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FIBREWISE CONVERGENCE

KYUNG CHAN MIN*, JIN WON PARK AND SEOK JONG LEE

1. Preliminaries

Recently fibrewise topology has been emerged as a subject in its own right. I. M. James has been promoting the fibrewise viewpoint systematically in topology [12-17]. As a matter of fact in many directions interests in research on fibrewise theory are growing now. In fibrewise homotopy theory, fibrewise exponential laws play central roles. Thus many attempts have been made to introduce a suitable category allowing fibrewise exponential laws [2-4,7,8,18,21,23,25]. Recently as a convenient category the authors [19] introduced the topological universe **Conv** of convergence spaces containing the category **Top** of topological spaces as a bireflective subcategory. It was shown that the category **Conv** allows various fibrewise exponential laws in a natural way (cf.[20]). In this paper, as a continuation of [19], we develop a general fibrewise theory in the category **Conv** of convergence spaces. The fibrewise notions of Hausdorffness, compactness and locally compactness are introduced as 'fibrewise properties'. A Tychonoff theorem is obtained using ultra b-filters and a fibrewise version of the one-point compactification is constructed as generalizations of those both for a convergence space and for a fibrewise topological space. For general categorical background we refer to H. Herrlich and G. E. Strecker [10], for the fibrewise theory to I. M. James [14] and for the convergence space to E. Binz [1]. Given a set X, a *convergence structure* is a map $c : X \to P(F(X))$, the power set of the set of all filters on X, that assigns to every point of X a collection of filters on X, subject to the following axioms;

(1) for any $x \in X$, $\dot{x} \in c(x)$, where $\dot{x}$ = the ultrafilter on X generated by $\{x\}$,

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(2) if $\mathcal{F} \in c(x)$, $\mathcal{F} \subseteq \mathcal{G}$, then $\mathcal{G} \in c(x)$,
(3) if $\mathcal{F}, \mathcal{G} \in c(x)$, then $\mathcal{F} \cap \mathcal{G} \in c(x)$.

The pair (X, c) is called a *convergence space*. The filters $\mathcal{F} \in c(x)$ are said to be *convergent to* x and x is called a *limit point* of $\mathcal{F}$. We usually write $\mathcal{F} \to x$ instead of $\mathcal{F} \in c(x)$. A map $f : (X, c) \to (Y, c')$ between convergence spaces is said to be *continuous* if for every $\mathcal{F} \in c(x)$, $f(\mathcal{F}) \in c'(f(x))$ for each $x \in X$. The category of convergence spaces and continuous maps is denoted by **Conv**. We say that a filter $\mathcal{F}$ has a *cluster point* x if there exists $\mathcal{G} \in c(x)$ with $\mathcal{G} \supseteq \mathcal{F}$. For a subset A of X, a point x in X is said to be *adherent to* A if there exists a filter $\mathcal{F}$ on X with $A \in \mathcal{F}$ convergent to x. Denote $\overline{A}$=the set of all points adherent to A. A subset A of X is said to be *closed* if $\overline{A} = A$. Given $B \in$**Conv**, let $\mathbf{Conv}_B$ be the comma category of **Conv** over B. An object (X, p) in $\mathbf{Conv}_B$ is called a *convergence space X over B* with a projection p. For a topological space B, the category $\mathbf{Conv}_B$ contains $\mathbf{Top}_B$ as a bireflective subcategory. Given a convergence space B, the category $\mathbf{Conv}_B$ has an initial structure over $\mathbf{Set}_B$. In fact, the initial structure in **Conv** serves for the initial structure in $\mathbf{Conv}_B$. Moreover since the category **Conv** is a topological universe, for each $B \in$ **Conv**, the category $\mathbf{Conv}_B$ is cartesian closed (cf.[9]). Let $\xi : B' \to B$ be a continuous map. A functor $\xi^* : \mathbf{Conv}_B \to \mathbf{Conv}_{B'}$ is defined by $\xi^* X = B' \times_B X$ with the first projection and $\xi^* f = 1_{B'} \times_B f$. We note that ξ^* has a left adjoint ξ_*, given by $\xi_*(X, p') = (X, \xi \circ p')$ and $\xi_* f = f$. Hence $\xi^*(\prod_B X_i) = \prod_{B'} \xi^* X_i$. Moreover it is easy to see that for continuous maps $\xi : B \to B'$ and $\xi' : B' \to B''$, $(\xi' \circ \xi)^*$ is naturally isomorphic to $\xi'^* \circ \xi^*$.

