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Generalized Frequency-wavenumber Migration Implemented by the Intrinsic Attenuation Effect

비탄성 매질의 진폭 감쇠 효과를 첨가한 일반화된 주파수-파수 구조보정

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Abstract: A method and results of computations are presented for the 2-D seismic migration process in the frequency-wavenumber domain for the laterally and vertically inhomogeneous medium. In order to take the intrinsic attenuation effect into account in the migration process the complex-valued wave velocity is used in the wavefield extrapolation operator, improving the generalized frequency-wavenumber migration technique. The imaginary part of the complex-valued wave velocity includes the seismic quality factor Q value. In derivation of the solution of the wave equation for the medium of inhomogeneous wave velocity and anelasticity, the inhomogeneous medium is mathematically converted to an equivalent system which consists of a homogeneous medium of averaged slowness and an inhomogeneous distribution of hypothetical wave source. The strength of the hypothetical wave source depends on the deviation of squared slowness from the averaged value of the medium. Results of numerical computation using the technique show more distinct geologic images than those using the convensional generalized frequency-wavenumber migration. Especially, the obscured images due to the wave attenuation by anelasticity are restored to show sharp boundaries of structures. The method will be useful in the imaging of the reflection data obtained in the regions of possible petroleum or natural gas reservoir and of fractured zone.

Key Words: generalized frequency-wavenumber migration, Q structure, inhomogeneous medium, complex-valued velocity, wave field extrapolation.

요 약

지진파의 전파속도와 감쇠정도가 불균질한 매질에서 2차원 지진파 단면자료에 대한 주파수-파수 영역에서의 구조보정 방법 및 그 결과를 제시한다. 파동전파의 감쇠효과를 포함시키기 위해 일반화된 주파수-파수 구조보정 방법을 개선하여 파동장의 상향 및 하향 외삽연산자에 복소수 전파속도를 사용한다. 이 복소수 전파속도의 허수 부분은 지진파 감쇠 척도인 Q 값을 포함하도록 한다. 불균질한 전파속도와 비탄성을 가진 매질 속에서의 전파방정식의 해를 얻기위해, 불균질한 매질자체를 일정한 전파속도와 비탄성을 갖는 평균적 매질과 가상적 파동원의 불균질한 분포로 규합된 등가의 시스템으로 취급한다. 가상적 파동원은 전파속도와 매질비탄성의 불균질 정도에 따라 그 세기가 좌우된다. 이 방법에 의해 수 개의 구조 모델에 대해 수치적으로 계산된 결과는 기존의 일반화된 파수-주파수 구조 보정 방법에 의한 것보다 더욱 선명한 단면 영상을 보여주며 파의 비탄성에 의해 불명확하게된 영상신호가 복원된다. 이 방법은 석유나 천연가스가 부존된 구조 또는 파쇄대가 존재하는 지역에서 획득된 자료를 구조보정하는데 유용하게 쓰일 것이다.

주요어: 일반화된 주파수-파수 구조보정, 불균질 매질, Q-구조, 복소수 전파속도, 파동외삽 연산자

INTRODUCTION

Among the four steps of the seismic process, i.e., data acquisition, data processing, migration and interpretation in the seismic reflection method, the migration is a process of imaging the geological subsurface structure and take an important role in qualifying the final data for the interpretation process. There are three different type of migration method depending on numerical techniques and procedures: Kirchhoff integral type of diffraction method, Fourier transform method, and finite difference method.

