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A NEW CLASS OF ALMOST CONTACT RIEMANNIAN MANIFOLDS

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(Dedicated to Professor Hisao Nakagawa on his sixtieth birthday)

1. Introduction

Among almost contact metric structures, the quasi Sasakian and the trans Sasakian structures were defined by D. E. Blair [1] and J. A. Oubina [5], which are normal almost contact metric structures containing both cosymplectic and Sasakian structures. Moreover, J. A. Oubina [5] showed that there is no inclusion relation between quasi Sasakian and trans Sasakian structures.

But we see that there is a new class of almost contact metric structures containing quasi Sasakian and trans Sasakian structures.

In this paper, we introduce such a new class of almost contact metric structures and study their essential examples as well as their fundamental properties.

2. Almost contact metric structures

Let M be an m-dimensional real differentiable manifold with almost contact metric structure (ϕ, ξ, η, g). Then the relations

$$\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1,$$
$$g(\xi, X) = \eta(X), \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

hold for any vector fields X and Y on M. The fundamental 2-form Φ is defined by $\Phi(X, Y) = g(\phi X, Y)$. It is known that the almost contact structure (ϕ, ξ, η) is normal if and only if

$$N(X, Y) = [\phi, \phi](X, Y) + 2d\eta(X, Y)\xi \tag{2.1}$$

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vanishes, where [ ] is a bracket operation and d denotes the exterior derivative.

An almost contact metric structure (ϕ, ξ, η, g) on M is said to be (cf. [2, 3, 4, 5])

(a) *quasi Sasakian* if Φ is closed and (ϕ, ξ, η) is normal,

(b) *trans Sasakian* if $d\Phi = \alpha(\Phi \wedge \eta)$, $d\eta = \beta\Phi$, $\phi^*(\delta\Phi) = 0$ and (ϕ, ξ, η) is normal,

(c) *cosymplectic* if Φ and η are closed and (ϕ, ξ, η) is normal, and

(d) *Sasakian* if $\Phi = d\eta$ and (ϕ, ξ, η) is normal,

where α and β are functions on M, and $\delta\Phi$ is the coderivative of Φ.

Throughout this paper we study a class of almost contact metric manifolds M which satisfies

$$(*) \qquad \begin{cases} \Phi \text{ is closed,} \\ \nabla_X \xi = \lambda \phi X \quad \text{for a smooth function } \lambda \text{ on } M \text{ and} \\ (\phi, \xi, \eta) \text{ is normal} \end{cases}$$

where ∇ denotes the Riemannian connection for g.

Briefly, we denote such a manifold by M^*.

PROPOSITION 1. *On M^*, we have*

$$(2.2) \qquad (\nabla_X \Phi)(Y, Z) = \lambda\{\eta(Y)g(X, Z) - \eta(Z)g(X, Y)\}.$$

Proof. In [6] under the assumptions Φ is closed, (ϕ, ξ, η) is normal and $\mathcal{L}_\xi g = 0$, it was proved that

$$(\nabla_X \Phi)(Y, Z) = \eta(Y)(\nabla_X \eta)(\phi Z) - \eta(Z)(\nabla_X \eta)(\phi Y),$$

where $\mathcal{L}_\xi$ is a Lie derivative with respect to ξ. Since $(\nabla_X \eta)Y = \lambda\Phi(X, Y)$, we get (2.2) and the proof is complete.

By the definitions (a) $\sim$ (d) and Proposition 1, M^* is quasi Sasakian and trans Sasakian. Moreover M^* becomes cosymplectic if $\lambda = 0$ and Sasakian if λ is non-zero constant.

PROPOSITION 2. *On M^*, the following relations*

$$R(X,\xi)Y = (X\lambda)(\phi Y) + \lambda^2\{\eta(Y)X - g(X,Y)\xi\}, \tag{2.3}$$

$$\xi\lambda = 0, \tag{2.4}$$

$$S(\xi, X) = (\phi X)\lambda + (m-1)\lambda^2\eta(X) \tag{2.5}$$

hold, where S is the Ricci curvature and R is the curvature tensor defined by

$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z.$$

Proof. Since ξ is Killing, we see that

$$\nabla_X(\nabla_Y\xi) - \nabla_{\nabla_X Y}\xi - R(X,\xi)Y = 0.$$

Hence, by use of $(*)_2$ and (2.2), we get (2.3). The relation (2.4) follows from (2.3). From the Ricci identity, $(*)_2$ and (2.2) reduce (2.5). This completes the proof of the proposition.

