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축방향으로 압축을 받는 GFRP 원통형 판넬의 유한요소 모델링

Finite Element Modelling of Axially Compressed GFRP Cylindrical Panels

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Abstract

In order to promote the efficient use of composite materials, effort is currently being directed at the development of design criteria for composite structures. Insofar as design against buckling is concerned, it is well known that, for metal shells, a key step is the definition of 'knockdown' factors on the elastic critical buckling stress accounting mainly for the influence of initial geometric imperfections. At present, the imperfection sensitivity of composite shells has not been explored in detail. Due to the large number of parameters influencing buckling response (considerably larger than for isotropic shells), a very large number of tests would be needed to quantify imperfection sensitivity experimentally. An alternative approach is to use validated numerical models for this task. Thus, the objective of this paper is to outline the underlying theory used in developing a composite shell element and to present results from a validation exercise and subsequently from a parametric study on axially loaded glass fibre-reinforced plastic (GFRP) curved panels using finite element modelling. Both eigenvalue and incremental analyses are performed, the latter including the effect of initial geometric imperfection shape and amplitude, and the results are used to estimate 'knockdown' factors for such panels.

요 지

복합재료를 효과적으로 사용하기 위하여 복합재료 구조물의 설계기준 개발을 위한 연구가 진행되고 있다. 금속 쉘의 좌굴에 대한 설계시에는 초기 결함의 영향과 탄성임계좌굴응력을 근거로 한 녹다운 계수(Knock-down factor)를 정의하는 것이 중요한 과정이나 복합재료 쉘의 좌굴에 대한 설계시에는 초기 결함에 대한 민감도가 거의 연구되어 있지 않은 실정이다. 복합재료 쉘의 좌굴거동에 영향을 주는 설계변수는 많기 때문에, 쉘의 설계시 이 변수들로 인한 초기 결함 민감도를 분석하기 위하여 많은 실험을 필요로 하고 있으며 실험 이외의 다른 방법으로서는 이미 검증된 수치모델을 사용하는 것이다.

본 논문에서는 복합재료 쉘요소를 개발하는데 사용된 이론을 요약, 정리 하였으며 수치예제를 통하여 본 연구에서 제안한 쉘요소의 정확성을 검증하였다. 그리고 축방향으로 압축을 받는 GFRP 곡선형

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판넬의 설계시 고려해야 하는 각 변수들을 다양하게 변화시키면서 좌굴거동에 미치는 영향을 유한요소 모델링에 의해 고찰하였다. 방법으로서 초기 결함 및 두께의 진폭을 고려한 비선형 해석과 고유치 해석을 수행하였으며 이 결과를 이용하여 녹다운 계수를 산출하였다.

1. Introduction

Composite materials are increasingly being used in a large variety of structures. Where high strength and stiffness to weight ratios are important, composite shell structures can be an attractive alternative to more conventional forms of consturction. Up to now, most of the work in this area, is concerned with 'advanced' carbon-fibre reinforced plastics (CFRP) used in the aerospace industry although systematic design data are still lacking for many practical cases and, in particular, for less costly laminates, such as glass-fibre reinforced plastics (GFRP), which are more likely to make an impact in general civil and offshore structural applications.

Most shell structures are thin, hence buckling response is a major design consideration. A considerable amount of design guidance is available for isotropic shell strucures but this is not the case for anisotropic shells. This is particularly true in the case of buckling strength prediction, where many problems are left to the designer and, therefore, high safety factors, that reduce the overall structural efficiency, are frequently used. In general, when results from experimental buckling studies of composite shells are compared with solutions from linear buckling theory, it is noted that the experimental results are lower than the corresponding theoretical predictions. At present, research is concerned with the influence of nonlinear pre-buckling and boundary conditions, as well as with post-buckling and boundary conditions, as well as with post-buckling behaviour and imperfection sensitivity, i.e. investigating the same factors that were found to be important in the behaviour of isotropic shells. At the same time, work is being carried out into effects that are of particular relevance to composite laminated structures, such as transverse shear deformation and failure related to material deterioration.