2. Fibrewise properties

We introduce notions of Hausdorffness, compactness and locally compactness as a fibrewise well-behaved properties. For a convergence space B, a property P_B of convergence spaces over B satisfying the following three conditions is called a *fibrewise property*:

CONDITION 1. If X, Y are isomorphic convergence spaces over B and if X has the property P_B then so does Y.

CONDITION 2. A convergence space X has the property P if and only if the convergence space X over the point $*$ has the property P_*.

CONDITION 3. If a convergence space X over B has the property P_B then the convergence space ξ^*X over B' has the property $P_{B'}$ for each convergence space B' and a continuous map $\xi : B' \to B$.

DEFINITION 2.1. A convergence space X over B is said to be *Hausdorff over B* if whenever a filter $\mathcal{F}$ on X converges to x and y with $x, y \in X_b$, $x = y$, where $X_b = p^{-1}(b)$.

PROPOSITION 2.2. *The property "Hausdorff over B" is a fibrewise property.*

Proof. Condition 1 and 2 follow obviously. For condition 3, suppose X is a Hausdorff convergence space over B. Let B' be a convergence space and $\xi : B' \to B$ a continuous map. Note that for any $b' \in B'$, $(\xi^*X)_{b'} = (B' \times_B X)_{b'} = b' \times X_{\xi(b')}$. Let a filter $\mathcal{F}$ on ξ^*X converge to (b', x) and (b', x') in $(\xi^*X)_{b'}$. Then $\mathrm{pr}_2(\mathcal{F}) \to \mathrm{pr}_2(b', x) = x$ and $\mathrm{pr}_2(\mathcal{F}) \to \mathrm{pr}_2(b', x') = x'$. Since x, x' belong to the same fibre $X_{\xi(b')}$ and X is Hausdorff over B, $x = x'$. Hence $(b', x) = (b', x')$. Thus ξ^*X is Hausdorff over B'.

By definitions it is easy to see that every subspace of a Hausdorff convergence space over B is Hausdorff over B and every product space over B of Hausdorff convergence spaces over B is Hausdorff over B. We remark that even though X is not Hausdorff over B, the space $X \times_B \{b\}$, which is empty or a singleton, is Hausdorff over B trivially.

PROPOSITION 2.3. *A convergence space X over B is Hausdorff over B if and only if the diagonal Δ in $X \times_B X$ is closed.*

Proof. Suppose X is Hausdorff over B and $(x, y) \in \overline{\Delta}$. Then there exists a filter $\mathcal{G}$ on $X \times_B X$ such that $\mathcal{G} \to (x, y)$ and $\Delta \in \mathcal{G}$. Since $\mathrm{pr}_1(\mathcal{G})$=$\mathrm{pr}_2(\mathcal{G})$, $x = y$. Conversely, let $x = y$ in X_b and $\mathcal{F} \to x, \mathcal{G} \to y$. We may assume that $\mathcal{F} \cap \dot{x} = \mathcal{F}$ and $\mathcal{G} \cap \dot{y} = \mathcal{G}$. Since $\mathcal{F} \times_B \mathcal{G} \to (x, y) \notin \Delta = \overline{\Delta}$, there exist $F \in \mathcal{F}$ and $G \in \mathcal{G}$ such that $(F \times_B G) \cap \Delta = \emptyset$ and hence $F \cap G = \emptyset$. Therefore $\mathcal{F} = \mathcal{G}$.