The Kirchhoff summation method (Schneider, 1971, 1978; French, 1974) in space-time domain uses the Kirchhoff integral technique to deal with laterally inhomogeneous media. The computational time of the method depends on the complexity of the slope structure of wave scatterers. The phase shift method of Gazdag (1978) does not have problems of such a slope structure. However, the method needs Fourier transform operation at every depth step of the wavefield extrapolation process. The F-K migration of Stolt (1978) takes short time of process, since it migrates the data down to the total depth at one step of computation. The vital flaw

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of the method is the limitation of the structural model to a laterally and vertically homogeneous medium. The migration of the finite difference method (Claerbout, 1970; Loewenthal *et al.*, 1976) can be applied to a laterally and vertically inhomogeneous medium. Disadvantage of the method is that it takes long computational time and that it does not work for a very steep distribution of wave scatterers. Most of the flaws of the above method can be removed by using the generalized F-K migration (Pai, 1988) for the laterally and vertically inhomogeneous medium. The method can handle with steep structures. It takes much smaller computational time than the migration by the finite difference method does.

Seismic or acoustic waves suffer attenuation of their amplitudes more or less depending on the type of media they propagate. The amplitudes of the waves are differentially attenuated by the inclusion of water, hydrocarbon or natural gas in the medium. Seismic records obtained from the reflection method often contain signals of reduced amplitudes caused by the intrinsic attenuation property of the medium. Such signals of reduced amplitudes cannot be restored by conventional migration methods. Thus the migration process results in obscured images of structures. The intrinsic attenuation of seismic or acoustic waves in a medium is caused by the anelasticity of the medium. Usually the seismic quality factor Q is used for the measure of the intrinsic attenuation. It is well known that the mathematical equivalence of the wave attenuation effect is aquired if the propagation velocity of the wave is allowed to be complex-valued. The imaginary part of the complex-valued velocity includes the seismic quality factor Q value. This type of velocity can be used in the wavefield extrapolation operator of the migration method for the restoration of obscured images caused by to the attenuation of signals.

In this study, a method is developed to restore the attenuated signal in migration process. The generalized F-K migration technique which is supposed to be superior to other methods as stated above is implemented by the attenuation effects. In the mathematical formulation of the wavefield extrapolation operator, the seismic quality factor Q value is included in the imaginary part of the complex-valued wave velocity. The structures of the laterally and vertically inhomogenous velocity and Q value are considered in the migration. Thus the restored images will show distinct shapes of geologic structures.

THEORY AND COMPUTATIONAL METHOD

The laterally inhomogeneous layered structure is assumed in the derivation of the theory to simulate the vertically and laterally inhomogeneous medium of the 2-D structure. Each layer is vertically homogeneous, but has lateral distribution of the wave velocity and anelasticity property. The anelasticity property is represented by the seismic quality factor Q.

In the frequency-wavenumber domain, the wave equation in the layer can be written by

$$\frac{\partial^2}{\partial z^2}\Psi(k;z)+\omega^2 S(k)*\Psi(k:z) - k^2\Psi(k;z) = 0 \tag{1}$$

where Ψ is the acoustic wave function, ω the angular frequency, k the horizontal wavenumber, and $S(k)$ the squared slowness in the wavenumber domain. The notation * indicates convolution operation in k variable. In order to take care of the wave attenuation due to the anelasticity, the complex-valued squared slowness (Carpenter, 1966; Aki and Richards, 1980).

$$S(x) = \frac{1}{c^2(x)}\left[1-sgn(\omega)\frac{i}{2Q(x)}\right]^2 \tag{2}$$

is used, where c is the wave propagation velocity, Q the seismic quality factor, i the unit imaginary value. The symbol $sgn(\omega)$ indicates the plus or minus sign of the angular frequency ω.

In order to get the solution of Eq. (1) in a easier way, $S(k)$ is decomposed into two parts.

$$S(k) = S_o+\Delta S(k) \tag{3}$$

where S_o indicates the squared slowness at zero wavenumber value, i.e., $S(0)$, and $\Delta S(k)$ is the series of $S(k)$ values in which the $S(0)$ value is replaced by zero. In the space domain, S_o and $\Delta S(x)$ imply the average and the deviation from the average value of the squared slowness $S(x)$, respectively. The substitution of Eq. (3) into Eq. (1).

$$\left[\frac{\partial^2}{\partial z^2}+(\omega^2 S_o-k^2)\right]\Psi(k;z) = \Omega(k;z) \tag{4}$$

where $\Omega(k;z) = -\omega^2\ \Delta S(k)*\Psi(k;z)$.