Shells are, in general, known to exhibit unstable post-buckling behaviour. However, cylindrical panels under compression may exhibit either a shell-type unstable behaviour or a stable plate-type behaviour depending on geometry and boundary conditions. As explained in Koiter's pioneering work,[1] the nature of the post-buckling behaviour may be determined by the narrowness or curvature parameter

$$\Theta_K = \frac{1}{2\pi}\left[12(1-\nu^2)\right]^{1/4}\frac{b}{\sqrt{Rt}} \tag{1}$$

In the range $0 \leq \Theta_K \leq 1$, the critical buckling stress increases from the value corresponding to a flat panel to that of a complete cylinder. A change from stable to unstable post-buckling behaviour must, therfore, occur within this range. For pinned supports along the longitudinal edges, the transition value is $\Theta_K \approx 0.64$. If $\Theta_K \geq 1$, the critical buckling stress of the panel is equal to that of a complete cylinder. The obove remarks apply to perfect isotropic panels but, in reality, panels will contain geometric imperfections, which reduce the buckling strength.

Early experimental studies[2,3] did not address in detail imperfection measurement or their effect on buckling strength. On the other hand, theoretical work[4,5] clearly demonstrated the effect of anisotropic properties on both critical buckling and limit loads of laminated curved panels. In particular, the effect of membrane-bending coupling in asymmetric laminates was shown to be similar to that of a geometric imperfection, insofar as it introduced bending from the beginning of loading. A comprehensive study on wide axially compressed CFRP panels is presented in.[6] The imperfection sensitivity was studied using numerical models and from several strategies adopted, the best comparison with experimental results was obtained by using imperfection shapes in he form of the critical eigenmode. Experimental knockdown factors (test load/linear buckling load) were about 80% or more, indicating reduced imperfection sensitivity compared to isotropic panels. More recently, analytical models for stiffened curved panels have been presented, thus allowing realistic boundary conditions to be applied,[7] and some design aspects for symmetric CFRP laminates have been considered.[8].

Finite element models, used in conjunction with a limited number of physical experiments, offer the best possible method for quantifying imperfection sensitivity and producing design data for composite shell panels. In turn, this entails the development of multi-layered composite shell elements accounting for geometric non-linearities and failure behaviour of composite materials. In many cases, this process involves the extension of isotropic elements, where the integration is carried out for each layer. This leads to excessive computational time for the nonlinear analysis of composite structures. Thus, it is essential to develop an efficient shell element for the nonlinear analysis of laminated composite structures. However, it is beyond the scope of this paper to present a review of finite element models for laminated plates and shells, for which the reader is referred to for example.[9,10]

In [11], a nonlinear composite shell element was developed and validated with reference to several experimental, analytical and numerical results. Using this element, an extensive parametric study on GFRP cylindrical panels under axial compression has been carried out for civil/marine structural applications and is reported herein. The results are presented in terms of "knock-down" factors which may be used in developing design recommendations.

2. Nonlinear Finite Element Analysis for Composite Shells

2.1 Brief description of Nonlinear Composite Shell Element

The finite element analysis was undertaken using a general purpose nonlinear FE package developed at Imperial College.[12] The program is particularly suited to structural stability problems (due to a number of strategies available for nonlinear solutions) and has been used extensively in the modelling of thin-walled structures. The finie element used in the current study[9] is an eight-noded isoparametric shell element with six degrees of freedom per node. The principle of minimum potential energy is employed for deriving the governing equations describing the behaviour of the composite shell element. The geometrical nonlinearity considered in the present work, which asumes small strain and large displacement analysis, is based on the updated Lagrangian method. In order to remove the rigid body rotations, the co-rotational method developed by Bates[13] is used. In this method, the displacement field is referred to a set of local co-rotational coordinates. The polar decomposition theory is used to derive the co-rotational formulation, in which the motion is decomposed into either a rigid translation followed by rigid rotation, or rigid rotation followed by rigid translation. In other words, the deformation can be isolated by removing the rigid body rotation from the total nodal displacements.

The laminate is treated as an equivalent single layer and it is assumed that this layer is elastic and that perfect bonding exists between layers. Based on laminate theory, the composite shell element is analytically integrated through the thickness.