Using this proposition, we have the following by routine work: Let X be a convergence space over B and Y a Hausdorff convergence space over B. For any pair of continuous maps $f, g : X \to Y$ over B, the set $\{x \in X \mid f(x) = g(x)\}$ is closed in X and for any continuous map

$f : X \to Y$ over B the graph $G(f) = \{(x, f(x)) \mid x \in X\}$ is closed in $X \times_B Y$.

Let (X, p) be a convergence space over B. A filter $\mathcal{F}$ on X is called a *b-tied filter* if the filter $p(\mathcal{F})$ has a cluster point b in B. A filter $\mathcal{F}$ on X is called a *b-filter* if the filter $p(\mathcal{F})$ has a limit point b in B. A filter $\mathcal{U}$ on X is called a *b-ultrafilter* if it is a maximal element in the set of b-tied filters with respect to the natural order. We remark that a filter $\mathcal{U}$ on X is a b-ultrafilter if and only if it is an ultrafilter in the ordinary sense and $p(\mathcal{U}) \to b$ in B. We say that a b-tied filter $\mathcal{F}$ on X has a *cluster point* x if $x \in X_b$ and it is a cluster point of $\mathcal{F}$ in X. Similarly, we say that a b-tied filter $\mathcal{F}$ on X has a *limit point* x, (or *converges to* x), if $x \in X_b$ and $\mathcal{F} \to x$ in X.

PROPOSITION 2.4. *Let $\mathcal{F}$ be a b-tied filter on a convergence space X over B and let $x \in X_b$. Then x is a cluster point of $\mathcal{F}$ if and only if x is a limit point of a b-tied filter $\mathcal{G}$ with $\mathcal{G} \supseteq \mathcal{F}$.*

Hence every cluster point of a b-ultrafilter is a limit point. Since every b-tied filter is contained in a b-ultrafilter, every b-ultrafilter converges if and only if every b-tied filter has a cluster point.

DEFINITION 2.5. A convergence space X over B is said to be *compact over B* if the projection $p : X \to B$ is *compact*, i.e., every b-ultrafilter converges.

We remark that this notion is a generalization of that of a *proper* map in [5] (cf.[11] also.). By a routine work it is shown that the property "compact over B" is a fibrewise property.

PROPOSITION 2.6. *Let $\{(X_i, p_i)\}_{i \in I}$ be a family of convergence spaces which are compact over B. Then $\prod_B X_i$ is compact over B.*

Proof. If $\prod_B X_i = \emptyset$, it is obvious. Suppose $\prod_B X_i = \emptyset$. Let $p : \prod_B X_i \to B$ be the projection and $q_i : \prod_B X_i \to X_i$ be the coordinate projections. Let $\mathcal{U}$ be a b-ultrafilter on $\prod_B X_i$. Since $p = p_i \circ q_i$ for all $i \in I$, $p_i(q_i(\mathcal{U})) \to b$. Since X_i is compact over B and $q_i(\mathcal{U})$ is a b-ultrafilter on X_i, there exists $x_i \in (X_i)_b$ such that $q_i(\mathcal{U}) \to x_i$. By the initial property of $\{q_i\}_{i\in I}$, $\mathcal{U} \to (x_i) \in p^{-1}(b)$. Thus $\prod_B X_i$ is compact over B.

PROPOSITION 2.7. *Let $f : X \to Y$ be a continuous map over B.*

(1) *If Y is compact over B and f is a closed embedding, then X is compact over B.*

(2) *If X is compact over B and f is surjective then Y is compact over B.*

Proof. The proofs are routine.

By straightforward adaptations of the definitions, we have the following: If $\xi : B' \to B$ is a compact map, then $\mathrm{pr}_2 : \xi^*X \to X$ is a compact map. For a compact convergence space X over B, if B is compact, then so is X. We note that every compact surjection over B is a closed map. Hence using Proposition 2.3. we can show that for a compact surjection $f : X \to Y$ over B if X is Hausdorff over B, then so is Y.