This is an inhomogeneous differential equation with a source distribution of $\Omega(k;z)$. The left hand side of the equation implies wave propagation in a homogeneous medium with the constant squared-slowness of S_o. Hence, the wave propagation in a laterally inhomogeneous layer governed by equation (1) is transformed to an equivalent system in which the wave propagates through a homogeneous layer with the acoustic source distribution. The concept and interpretation of the transformation is illustrated in Figure 1. The wave amplitude attenuates even in the homogeneous layer since the averaged squared-slowness can comprise the non-zero Q value. Unfortunately the source distribution $\Omega(k;z)$ is inhomogeneous vertically as well as laterally in the layer. This fact limits the depth interval to a small amount in the wavefield extrapolation scheme of migration process.

The solution of Eq. (4) consists of the homogeneous equation's solution $\varphi(k,z)$ and the paticular solution. The total

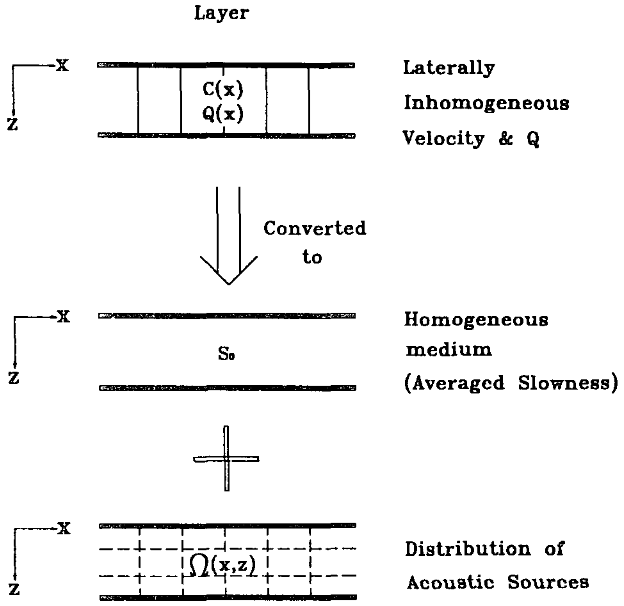

Fig. 1. Schematic diagram for the explanation of the main concept of the generalized frequency-wavenumber migration implimented by the intrinsic attenuation effect. C(x) and Q(x) represent horizontal distribution of the wave velocity and the seismic quality factor, respectively. S_o indicates the horizontally averaged squared-slowness of complex value. $\Omega(x,z)$ represents ficticious mathematical source distribution in the layer. Thus the wave propagation in a laterally inhomogeneous layer is transformed to an equivalent system in which the wave propagates through a homogeneous layer with an acoustic source distribution.

solution becomes

$$\Psi(k;z) = \varphi(k;z) + G(k;z,z')\ \Omega(k;z')\ \Delta z \tag{5}$$

using the Born approximation and small depth interval Δz $(=z-z_o)$. Here, G is the Green's function for source distribution of the delta function $\delta(z-z')$. Since the solution $\varphi(k;z)$ is composed of the wave propagation part, i.e., wave function $P(k;z,z')$ and the boundary value $\varphi(k;z')$, the total solution can be written as

$$\Psi(k;z) = P(k;z,z')\ \varphi(k;z')\ +\ G(k;z,z')\ \Omega(k;z')\ \Delta z \tag{6}$$

Using the property of wave function $P(k;z',z')=1$, we obtain the relation $\varphi(k;z')=\Psi(k;z')$ by substitution of zero value of Δz to Eq. (6). Therefore, Eq. (6) becomes

$$\Psi(k;z) = [P(k;z,z') - \Delta z\ \omega^2\ G(k;z,z')\Delta S(k) *]\Psi(k;z') \tag{7}$$

The part in the bracket parenthesis in this equation works as the wavefield extrapolation operator. In practical computations, the convolution operation $\Delta S(k) * \Psi(k,z')$ in k-domain can be done in x-domain by direct multiplication operation $\Delta S(x)\Psi(x,z')$ in order to reduce computational time. And then the result is transformed to k-domain to continue the next operation of the multiplication by the Green's function.