Let X be the unit vector field orthogonal to ξ. Then, by use of (2.3) and (2.4), we obtain $R(\xi,X)\xi = -\lambda^2 X$. Therefore $g(R(\xi,X)X,\xi) = \lambda^2$. Thus we have

PROPOSITION 3. *On M^*, the sectional curvature of all plane section containing ξ is λ^2.*

If the Ricci curvature S is parallel on M^*, then we get

$$\begin{aligned}\lambda S(\phi X, Y) =& X((\phi Y)\lambda) - (\nabla_{\phi X}Y)\lambda + (2m-1)\lambda(X\lambda)\eta(Y) \\ &+ (m-1)\lambda^3\Phi(X,Y)\end{aligned} \tag{2.6}$$

by use of (2.2) and (2.4). Putting $Y = \xi$ in (2.6) and using (2.4) and (2.5), we have $\lambda(X\lambda) = 0$, that is, λ^2 is constant on M^*. Since M^* is connected and λ is a smooth function on M^*, we see that λ is constant. Thus we have

PROPOSITION 4. *If the Ricci curvature S is parallel on M^*, then the function λ is constant.*

REMARK. If the Ricci curvature S is parallel on M^*, then λ is constant by virtue of the proposition 4. Using Ricci identity, (2.3) and (2.5), we can see that M^* is Einstein when λ is non-zero constant.

3. Examples

We denote Cartesian coordinates in a 3-dimensional Euclidean space E^3 by (x_1, x_2, x_3) and define a symmetric tensor field g by

$$g = e^{2f} \begin{pmatrix} 1+\sigma^2 & 0 & -\sigma \\ 0 & 1 & 0 \\ -\sigma & 0 & 1 \end{pmatrix},$$

where σ and f are functions on E^3. Then g is positive definite Riemannian metric. The inverse matrix of g is given by

$$g^{-1} = e^{-2f} \begin{pmatrix} 1 & 0 & \sigma \\ 0 & 1 & 0 \\ \sigma & 0 & 1+\sigma^2 \end{pmatrix},$$

We define an almost contact structure (ϕ, ξ, η) on E^3 by

$$\phi = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -\sigma & 0 \end{pmatrix},$$

$$\xi = e^{-f} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$\eta = e^f(-\sigma, 0, 1).$$

Then (ϕ, ξ, η, g) constitutes an almost contact metric structure on E^3. The fundamental 1-form η and 2-form Φ have the forms

$$\eta = e^f(-\sigma dx_1 + dx_3) \text{ and } \Phi = e^{2f} dx_1 \wedge dx_2$$

respectively, and hence

$$\begin{aligned} d\eta =& e^f\{(\sigma f_2 + \sigma_2) dx_1 \wedge dx_2 + f_2 dx_2 \wedge dx_3 \\ &+ (\sigma_3 + \sigma f_3 + f_1) dx_1 \wedge dx_3\}, \end{aligned}$$

$$d\Phi = 2f_3 e^{2f} dx_1 \wedge dx_2 \wedge dx_3,$$

where $f_i = \partial f/\partial x_i$ and $\sigma_i = \partial\sigma/\partial x_i$. The equation (2.1) can be written as local components

$$\begin{aligned} N_{kj}{}^i =& \phi_k{}^h(\partial_h\phi_j{}^i - \partial_j\phi_h{}^i) - \phi_j{}^h(\partial_h\phi_k{}^i - \partial_k\phi_h{}^i) \\ &+ \eta_k(\partial_j\xi^i) - \eta_j(\partial_k\xi^i), \end{aligned}$$

where the indices h, i, j and k run over the range 1,2,3. Then non-trivial components of $N_{kj}{}^i$ are given by

$$N_{12}{}^3 = \sigma f_2, \ N_{13}{}^3 = \sigma_3 + \sigma f_3 + f_1, \ N_{23}{}^3 = f_2.$$