PROPOSITION 2.8. *Let a convergence space X be compact over B and a convergence space Y be Hausdorff over B. Then every continuous map $f : X \to Y$ over B is compact.*

Proof. The proof is routine by definitions.

PROPOSITION 2.9. *Let $f : X \to Y$ be compact and bijective. If X is pseudo-topological, then f is a homeomorphism.*

Proof. Suppose $\mathcal{F} \to y$ in Y and take any ultrafilter $\mathcal{U}$ on X with $g(\mathcal{F}) \subseteq \mathcal{U}$. Then $f(\mathcal{U})$ contains $\mathcal{F}$ and hence $f(\mathcal{U}) \to y$ in Y. Since f is compact, $\mathcal{U} \to x$ in X and $f(x) = y$ for some x in X. Since f is 1-1 and X is pseudo-topological, $g(\mathcal{F}) \to x$ in X. Note that $x = g(y)$. Therefore g is continuous.

By combining Propositions 2.8. and 2.9., we have the following result which is a fibrewise and improved version of Theorem 10 in [6].

COROLLARY 2.10. *Let a pseudo-topological space X be compact over B and a convergence space Y be Hausdorff over B. Then every bijective continuous map $f : X \to Y$ over B is a homeomorphism.*

DEFINITION 2.11. A convergence space X over B is *locally compact over* B if the projection $p : X \to B$ is locally compact, i.e., for any convergent filter $\mathcal{F}$ on X, there exists $K \in \mathcal{F}$ such that the restriction map $p : K \to B$ is compact.

REMARK. In this definition, if X is Hausdorff over B, then we can show that K is a closed subset of X. Hence for a Hausdorff convergence space over B, our notion 'locally compact over B' is a generalization of that of James in [13]. He defined that a topological space (X, p) over B is locally compact over B if for each $x \in X$ there exists a neighborhood U of x such that the restriction map $p : \overline{U} \to B$ is compact.

PROPOSITION 2.12. *The property "locally compact over B" is a fibrewise property.*

Proof. Condition 1 and 2 are obvious. For condition 3, suppose that a convergence space X is locally compact over B. Let B' be any convergence space and $\xi : B' \to B$ any continuous map. Let a filter $\mathcal{G} \times_B \mathcal{H} = \{G \times_B H \mid G \in \mathcal{G}, H \in \mathcal{H}\}$ converges to $(b', x) \in B' \times_B X$. Then $\mathcal{H}$ converges to x. But X is locally compact over B, thus there exists $A \in \mathcal{H}$ such that $p|_A : A \to B$ is compact. Then $B' \times_B A \in \mathcal{G} \times_B \mathcal{H}$. By Proposition 2.6. $pr_1|_{B' \times_B A} : B' \times_B A \to B'$ is compact. Hence $\xi^* X$ is locally compact over B'.

REMARK. If a convergence space X is compact over B, then X is locally compact over B. Moreover for a locally compact map $\xi : B' \to B$, $\mathrm{pr}_2 : \xi^* X \to X$ is a locally compact map. By Proposition 2.6., we can show that if $\{(X_i, p_i)\}_{i \in I}$ is a family of convergence spaces which are locally compact over B and all but finitely many X_i are compact over B, then $(\prod_B X_i, p)$ is locally compact over B.

3. One-point compactification

We construct a fibrewise version of one-point compactification for a convergence space over B, simultaneously generalizing those notions for a topological space over B and a convergence space.