MODEL STUDY

In order to validate the method presented in the previous section, synthetic sections are migrated for three simple but plausible model structures. The model shown in Figure 2c consists of a plane reflector at 10 km depth and two different media with a vertical contact. Both of the two media have the same wave velocity, 5 km/sec. However, the media on the left and right sides are assigned Q values of 900 and 50, respectively. The zero-offset synthetic seismogram (Fig. 2a) shows attenuated signals at the 2 seconds position of one way travel time in the horizontal range between 10 km and 20 km due to the low Q value of 50 in the medium of the right side. The wave travelled through the right side of the meditm suffers severe loss of energy. The migrated section (Fig. 2b) obtained by the conventional method could not restore the image of obscured reflector signals, even though the amplitude of signals are a little more enhanced than those in the synthetic seismogram. On the other hand the migration result (Fig. 2d) computed by the technique presented in this paper gives us a full recovery of the strong reflector at 10 km depth.

Figure 3c shows a little more complicated model than that of Figure 2c. The box structure in the middle of the model has a relatively low Q value of 20 compared to the value 900 of the surrounding material. In the synthetic seismogram (Fig. 3a) the waves reflected from the top of the box structure show diffraction curves on the left and right edges. The signals travelled through the box and reflected from the bottom of the box structure have relatively low amplitudes and earlier arrival times compared with other signals at 2 seconds of one way arrival time. The phenomenon is caused by the fact that the box structure has a higher velocity of 5.2 km/sec and a lower Q value of 20 compared with the surrounding medium. The migrated section (Fig. 3b) generated by the conventional method shows a smeared shape of reflector at the bottom of the box structure since the attenuation effect is not considered in the migration process. However, the migrated section in Figure 3d which is computed by the technique given in this paper reveals a complete reflector shape at the depth of 10 km.

A more realistic model of geologic structure is prepared in Figure 4c. The structure consists of sedimentary strata, a reverse fault at center of the figure, a natural gas or petroleum reservoir under the fault, and a cut-and-filling structure at about 4 km depth. The suspicious gas or petroleum reservoir has a little low velocity and a very low Q value of 10 compared with surrounding materials. The synthetic shown in Figure 4a has complicated diffraction signals preventing us from direct interpretations. The time section distorts the true shape of the reservoir. Especially the signal corresponding to the bottom of the reservoir cannot be seen clearly