Transverse shear deformation effects are included using a first-order (Mindlin-Reissner) theory, in order to allow modelling of relatively thick plates and shells. However, it is generally accepted that the effect of transverse deformations is more significant in laminates compared to isotropic structures due to high moduli ratios (typically $E/G \approx 2.6$ for isotropic material but $E_L/G_{LT} \approx 6$ for GFRP laminae and $E_L/G_{LT} \approx 80$ for GFRP laminae). Ziegler[14] showed that the presence of transverse shear decreases the buckling load of an isotropic beam. Noor[15] compared the buckling solution of a composite plate based on classical plate theory with the exact solution considering transverse shear deformation. He concluded that the error introduced by classical plate theory is strongly dependent on the thickness ratio and that classical plate theory is not adequate for uniaxial buckling with thickness/length>0.05. Khdeir[16] presented similar results on the shear deformation effects of anti-symmetric angleply laminates using a Levy-type classical solution. By comparing the buckling solutions corresponding to classical theory and first-order shear deformation theory, he concluded that the results obtained by classical theory can produce significant errors in buckling problems. In this respect, for buckling analysis of laminated structures, the transverse shear effects must be included in both the linearized and the geometric stiffness matrix.

On the other hand, in order to avoid shear locking in very thin plates and shells, reduced integration and so-called "special energy balance" techniques are used to control the shear strain energy at the Gaussian integration points. By using six degrees of freedom, the present element can model stiffened composite plate and shell structures, composite plates and shells with isotropic and/or composite stiffeners, folded plates, shell junctions, box-junctions, etc.

Using the small strain assumption,[17] the updated Lagrangian form of the total potential energy ina co-rotational coordinate system has the following form

$$\Pi=\frac{1}{2}\int(\Delta^{t}N\Delta^{t}\hat{e}_{m}+\Delta^{t}M\Delta^{t}\hat{e}_{b}+\Delta^{t}Q\Delta^{t}\hat{e}_{s})dS$$

$$+\frac{1}{2}\int({}^{t}N\Delta^{t}\hat{\eta}_{m}+{}^{t}M\Delta^{t}\hat{\eta}_{b}+{}^{t}Q\Delta^{t}\hat{\eta}_{s})dS \qquad (2)$$

$$+\frac{1}{2}\int({}^{t}N\Delta^{t}\hat{e}_{m}+{}^{t}M\Delta^{t}\hat{e}_{b}+{}^{t}Q\Delta^{t}\hat{e}_{s})dS-{}^{t+\Delta t}\Delta V$$

where t and $t+\Delta t$ denote the current and next configuration respectively. Using the principle of stationary potential energy ($\delta\Pi=0$), the incremental equilibrium equation, from which the total tangent stiffness matrix and internal forces are derived, is obtained. The full formulation is presented in[11].

2.2 Linear Buckling Analysis

If the deformations up to the critical point are neglected then the initial displacement matrix a in the following equation (3) is identically equal to zero. As the response of the structure prior to buckling is limited to a linear elastic behaviour, all stresses vary proportionally to a loading parameter. The stability condition may then be written as follows:

$$(K_{O}+K_{G})\ a=0 \qquad (3)$$

By applying to the structure a reference level of loading and carrying out a linear eigenvalue analysis, the critical buckling load is obtained. In adition, eigenvector displacements describe any possible buckling mode which the structure may wish to adopt.

2.3 Nonlinear Buckling Analysis

In nonlinear solution, the equilibrium equation must be satisfied throughout the complete history of loading and the nonlinear processing will be stopped only when the out of balance forces are negligible within a certain convergence limit. Solving for the displacement, the displacement must be updated using the current incremental displacement. The incremental nonlinear equilibrium equation can be written as

$$({}^{t}K_{0}+{}^{t}K_{G})\Delta\hat{a}={}^{t+\Delta t}F-{}^{t}f \qquad (4)$$

In general, it is necessary to apply an iterative technique derived from Newton-Raphson method for the solution of the system of nonlinear equations. However, it is impossible to solve the post-buckling problem with snap-through behaviour through a general nonlinear solution algorithm based on Newton-Raphson method. This is due to the singularity which arises in the tangent stiffness matrix near the limit point. To extend the stability analysis beyond the limit point, i.e. in the post-buckling range, more appropriate procedures must be applied. One approach is to use the arc length control method in conjunction with Newton-Raphson method.[18]. In the finite element analysis package used, such an algorithm has been implemented together with alternative strategies

Table 1. Non-dimensionalized buckling loads $\left(\frac{N_{cr} b^2}{E_T h^3}\right)$ for cross ply plates (a/h=10) with various number of layers