Therefore we can see that (ϕ, ξ, η) is normal if and only if $f_2 = 0$ and $\sigma_3 + \sigma f_3 + f_1 = 0$. Hence, by a simple calculation, we see that E^3 with (ϕ, ξ, η, g) is

(1) quasi Sasakian if $f_2 = 0,\ f_3 = 0$ and $f_1 + \sigma_3 = 0$,

(2) trans Sasakian if $f_2 = 0, f_3 = \frac{1}{2}\alpha e^f, \sigma_2 = \beta e^f$ and $\sigma_3 + \sigma f_3 + f_1 = 0$,

(3) $(*)$-structure if $f_2 = 0,\ f_3 = 0,\ f_1 + \sigma_3 = 0$ and $\sigma_2 = -2\lambda e^f$,

(4) Sasakian if $f_2 = 0,\ \sigma_3 + \sigma f_3 + f_1 = 0$ and $\sigma_2 = e^f$,

and

(5) Cosymplectic if $f_2 = 0,\ f_3 = 0,\ \sigma_3 + f_1 = 0$ and $\sigma_2 = 0$,

where $(*)$-structure is the structure satisfies $(*)$-conditions and α, β are the functions appeared in the section 2. Henceforth, we can construct a non-trivial $(*)$-structure on E^3.

For example

$(1)'$ $f = x_1{}^2,\ \sigma = -2x_1x_3$,

$(2)'$ $f = \frac{1}{2}x_3,\ \sigma = x_2e^{-\frac{1}{2}x_3},\ \alpha = e^{-\frac{1}{2}x_3},\ \beta = e^{-x_3}$,

$(3)'$ $f = x_1,\ \sigma = -2x_2 - x_3,\ \lambda = e^{-x_1}$,

$(4)'$ $f = x_1,\ \sigma = x_2e^{x_1} - x_3$,

and

$(5)'$ $f = x_1,\ \sigma = -x_3$.

We can construct further examples on a $(2n+1)$-dimensional Euclidean space E^{2n+1} by the similar way.

4. Curvature tensor

Let M^* be an m-dimensional almost contact metric manifold with $(*)$-conditions.

First of all, we prepare

PROPOSITION 5. *Let S be the Ricci curvature of M^*. Then we have*

$$\begin{aligned}(4.1)\ & R(X,Y)\phi Z - \phi R(X,Y)Z \\ &= (Y\lambda)\{g(X,Z)\xi - \eta(Z)X\} - (X\lambda)\{g(Y,Z)\xi - \eta(Z)Y\} \\ &\quad + \lambda^2\{\Phi(X,Z)Y - \Phi(Y,Z)X + g(X,Z)\phi Y - g(Y,Z)\phi X\},\end{aligned}$$

$$\begin{aligned}(4.2)\ & g(R(\phi X,\phi Y)Z,W) \\ &= g(R(X,Y)Z,W) + (Z\lambda)\{\eta(Y)\Phi(W,X) + \eta(X)\Phi(Y,W)\} \\ &\quad - (W\lambda)\{\eta(Y)\Phi(Z,X) + \eta(X)\Phi(Y,Z)\} + \lambda^2\{\Phi(Z,X)\Phi(Y,W) \\ &\quad - \Phi(W,X)\Phi(Y,Z) + g(Z,X)g(Y,W) - g(W,X)g(Y,Z)\},\end{aligned}$$

$$\begin{aligned}(4.3)\ & g(R(\phi X,Y)\phi Z,W) - g(R(\phi X,Y)\phi W,Z) \\ &= (Y\lambda)\{\Phi(X,Z)\eta(W) + \Phi(W,X)\eta(Z)\} \\ &\quad + ((\phi X)\lambda)\{g(W,Y)\eta(Z) - g(Y,Z)\eta(W)\} \\ &\quad - \lambda^2\{\Phi(X,Z)\Phi(W,Y) - \Phi(Z,Y)\Phi(X,W) + g(X,Z)g(W,Y) \\ &\quad - g(Z,Y)g(X,W) + g(Z,Y)\eta(X)\eta(W) - g(W,Y)\eta(X)\eta(Z)\},\end{aligned}$$