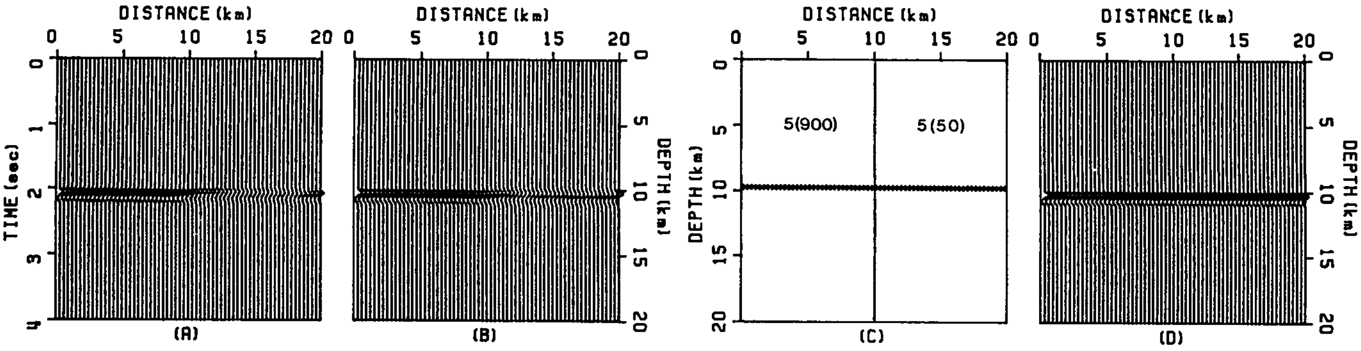

Fig. 2. Comparison of migration results for a model of a plane reflector below two different Q structures.
(A) Zero-offset synthetic seismogram. Scale in the time axis is the one-way travel time in second.
(B) Migration result of the synthetic given in (A) using conventional method without consideration of the intrinsic attenuation effect.
(C) Structural model. The dotted line indicates the reflector. Numbers outside and inside of the parenthesis indicate the wave propagation velocities in km/sec and Q values, respectively.
(D) Migration results of the synthetic given in (A) using the migration technique with consideration of the intrinsic attenuation effect.

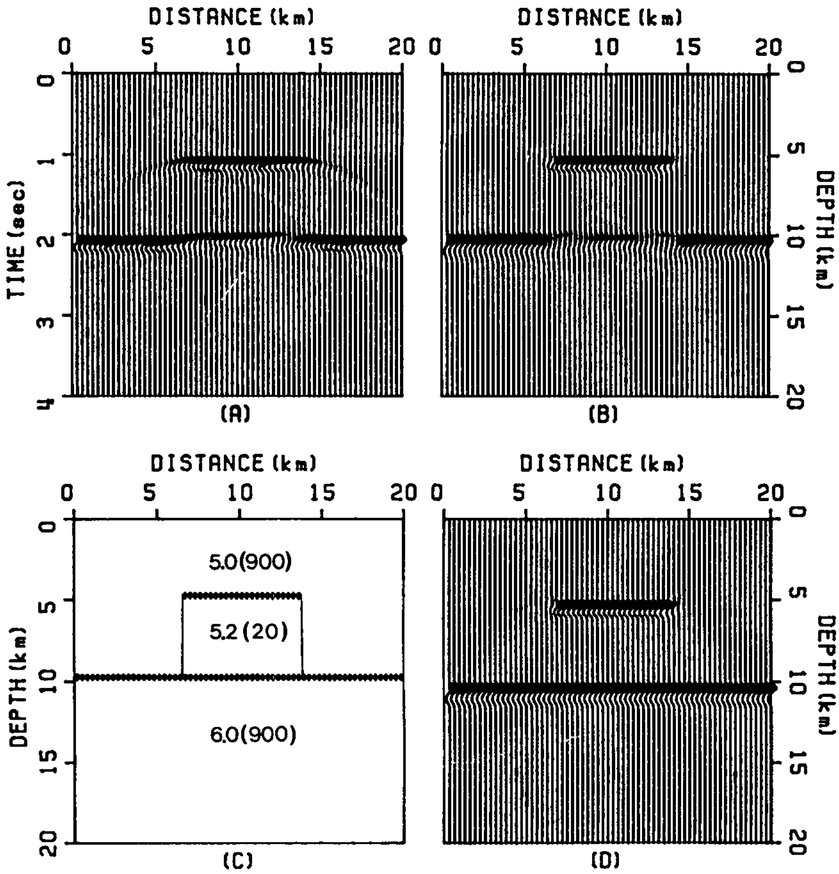

Fig. 3. Comparison of migration results for a box model structure with a low Q value. Details of explanations on sections (A)-(B) are given in Fig. 2.

due to the loss of energy during the propagation of the wave through the reservoir with a very low Q value. Therefore it is not easy to interprete the time section directly without a migration processing. Even after processing the migration using the conventional method (Fig. 4b) one may not realize the importance of the lower boundary of the reservoir due to obscured shape of signal. However, most of the features of the model structure are imaged well by the migration