Let X be locally compact Hausdorff over B, but not compact over B. Let $X_B^+ = X + B$, disjoint union with the natural projection r. We define a map $c : X_B^+ \to \mathcal{P}(\mathcal{F}(X_B^+))$ as follows; For $x \in X$, $\mathcal{F} \in c(x)$ if

$r(\mathcal{F}) \to r(x)$ in B and $X \in \mathcal{F}$ and $\mathcal{F} \cap X \to x$ in X and for $b \in B$, $\mathcal{F} \in c(b)$ if $\mathcal{F} \supseteq \mathcal{G} \cap \dot{b}$, for some b-filter $\mathcal{G}$ on X_B^+ such that $\mathcal{G} \cap X$ has no cluster point. We remark that if $\mathcal{G}_1 \cap \mathcal{G}_2$ has a cluster point, then $\mathcal{G}_1$ or $\mathcal{G}_2$ has a cluster point. Hence it is easy to see that (X_B^+, c) is a convergence space over B and the inclusion map $\iota : X \to X_B^+$ is an embedding over B.

PROPOSITION 3.1. (1) *(X_B^+, c) is compact Hausdorff over B.*
(2) *X is a dense subspace of X_B^+ over B.*

Proof. (1) Let $\mathcal{U}$ be a b-ultrafilter in X_B^+. Then $r(\mathcal{U}) \to b$ in B. We show that $\mathcal{U}$ has a limit point. Suppose $\mathcal{U} \cap X$ has a cluster point x in X, where $x \in X_b$. Then the b-ultrafilter $\mathcal{U} \cap X \to x$ in X and hence $\iota(\mathcal{U} \cap X) = \mathcal{U} \to x$ in X_B^+. Suppose $\mathcal{U} \cap X$ has no cluster point. Then $\mathcal{U} \supseteq \mathcal{U} \cap \dot{b}$ and hence $\mathcal{U} \to b$ in X_B^+. Therefore X_B^+ is compact over B.

Take $x \in X$ and $b \in B$. Suppose $\mathcal{F} \to x, b$ in X_B^+ with $p(x) = b$. Then $\mathcal{F} \cap X \to x$ in X and there exists a b-filter $\mathcal{G}$ in X_B^+ such that $\mathcal{F} \supseteq \mathcal{G} \cap \dot{b}$ and $\mathcal{G} \cap X$ has no cluster point. Since $\mathcal{F} \cap X \supseteq \mathcal{G} \cap X$, $\mathcal{G} \cap X$ has a cluster point x, which is a contradiction. Therefore X_B^+ is Hausdorff over B.

(2) Note that for each $b \in B$, $(X_B^+)_b$ is the one-point compactification of the locally compact Hausdorff convergence space X_b. Hence X_b is dense in $(X_B^+)_b$.

For $B = *$, it is easy to see that X_*^+ is the smallest Hausdorff compactification for a 'locally compact' Hausdorff non-compact convergence space in [22].

In [12], James constructed a fibrewise one-point compactification X_B^* for a locally compact Hausdorff topological space X over B as follows: He takes $X_B^* = X + B$ as we constructed and gives the following topology on it. The generating open sets, before supplementation, are of two kinds. The first kind is the open sets of X, regarded as subsets of X_B^*. The second kind is the complements in X_B^* of the subsets of X which are compact over B. As a matter of fact, this construction coincides with ours in the case of topological spaces.

PROPOSITION 3.2. *Let B be a topological space and X a locally compact Hausdorff topological space over B. Then $X_B^* = X_B^+$ as a topological spaces.*

