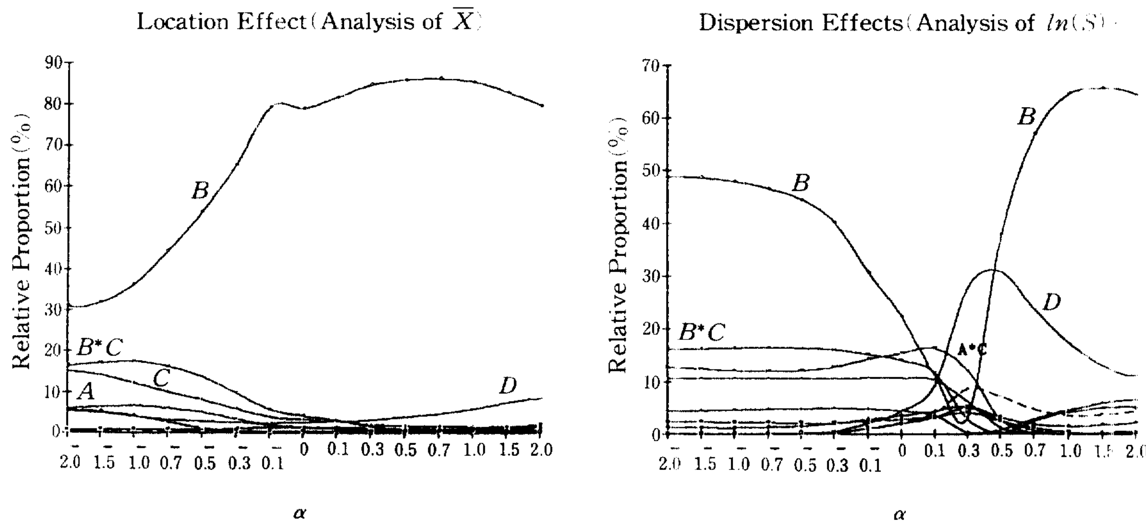
Column Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Assigned Effect	A	C	$A\times C$	B	e	$B\times C$	e	D	$A\times D$	$D\times F$	F	e	e	e	$B\times F$

〈 Table 4 : Data and SN ratio of the Tensil Strenth Expereiment 〉

Run	Data								SN ratio
	1	2	3	4	5	6	7	8	
1	135 115	195 170	50 1	10 30	15 20	55 80	155 120	110 105	11.92
2	65 155	155 130	25 5	35 50	95 1	140 105	280 370	95 205	11.86
3	210 445	395 335	25 25	70 110	30 75	220 375	445 460	300 185	35.14
4	530 370	65 300	70 50	438 385	50 75	20 65	320 370	465 390	35.79
5	95 70	160 100	30 40	40 50	25 35	35 25	390 230	120 80	32.85
6	115 65	325 230	115 110	35 10	50 15	175 475	395 305	70 205	29.88
7	185 375	315 280	175 130	380 135	80 70	100 85	350 405	365 325	42.78
8	410 210	555 485	80 75	550 460	60 110	25 60	365 390	520 485	37.83
9	75 80	125 185	70 10	50 1	1 1	145 105	195 295	165 85	7.25
10	235 110	260 130	120 110	30 50	35 1	30 20	305 245	70 105	12.01
11	500 325	240 130	110 70	220 140	85 40	140 195	315 390	440 220	40.80
12	580 510	280 330	60 110	145 125	85 120	20 25	360 360	400 505	35.20
13	100 145	220 230	70 70	10 1	50 75	65 55	270 285	100 105	11.99
14	200 115	215 145	5 25	40 35	60 30	415 245	340 320	95 85	25.52
15	685 330	260 330	15 5	115 95	85 80	135 140	355 275	330 395	25.50
16	310 460	440 480	5 30	95 105	100 110	65 5	365 270	325 375	22.91

Schild and Pishko analyzed the SN ratio (2.3) and found that two effects A, B and two interactions $B\times C$, $A\times C$ were significant on the SN ratio. Moreover two factors D and F were identified as influential effects in the ANOVA procedure of mean values. Therefore, they identified factors A, B, C as dispersion effects, and identified D, F as location effects. Figure 2 shows the plottings made by same procedure explained in the Example 1. From the Figure 2 we can infer that the appropriate value of α is about $\alpha = 0.3$. For such transformed data, we can see that the single factor B has influencd on the mean, and factor D (and possibly one interaction $A \times C$) are significant on the dispersion.

According to these results, the optimal setting of control factors can be obtained for the transformed data by identifying the levels of D and B which give the minimum variance and the maximum mean, respectively. From this example, we can also gain some evidences that the SN ratio (2.3) might be inappropriate for analyzing this experiment of parameter design.



⟨ Figure 2 Plots of Relative Proportions in Tensil Strength Experiment ⟩

6. Concluding Remarks

During the time of preparing this paper, many other empirical data have been examined in evaluating the appropriateness of Taguchi SN ratios. The other results are not presented here, because of lack of space. However most of results reveal that the reckless use of SN ratios suggested by Taguchi is liable to mislead.

Therefore, in order for parameter design to be successful, we should be very cautious in choosing the suitable statistic to be analyzed. For deciding the proper statistic, some other simple statistical techniques such as exploratory data analysis method can be employed to obtain information about the relationship between the mean and the variance of the data.

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