NL (Number of Layers)	3-D Elasticity (Ref. 15)	HSDT (Ref. 16)	FSDT (Ref. 16)	CPT (Ref. 16)	Present
3	22.8807	22.312	22.315	26.160	22.3182
5	24.5929	24.727	24.574	36.160	24.5501
9	25.3436	25.671	25.495	36.160	25.4984

based on displacement rate control.[19]

2.4 Numerical Examples

As a first validation example, the buckling load of a moderately thick composite plate (thickness ratio a/h=10) with 3, 5 and 9 layers (0°/90°/0°…) obtained using the above element is compared with the results from first-order transverse shear deformation theory (FSDT), higher order transverse shear deformation theory (HSDT), 3-D elasticity theory and classical plate theory (CPT). Comparison of results is given in Table 1, and satisfactory agreement is obtained for the cases considered.

Furthermore, the results of an incremental nonlinear analysis for an asymmetric cross-ply (0°/90°) boron-epoxy laminate (thickness ratio a/h=100) are presented (Fig. 1). The reference solution is the analytical approach followed by Zhang and Matthews[4,5]. Due to asymetry about the middle-surface, bending due to membrane-bending mechanical coupling is induced from the beginning of loading. The results shown in Fig. 1 reveal good agreement, in geneal, but exhibit some small discrepancy mainly in the vicinity of the critical load. As pointed out by Sheinman and Frostig,[7] this may be due to the type of Galerkin solution adopted in [5]. In fact, comparison with the limit load obtained using a modified Galerkin solution, which minimizes the error introduced by the truncation of the series and the partial fulfilment of the boundary conditions,[7] is in very good agreement with the finite element solution.

3. Numerical Modelling of GFRP Curved Panels

As mentioned before, shell structures under inplane loading lose stability at much lower loads

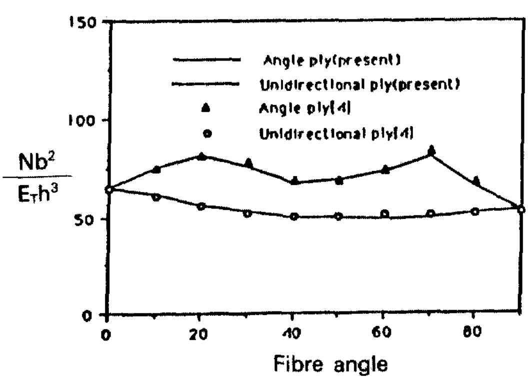

Fig. 1. Comparison of finite element results with analytical solutions (boron-epoxy cross ply panel)

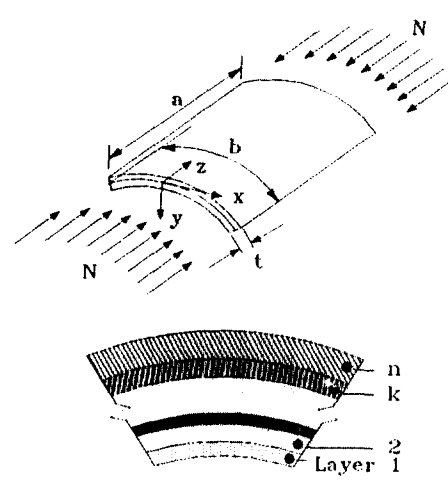

Fig. 2. Geometry and ply numbering

than those calculated from linear buckling analysis. Calculating the critical buckling load by eigenvalue analysis as well as tracing the full equilibrium path using nonlinear analysis have been among the most important and challenging finite element problems in structural engineering. In the following, the behaviour of axially compressed GFRP panels is investigated using the shell finite element described above.

The overall geometry of and loading on the curved panel are shown in Fig. 2. The aspect ratio was kept constant ($a/b=1$) and the total thickness was also kept fixed at 2.5 mm. Four curvature values were investigated, namely $R/t=100$, 200, 400 and 1000. In view of the constant aspect ratio, the arc angle varied between 57.3° and 5.7° respectively. The curvature parameter defined by Koiter[1] varied between 2.9 and 0.9, implying that if the panels were isotropic, they should all exhibit unstable post-buckling behaviour. The boundary conditions on the panels were specified using local coordinates to enable tangential and radial constraints to be imposed and, with reference to a typical set of local axes shown in Fig. 2, are given by:

$$u_x = u_y = \Psi_y = \Psi_z = 0 \quad \text{(Curved edges)}$$
$$u_x = u_y = \Psi_x = \Psi_y = 0 \quad \text{(Straight edges)}$$

Thus, the panels are assumed to be simply supported and restrained. In order to avoid the limitations imposes by symmetry lines, the entire panel was modelled and the load was applied on both sides, with appropriate constraints introduced to avoid rigid body motions.