$$\begin{aligned}(4.4)\qquad & S(X,Y) + \frac{1}{2}\sum_{i=1}^{m} g(R(e_i,\phi e_i)X,\phi Y) \\ &= ((\phi X)\lambda)\eta(Y) + (m-2)\lambda^2 g(X,Y) + \lambda^2\eta(X)\eta(Y),\end{aligned}$$

where $\{e_i\} = \{e_1,\ldots,e_{m-1},e_m = \xi\}$ is a local orthonormal frame field.

Proof. (4.1) follows from (2.2) and Ricci's identity. By use of (2.3) and (4.1), we can prove (4.2). (4.3) follows from (4.2). Finally, using (4.1) and the first Bianchi's identity, we get (4.4).

PROPOSITION 6. *On M^*, λ is constant if and only if the Ricci curvature tensor S and ϕ are commute.*

Proof. From (4.1) and the first Bianchi's identity, we have

$$S(X,\phi Y)+\frac{1}{2}\sum_{i=1}^{m} g(R(e_i,\phi e_i)X,Y) \\ = -(m-2)(X\lambda)\eta(Y)-(m-2)\lambda^2\Phi(X,Y).$$

Taking the skew-symmetric part in (4.5) with respect to X and Y, it follows that

$$S(\phi X,Y)+S(X,\phi Y)=(m-2)\{(X\lambda)\eta(Y)+(Y\lambda)\eta(X)\}.$$

Hence we can get the desired result and the proof is complete.

The Weyl conformal curvature tensor C is defined by

$$(4.6)\ C(X,Y)Z-R(X,Y)Z \\ = -\frac{1}{m-2}\{S(Y,Z)X-S(X,Z)Y+g(Y,Z)AX \\ -g(X,Z)AY\}+\frac{r}{(m-1)(m-2)}\{g(Y,Z)X-g(X,Z)Y\},$$

where S and r are the Ricci tensor and the scalar curvature respectively, and we have put $g(AX,Y)=S(X,Y)$. At first we shall prove

PROPOSITION 7. *If M^* is conformally flat and $m>3$, then M^* is a space of constant curvature.*

Proof. Since M^* is conformally flat, the curvature tensor R is

$$(4.7)\ R(X,Y)Z \\ = \frac{1}{m-2}\{S(Y,Z)X-S(X,Z)Y+g(Y,Z)AX \\ -g(X,Z)AY\}-\frac{r}{(m-1)(m-2)}\{g(Y,Z)X-g(X,Z)Y\}.$$

From (4.7), we get

$$(4.8)\ g(R(X,\phi X)Y,\phi Z)$$
$$= \frac{4}{m-2}\{-S(Y,Z) + ((\phi Y)\lambda)\eta(Z) + (m-1)\lambda^2\eta(Y)\eta(Z)\}$$
$$+ 2((\phi Z)\lambda)\eta(Y) + \frac{2r}{(m-1)(m-2)}\{g(Y,Z) - \eta(Y)\eta(Z)\}.$$

On the other hand, by use of (4.5), the left hand side of (4.8) becomes

$$-2S(Y,Z) + 2((\phi Y)\lambda)\eta(Z) + 2(m-2)\lambda^2 g(Y,Z) + 2\lambda^2\eta(Y)\eta(Z).$$

Henceforth, we can get

$$r = m(m-1)\lambda^2 \tag{4.9}$$

and that

$$((\phi X)\lambda)\eta(Y) = ((\phi Y)\lambda)\eta(X) \tag{4.10}$$

for $m > 3$. If we substitute (4.9) into (4.7) and make use of (2.3) and (4.10), then it follows

$$S(X,Y) = 2((\phi X)\lambda)\eta(Y) + (m-1)\lambda^2 g(X,Y).$$

Therefore (4.7) becomes

$$R(X,Y)Z = \lambda^2\{g(Y,Z)X - g(X,Z)Y\},$$

that is, M^* is a space of constant curvature when $m > 3$. This completes the proof.