Proof. Let $x \in X$ and $\mathcal{N}_x$ the neighborhood filter of X in X^*_B. Note that $\mathcal{N}_x$is the filter on X^*_B generated by the family $\{U \cap r^{-1}(W) \mid U$ is an open neighborhood of x in X and W is an open neighborhood of $r(x)$ in $B\}$. Hence $\mathcal{N}_x \cap X \to x$ in X and so $\mathcal{N}_x \to x$ in X^+_B. Let $b \in B$ and let $\mathcal{N}_b$ be the neighborhood filter of b in X^+_B. Then $\mathcal{N}_b$ is the filter on X^*_B generated by the family $\{(X^*_B \backslash K) \cap r^{-1}(W) \mid K$ is a subset of X which is compact over B and W is a open neighborhood of b in $B\}$. Clearly $\mathcal{N}_b$ is a b-filter. Moreover $\mathcal{N}_b \cap X$ has no cluster point since X is locally compact over B. Hence $\mathcal{N}_b \to b$ in X^+_B. Therefore $id : X^*_B \to X^+_B$ is continuous. For the continuity of $id : X^+_B \to X^*_B$, let $x \in X$ and $\mathcal{F} \to x$ in X^+_B. Since $X \in \mathcal{F}$ and $\mathcal{F} \cap X \to x$ in X, $\mathcal{F} \supseteq \mathcal{N}_x$. Let $b \in B$ and $\mathcal{F} \to b$ in X^+_B. Then there exists a b-filter $\mathcal{G}$ in X^+_B such that $\mathcal{F} \supseteq \mathcal{G} \cap \dot{b}$ and $\mathcal{G} \cap X$ has no cluster point. We note that for any neighborhood W of b there exists $G \in \mathcal{G}$ with $G \subseteq r^{-1}(W)$ since $r(\mathcal{G}) \to b$ in B. To prove $\mathcal{F} \supseteq \mathcal{N}_b$, we show that for any $K \subseteq X$ which is compact over B there exists $G \in \mathcal{G}$ with $X^+_B \backslash K \supseteq G$. Suppose not. Then there exists such K with $G \cap K = \emptyset$ for all $G \in \mathcal{G}$. Thus we have a b-filter $\mathcal{G} \cap K$ on K. Since K is compact over B, $\mathcal{G} \cap K$ has a cluster point x in K. In fact, this point x is a cluster point of the b-filter $\mathcal{G} \cap X$. This is a contradiction.

REMARK. This proposition gives a direct proof of Theorem 1.4. in [22]. We note that a Hausdorff topological space is locally compact iff it is open in each Hausdorff compactification. (cf. [24])

Let X and Y be locally compact Hausdorff convergence spaces over B. A function $f : X \to Y$ over B determines a section preserving function $f^+ : X^+_B \to Y^+_B$ over B, and vice versa. Note that $\iota_Y \circ f = f^+ \circ \iota_X$.

PROPOSITION 3.3. *A map $f : X \to Y$ is compact (consider Y as B in Definition 2.5.) if and only if $f^+ : X^+_B \to Y^+_B$ is continuous.*

Proof. let p, q, r, s be projections for X, Y, X^+_B, Y^+_B, respectively. Suppose f is compact. If $\mathcal{F} \to x$ in X^+_B, then $f^+(\mathcal{F}) \to F^+(x)$ in Y^+_B, obviously. Let $\mathcal{F} \to b$ in X^+_B. Then there exists a b-filter $\mathcal{A}$ on X^+_B such that $\mathcal{F} \supseteq \mathcal{A} \cap \dot{b}$ and $\mathcal{A} \cap X$ has no cluster point. Moreover, $f^+(\mathcal{F}) \supseteq f^+(\mathcal{A}) \cap \dot{b}$. Hence $f^+(\mathcal{F}) \to b$ in Y^+_B. Therefore f^+ is continuous. Conversely, let $\mathcal{U}$ be an ultrafilter such that $f(\mathcal{U}) \to y$ in Y and $b = q(y)$. Then $r \circ \iota_X(\mathcal{U}) = p(\mathcal{U}) = q \circ f(\mathcal{U}) \to b$ and hence $\iota_X(\mathcal{U})$ is a b-ultrafilter in

X_B^+. Suppose $\iota_X(\mathcal{U}) \to x$ in X_B^+. Then $\mathcal{U} \to x$ in X and $f(x) = y$, since f is continuous and Y is Hausdorff over B. Suppose $\iota_X(\mathcal{U}) \to b$ in X_B^+. Then $f^+ \circ \iota_X(\mathcal{U}) = \iota_Y \circ f(\mathcal{U}) \to b, y$ in Y_B^+, which is a contradiction. Hence f is compact.