In terms of material modelling, each panel is composed of twenty plies (ply thickness=0.125 mm), whth the following properties assumed for each ply (typical GFRP material):

$$E_L=5.38\times10^4\ \text{N/mm}^2;\ E_T=1.79\times10^4\ \text{N/mm}^2;$$
$$G_{LT}=8.9\times10^3\ \text{N/mm}^2;\ V_{LT}=0.25$$

The lay-up configurations considered are:

(a) Angle-ply $(\theta/-\theta)_{5s}$, where $\theta=0°$, 15°, 30°, 45°, 60°, 75°, 90°

(b) Cross-ply $(90°/0°)_{5s}$

(c) Quasi-isotropic $(0°/18°/-18°/36°/-36°/54°/-54°/72°/-72°/90°)_s$

The last lay-up is considered a 'quasi-isotropic' configuration on the basis that the extensional stiffness {A} satisfies the following conditions $A_{11}=A_{22}$, $A_{11}-A_{22}=2A_{66}$ and $A_{16}=A_{26}=0$. However, it is clear that the bending stiffness {D} does not satisfy isotropic criteria and that bending-twisting coupling exists. This coupling is also present in angle-ply cases but vanishes in the cross-ply configuration. All laminates are symmetric about the middle-surface in order to eliminate the effect of mechanical membrane-bending coupling, i.e. $\{B\}=0$.

Before proceeding with the full parametric study, the influence of the finite element mesh was quantified. Several cases were studied in order to examine the effect of boundary conditions (restrained vs. unrestrained), panel curvature ('shell' type vs. 'plate' type) and convergence of eigenvalue clusters. As expected, panels with high curvature required a finer mesh and the same was true in the case of restrained panels (compared to unrestrained panels). As a result, a 12×12 mesh was adopted for $R/t=100$ and a 8×8 mesh for all other R/t values.[11]

4. Analysis of Results

4.1 Linear Buckling Analysis

The first step in the parametric study consisted of eigenvalue analyses for the various panels examined. This type of analysis has several limitations in shell buckling problems but can still provide some useful information, first as a preliminary assessment of ply orientation on buckling strength and, secondly, as a guide in selecting appropriate imperfection modes for nonlinear analysis.

The results show that for constant R/t values, the effect of ply orientation on the critical load of angle-ply panels is, in general, fairly small but more noticeable for low R/t. Thus, the range of critical buckling loads at $R/t=100$ was between 409~436 N/mm, with $\theta=45°$ corresponding to the

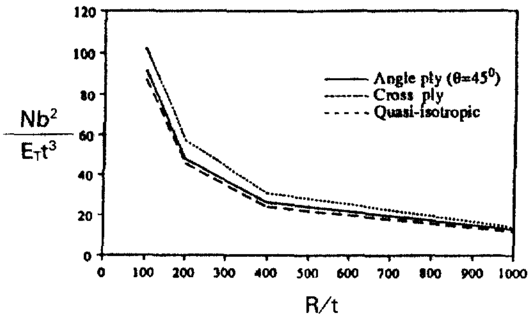

Fig. 3. Effect of R/t ratio on critical buckling load of GFRP curved panels

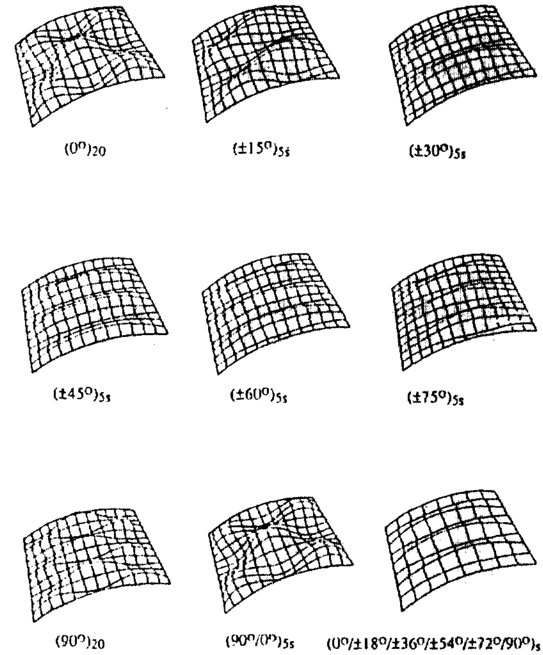

Fig. 4. Buckling mode shape of GFRP curved panels (R/t=100)