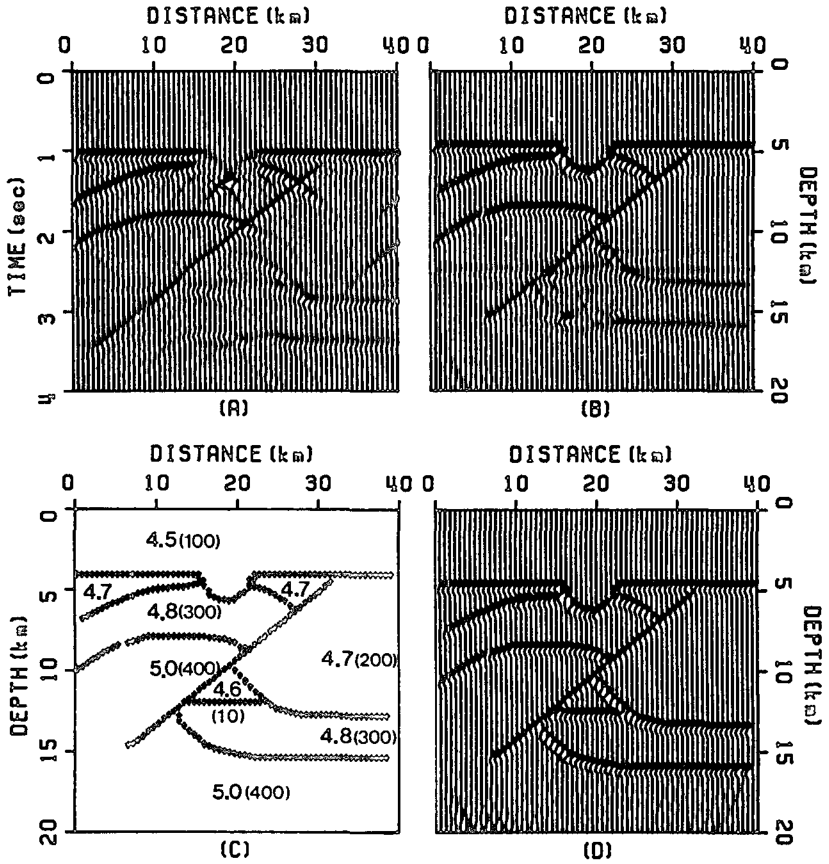

Fig. 4. Comparison of migration results for a geological model. The structure consists of sedimentary strata, a reverse fault, a natural gas or petroleum resorvoir and a cut-and-filling structure. Details of explanations on sections (A)-(B) are given in Fig. 2.

technique presented in this paper which takes the intrinsic absorption effect into account (Fig. 4d). Thus the migration with consideration of the Q value effect recovered the image of the bottom boundary of the highly attenuating reservoir medium and the structure below it.

DISCUSSION AND CONCLUSION

A technique for restoration of smeared image due to the intrinsic attenuation of the signal is applied to the conventional generalized frequency-wavenumber migration of the seismic section. The medium is allowed to be a vertically and laterally inhomogeneous velocity and Q structure. In order to take the intrinsic attenuation effect into account the complex-valued wave velocity is used in the wave field extrapolation operator. The imaginary part of the complex-valued wave velocity includes the seismic quality factor Q value.

Results of numerical computations using the technique for three model structures show more distinct geologic images than those using conventional generalized frequency-wavenumber migration. Especially the obscured images due to the highly attenuating medium of the petroleum or gas reservoir are restored to show sharp boundaries of structures. The method will be useful for the imaging of the reflection data obtained in the regions of possible petroleum or natural gas reservoir and of fractured zones.

In practical application of the method, however, priori information on the approximate geologic structure should be known before the application of the new technique. Therefore, the trial migration processes of a conventional method is recommended to be done first. Then the expected parameters on the structure and properties of the media obtained from the first step of migration are used as input conditions in the final migration process using the method presented in this paper.

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