In a manifold M^*, a sectional curvature

$$H = -\frac{g(R(\phi X,\phi^2 X)\phi X,\phi^2 X)}{g(\phi X,\phi X)g(\phi^2 X,\phi^2 X)} \tag{4.11}$$

determined by two orthogonal vectors ϕX and $\phi^2 X$ for the unit vector X is called the ϕ-holomorphic sectional curvature.

Now if we assume that H is independent of the choice of ϕ-holomorphic section at $p \in M^*$, then we have

$$
\begin{aligned}
(4.12)\quad & 2\{R(\phi X, Y, \phi Z, W) + R(\phi X, W, \phi Y, Z) + R(\phi X, Z, \phi W, Y)\} \\
& - 2\lambda^2\{\Phi(X,Z)\Phi(Y,W) - \Phi(X,Y)\Phi(Z,W) - \Phi(Y,Z)\Phi(X,W)\} \\
& + 2(\lambda^2 - H)\{g(X,Y)\eta(W)\eta(Z) + g(X,Z)\eta(Y)\eta(W) \\
& + g(X,W)\eta(Y)\eta(Z)\} - 6(\lambda^2 - H)\eta(X)\eta(Y)\eta(Z)\eta(W) \\
& + 2H\{g(X,Y)g(Z,W) + g(X,W)g(Y,Z) + g(X,Z)g(Y,W)\} \\
& + 2((\phi X)\lambda)\{g(Y,Z)\eta(W) + g(Z,W)\eta(Y) + g(Y,W)\eta(Z) \\
& - 3\eta(Y)\eta(Z)\eta(W)\} + (W\lambda)\{\Phi(X,Z)\eta(Y) - \Phi(X,Y)\eta(Z)\} \\
& + (Z\lambda)\{\Phi(X,Y)\eta(W) - \Phi(X,W)\eta(Y)\} + (Y\lambda)\{\Phi(X,W)\eta(Z) \\
& - \Phi(X,Z)\eta(W)\} - 2H\{g(Z,W)\eta(X)\eta(Y) + g(Z,Y)\eta(X)\eta(W) \\
& + g(Y,W)\eta(X)\eta(Z)\} = 0,
\end{aligned}
$$

by virtue of (4.2), (4.3) and (4.11), where $R(X,Y,Z,W) = g(R(X,Y)Z, W)$.

Replacing X and Z with ϕX and ϕZ respectively, we obtain

$$
\begin{aligned}
& 4R(X,Y,Z,W) + 2R(X,Z,W,Y) - 2R(\phi Y, W, X, \phi Z) \\
& + (Y\lambda)\{6\Phi(X,W)\eta(Z) + \Phi(Z,X)\eta(W) - 2\Phi(Z,W)\eta(X)\} \\
& - (X\lambda)\{4\Phi(Y,W)\eta(Z) + 4\Phi(Z,Y)\eta(W) + 2\Phi(Z,W)\eta(Y)\} \\
& - (W\lambda)\{2\Phi(X,Y)\eta(Z) + 8\Phi(Y,Z)\eta(X) + \Phi(X,Z)\eta(Y)\} \\
& + 4(Z\lambda)\Phi(Y,W)\eta(X) + ((\phi Z)\lambda)\{g(X,Y)\eta(W) - g(X,W)\eta(Y)\} \\
& + 2((\phi Y)\lambda)\{g(Z,W) - \eta(Z)\eta(W)\}\eta(X) \\
& + 2\lambda^2\{\Phi(W,X)\Phi(Z,Y) - g(X,Y)g(Z,W) + g(W,X)g(Y,Z) \\
& + g(Y,W)g(X,Z) + g(X,Z)\eta(Y)\eta(W) - g(X,W)\eta(Y)\eta(Z) \\
& - g(Y,Z)\eta(X)\eta(W) + g(Y,W)\eta(X)\eta(Z) - \eta(X)\eta(Y)\eta(Z)\eta(W)\} \\
& + 2H\{\Phi(X,Y)\Phi(Z,W) + \Phi(X,W)\Phi(Z,Y) + g(Y,W)g(X,Z) \\
& - g(Y,W)\eta(X)\eta(Z) - g(X,Z)\eta(Y)\eta(W) + \eta(X)\eta(Y)\eta(Z)\eta(W)\} = 0,
\end{aligned}
$$

by means of (2.3), (2.4), (4.2) and (4.3).