lowest value and $\theta=15°$, $75°$ to the highest. The cross-ply panel gave the highest critical load (445 N/mm), whilst the quasi-isotropic configuration resulted in the lowest value (389 N/mm).

The effect of radius to thickness ratio for typical cases is shown in Fig. 3 and follows the expected trend. However, it is worth pointing out that all the panels were analysed with constant width and as a result the narrowness parameter Θ_K changes from 2.9 to 0.9 as R/t varies from 100 to 1000. Additional runs were carried out to quantify the reduction in buckling load when the narrowness parameter is kept constant at 2.9 and it was found that this is between 5~15% for the panels examined.

The rather small variations at constant R/t for different lay-ups are expected in GFRP panels, since the ratio of moduli is small compared to other composite materials. On the other hand, it is interesting to note that despite the samall differences in the critical buckling load, the effect on the corresponding buckling mode is significant. It may be observed that GFRP panels with R=250 mm exhibit shell-type buckling behaviour (i.e. "chess-board" modes and "axi-symmetric" modes) depending on the fibre orientation. Fig. 4 shows the mode associated with the lowest eigenvalue for a typical selection of panels with R/t=100. Returning to panels with shell-type behaviour (R=250 mm) it is interesting to note that for $\theta=45°$ the buckling mode is similar to the one corresponding to an isotropic panel. In this case $D_{11}=D_{22}$ and $D_{16}=D_{26}$. For other angle-ply cases, these relations do not hold and this may explain the reason for the chess-board type modes, coupled in some case (e.g. $\theta=15°$) with a degree of skewness. Table 2 summarizes the properties of the bending stiffness D in terms of the above mentioned ratios.

Table 2. Stiffness ratios of GFRP lay-ups

Lay-up	D_{11}/D_{22}	D_{16}/D_{11}	D_{26}/D_{22}
$(45/-45)_{5s}$	1	0.0466	0.0466
$(15/-15)_{5s}$	0.37	0.012	0.023
$(75/-75)_{5s}$	2.68	0.023	0.012
Cross ply	0.85	0	0
Quasi-isotropic	1.97	0.022	0.003

These results also indicate that the critical imperfection shapes in composite panels may vary substantially depending on ply orientation. It is, therefore, essential to carry out detailed imperfection surveys on experimental models and, if possible, on real full-scale shells, so that dominant modes can be identified. As a result, characteristic imperfection models can gradually be developed for different manufacturing methods and lay-up configurations to include not only 'critical' modes but also modes with large mean amplitudes.[20]

4.2 Non-linear Analysis

As mentioned avove, linear eigenvalue analysis is only the first step in studying the buckling response of composite panels. A non-linear incremental analysis is required even for symmetrically layered panels, in order to quantify imperfection sensitivity. Apart from computational problems, the choice of a suitable imperfection shape is the most difficult task in deriving sensible results from this type of analysis. One possible approach would be to select modes on the basis of the following two criteria, based on procedures developed for isotropic shells where imperfection sensitivity and the selection of relevant modes has occupied researchers for many years:

(a) criticality; for example, an imperfection mode affine to the buckling mode obtained from linear analysis (although, due to non-linear pre-buckling, this is not necessarily the mode associated with the highest imperfection sensitivity); alternatively, a mode that is found to grow significantly in the pre-buckling stage of a non-linear analysis may be identified and selected as a 'critical' imperfection mode; finally, the effect of localised imperfection modes needs to be investigated.

(b) large amplitude; this would involve either direct measurement of imperfections on panels followed by harmonic analysis or information available on characteristic models from imperfection databanks.