REMARK. In [17] James introduced a weaker notion of fibrewise locally compactness than the corresponding notion in [12]: A topological space (X, p) over B is fibrewise locally compact if for each $x \in X_b$ there exists a neighborhood W of b and a neighborhood $U \subseteq X_W$ of x such that the restriction map $p : X_W \cap \overline{U} \to W$ is compact, where $X_W = p^{-1}(W)$. In fact, by some modification, we can also obtain corresponding results on the weaker notion including the fibrewise Alexandroff compactification in [17].

References

1. E. Binz, *Continuous Convergence on $C(X)$*, Lecture Notes in Mathematics, vol. 469, Springer-Verlag, Berlin, 1975.
2. P. I. Booth, The exponential law of maps I, *Proc. London Math. Soc.* (3) **20** (1970), 172–192.
3. ______, On the application of fibred mapping spaces to exponential laws for bundles, ex-spaces and other categories of maps, *Gen. Top. and its Appl.* **8** (1978), 165–179.
4. P. I. Booth and R. Brown, Spaces of Partial maps, fibred mapping spaces and compact-open topology, *Gen. Top. and its Appl.* **8** (1978), 181–195.
5. N. Bourbaki, *General Topology*, Part 1 (transl.), Addison-Wesley, Reading, 1966.
6. H. R. Fischer, Limesräume, *Math. Ann.* **137** (1959), 269–303.
7. D. B. Fuks, On duality in homotopy theory, *Soviet Math. Dokl.* **2** (1961), 1575–1578.
8. ______, Eckmann-Hilton duality and the theory of functors in a category of topological spaces, *Uspehi Mat. Nauk* **21** (1966), 3–40.
9. H. Herrlich, Topological improvements of categories of structured sets, *Topology Appl.* **27** (1987), 145–155.
10. H. Herrlich and G. E. Strecker, *Category Theory* Heldermann, Berlin, 1979.
11. R. A. Herrmann, Perfect maps on convergence spaces, *Bull. Austral. Math. Soc.* **20** (1979), 447–466.
12. I. M. James, *General Topology and Homotopy Theory*, Springer-Verlag, New York, 1984.
13. ______, General topology over a base, *Aspect of Topology*, London Math. Soc. Lecture Notes, vol. 93, 1984.
14. ______, Uniform spaces over a base, *J. London Math. Soc.* (2) **32** (1985), 328–336.
15. ______, Spaces, *Bull. London Math. Soc.* **18** (1986), 529–559.

16. ———, *Topological and Uniform Spaces*, Springer-Verlag, New York, 1987.
17. ———, *Fibrewise Topology*, Cambridge University Press, London, 1989.
18. L. G. Lewis Jr., Open maps, colimits and a convenient category of fibre spaces, *Topology Appl.* **19** (1985), 75–89.
19. K. C. Min and S. J. Lee, Fibrewise convergence and exponential laws, *Tsukuba J. Math.* **16** (1992), 53–62.
20. K. C. Min and Y. S. Kim, Fibrewise exponential laws in topological universes (1992), preprint.
21. S. B. Niefield, Cartesianness: Topological spaces, uniform spaces, and affine schemes, *J. Pure Applied Alg.* **23** (1982), 147–168.
22. C. J. M. Rao, On smallest compactification for convergence spaces, *Proc. Amer. Math. Soc.* **44** (1974), 225–230.
23. N. E. Steenrod, A convenient category of topological spaces, *Michigan Math. J.* **14** (1967), 133–152.
24. Vinod-Kumar, Compactification of a convergence space, *Proc. Amer. Math. Soc.* **73** (1979), 256–262.
25. R. M. Vogt, Convenient categories of topological spaces for homotopy theory, *Arch. Math.* XXII (1971), 545–555.

Department of Mathematics,
Yonsei University,
Seoul 120-749, Korea

Department of Mathematics,
Chungbuk National University,
Cheongju 360-763, Korea