Taking the skew symmetric part of this equation with respect to Y and Z, we get

$$
\begin{aligned}
(4.13)\ & 8R(X,Y,Z,W) \\
&= 2(H+3\lambda^2)\{g(X,W)g(Y,Z) - g(Y,W)g(X,Z)\} \\
&\quad + 2(H-\lambda^2)\{g(Y,W)\eta(X)\eta(Z) - g(X,W)\eta(Y)\eta(Z) \\
&\quad + g(X,Z)\eta(Y)\eta(W) - g(Y,Z)\eta(X)\eta(W) + \Phi(Z,X)\Phi(Y,W) \\
&\quad - \Phi(Z,Y)\Phi(X,W) - 2\Phi(Z,W)\Phi(X,Y)\} + (Z\lambda)\{8\Phi(X,Y)\eta(W)\} \\
&\quad - (W\lambda)\{16\Phi(X,Y)\eta(Z) + \Phi(X,Z)\eta(Y) - \Phi(Y,Z)\eta(X)\} \\
&\quad - (Y\lambda)\{6\Phi(Z,W)\eta(X) + \Phi(X,Z)\eta(W) + 6\Phi(W,X)\eta(Z)\} \\
&\quad + (X\lambda)\{6\Phi(Z,W)\eta(Y) + \Phi(Y,Z)\eta(W) - 6\Phi(Y,W)\eta(Z)\} \\
&\quad - ((\phi X)\lambda)\{g(Y,Z)\eta(W) - g(Z,W)\eta(Y) - 2g(Y,W)\eta(Z) \\
&+ 2\eta(Y)\eta(Z)\eta(W)\} + ((\phi Y)\lambda)\{g(X,Z)\eta(W) - g(Z,W)\eta(X) \\
&\quad - 2g(X,W)\eta(Z) + 2\eta(X)\eta(Z)\eta(W)\}.
\end{aligned}
$$

Replacing X with ξ in (4.12) and using (2.3), we have $(\phi Y)\lambda = 0$. From this fact and (2.4) implies that λ is constant. Thus the equation (4.13) is reformed to

$$
\begin{aligned}
(4.14)\ 4R(X,Y)Z =& (H+3\lambda^2)\{g(Y,Z)X - g(X,Z)Y\} \\
& + (H-\lambda^2)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
& + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi - \Phi(X,Z)\phi Y \\
& + \Phi(Y,Z)\phi X - 2\Phi(X,Y)\phi Z\}.
\end{aligned}
$$

Hence the Ricci tensor and scalar curvature are given by

$$
\begin{aligned}
(4.15)\qquad 4S(X,Y) =& \{(m+1)H + (3m-5)\lambda^2\}g(X,Y) \\
& - (m+1)(H-\lambda^2)\eta(X)\eta(Y),
\end{aligned}
$$

$$
(4.16)\qquad r = \frac{1}{4}(m-1)\{(m+1)H + (3m-1)\lambda^2\},
$$

respectively. From (4.15), (4.16) and using $\delta S = \frac{1}{2}\nabla r$, we find H is constant if $m \neq 3$.

Thus we have

THEOREM 8. *If, in M^*, ϕ-holomorphic sectional curvature is independent of ϕ-holomorphic section at a point, then λ is constant and the curvature tensor has the form* (4.14), *where H is a constant if $m \neq 3$.*

A manifold M^* is said to be of constant ϕ-holomorphic sectional curvature if the ϕ-holomorphic sectional curvature is constant on the manifold.

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