The latter is not currently available for composite shells, although recent work has shown that valuable information can be obtained about dominant modes and their statistical dependence if imperfection data on groups of similarly manufactured specimens is properly recorded and analysed.[20] For the purposes of the present study, it was decided to select imperfection shapes only on the bais of criticality as determined from linear eigen-

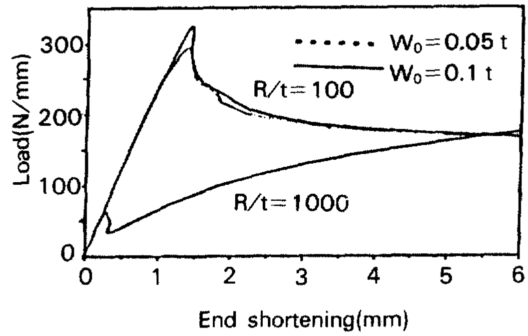

Fig. 5. Load-end shortening response of angle-ply $(45°/-45°)_{5s}$ panels

value analysis. A further problem that had to be resolved was to determine the maximum imperfection amplitude that can be allowed in the panel. It is, of course, inappropriate to use tolerance values from steel codes due to the entirely different manufacturing methods used in producing composite panels. On the basis of previous studies that have been carried out on aircraft structures,[6] it was decided to specify the maximum allowable amplitude as a fraction of the thickness. Thus, a maximum value of 10% of the thickness was adopted, although this is probably too stringent for less critical applications. Finally, it is worth noting that in this study, the load is applied on one of the two curved edges and that control on the fastest growing nodal displacement is enforced in order to follow the equilibrium path in the post-buckling range.

Fig. 5 shows the load-end shortening response obtained for $(\pm 45°)_{5s}$ angle-ply for two different R/t values. As expected, the curves show unstable behaviour after the limit load is attained, typical of wide panels. For the angle-ply panel with R/t=100, the numerical knockdown factor (defined here as the limit load obtained from non-linear analysis divided by the critical buckling load from linear eigenvalue analysis) is 0.80 for an imperfection amplitude equal to 5% of the thickness (not shown on this plot but estimated from an additional finite element analysis) and drops to 0.71 for 10%. For R/t=1000, the limit load is higher than the critical buckling load resulting in a 'knock-

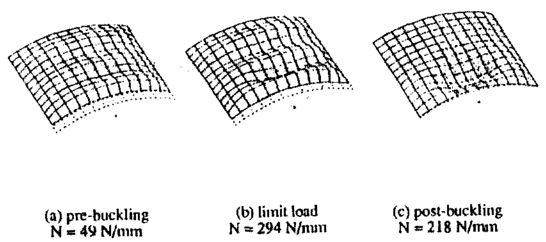

Fig. 6. Deflected shapes of angle-ply $(45°/-45°)_{5s}$ panel ($R/t=100$, $w_o=0.1$ t)

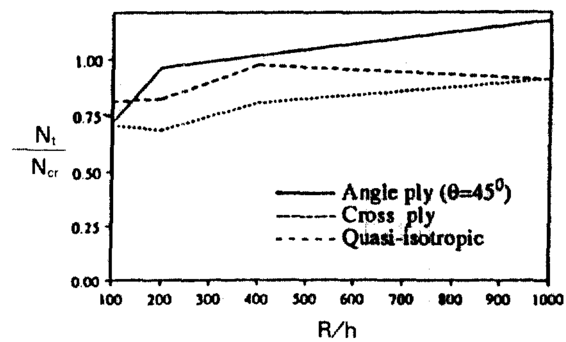

Fig. 7. Knock-down factors for GFRP curved panels

down' factor of about 1.2. It is worth noting that results for geometrically identical cross-ply panels reveal 'knock-down' factors which are consistently below unity, namely 0.70 for R/t=100 and 0.91 for R/t=1000.

In order to understand the features of non-linear response, it is also helpful to examine the deflected shapes at various stages of loading (pre-buckling, limit load and post-buckling), typically shown in Fig. 6 for an angle-ply panel with R/t=100 and a 10% maximum imperfection. As can be seen, in the pre-buckling range, the deflection grows outwards in a mode sympathetic to the imperfection imposed (i.e. with three axial waves and a single circumferential half-wave) and this mode is prevalent also at the limit load, although evidence of larger growth close to the edges is shown. The latter is more clearly observed in the post-buckling deflected shape. The magnification factor applied to these diagrams is, of course, different. It may be concluded, that in this case, although there are clear differences between pre- and post-buckled shapes, the interaction between imperfection, pre-buckling and buckling mode shapes results in a smooth transition, which as expected, is associated with a degree of imperfection sensitivity.

Similar plots for the angle-ply panel with R/t=1000 reveal that the change in deflected shape from pre- to post-buckling stage is more sudden and this explains the value of the "knockdown' factor being above unity, since it appears that there is no sensitivity to the critical mode shape. This, of course, does not imply absence of imperfection sensitivity but, rather, that the selection of imperfection modes on the basis of linear eigenvalue analysis is not always appropriate and more eleborate criteria need to be developed.

Fig. 7 summarizes the information regarding knockdown factors for GRFP curved dpanels under axial compression and the following comments are appropriats:

(i) in general, imperfection sensitivity in symmetrically layered composite panels is lower than in isotropic panels (cf. a wide panel exhibiting cylinder-type behaviour for which the knockdown factor for an imperfection amplitude equal to 10% of the thickness is about 0.55).

(ii) imperfection sensitivity is higher for low R/t ratios and high Θ_K values, as expected.

(iii) the results are in line with the conjecture that 'optimized' panels show higher imperfection sensitivity, since the cross-ply panels give rise to the lowest set of knockdown factors.

(iv) the reason for the angle-ply results has been explained above and further studies will be performed in order to quantify the observed trend.

5. Conclusions

Results have been presented from a parametric study on axially compressed GFRP curved panels. The effect of ply orientation on critical buckling loads was found to be small for various angle-ply panels at constant curvature, although when cross-ply and quasi-isotropic lay-ups are considered the variation in critical loads increases to about 12%. More significantly, a large range of buckling mode shapes was observed which has implications for both imperfection sensitivity studies and the specification of tolerances in structural codes.

Incremental nonlinear analyses were also carried out for a number of cases in order to quantify imperfection sensitivity and estimate knockdown factors. Imperfection modelling has been based on the critical mode determined from linear eigenvalue analysis and the limitations associated with this approach have been outlined. It has been shown that symmetrically layered wide composite curved panels (i.e. exhibiting unstable shell-type behaviour) have relatively high knockdown factors compared to their isotropic counterparts. The smallest knockdown factor is about 0.70 and, in general, factors were lower for low R/t ratios, which is in agreement with similar results found in a previous study on CFRP panels[6].

However, further work is needed in this area, especially in experimental studies to validate results which are so far based only on numerical analysis. Theoretical work to determine the characteristics of post-buckling behaviour (for example, in terms of the *b*-coefficients derived from initial post-buckling analysis) are also required for composite panels. The specification of imperfection tolerances is another area that needs to be investigated by quantifying the effect of manufacturing methods on final geometry. Finally, appropriate failure criteria should be developed using, as far as possible, experimentally observed failure modes. All these are essential ingredients in order to promote structural applications of composite materials in the offshore field and to move towards the development of validated limit state design criteria for composite shells.

Notation

Π	: total potential energy
V	: potential energy
$\wedge$	: Co-rotational value attributed to pure deformation
N, M, N	: resultant membrane force, bending moment and transverse shear force
$\hat{e}_m$, $\hat{e}_b$, $\hat{e}_s$	: linear membrane, bending and transverse shear strain
$\hat{\eta}_m$, $\hat{\eta}_b$, $\hat{\eta}_s$	: non-linear membrane, bending and transverse shear strain
K_0	: global linear elastic stiffness matrix,
K_G	: global geometric stiffness matrix
${}^{t+\Delta t}F$	: vector of externally applied loads and
${}^{t}f$	: internal incremental forces.
a	: eigen vector defining the buckling mode shape
λ_{cr}	: eigenvalue
a	: panel length
b	: panel width
t	: panel thickness (total)
w_o	: initial imperfection amplitude
E_L	: Young's modulus parallel to fibre direction
E_T	: Young's modulus perpendicular to fibre direction
G_{LT}	: Shear modulus
N	: axial compressive load
N_{cr}	: critical buckling load (eigenvalue analysis)
N_l	: limit load (non-linear analysis)
R	: radius of curvature
θ	: fibre angle
ν, ν_{LT}	: Posson's ratio
Θ_K	: Koiter's curvature or narrowness parameter

Units used throughlut are N, mm.

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