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심해파의 불안정성에 관한 실험 연구

제 2 부 : 초기변형파의 불안정성

Experimental Study of Deep-Water Wave Instability : Part 2. Evolution of The Initially-Modulated Wave Train

조 원 철*
Cho, Won Chul

Abstract

Experiment on the instability and breaking of the initially modulated deep-water wave train (in wave amplitude or in wave frequency) is performed to investigate the effect of the initial modulation on nonlinear wave evolution. Wave amplitude and frequency modulations are developed earlier and larger than in the case of the uniform deep-water wave trains. However, for smalll wave steepness in the initially amplitude-modulated wave train, the wave train becomes demodulated and nearly returns to the original wave form at the end of the wave evolution far downstream from the breaking region, with energy returning to the fundamental wave frequency.

요 지

초기 변형파(파고 또는 주파수에의 변형)의 실험이 파랑의 전개에 있어 초기 변형의 영향을 살펴보기 위해 행하여졌으며, 이로부터 초기 변형파의 파고와 주파수의 변형이 정상파의 전개에 있어서 보다 더 크고 빨리 일어남이 관찰되었다. 그러나 작은 파형경사를 가진 초기 파고변형파의 전개에 있어서는 초기에서 중기까지는 파랑의 파고와 주파수의 변형이 빠르게 일어남이 관찰되었으나 말기에는 이 변형파의 최초의 형태나 주파수로 다시 전환됨이 관찰되었다.

1. Introduction

After Benjamin and Feir(1967) discovered that a uniform deep-water wave train with constant wave frequency becomes unstable and finally breaks far from its origin due to modulational perturbations of side-band frequency components, many studies(refer Part 1.'Introduction') have been conducted to understand the evolution of nonlinear deep-water wave train. In 1977, Lake et. al. performed an experimental work on the evolution of the nonlinear continuous wave train in deep water. Since their laboratory wave channel was not long enough(40 feet) to observe the entire evolution of a uniform wave trains, They generated two types of initially uniform wave train to investigate the initial stage of wave evolution, and used imposed initial amplitude modulations to investigate long-time wave evolution. In the initial

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* 정회원 · 중앙대학교 기술과학연구소 선임연구원

stage of wave evolution, the wave field was chatacterized by the Benjamin-Feir instbility, so that a single pair of side-band frequency components appeared and an amplitude modulation developed and grew exponentially. Later, side-band frequency components continued to grow and underwent strong modulations, with a spreading of wave energy over many frequency components. However, at the end stage, the instability did not lead to wave-train disintegration, but rather the wave train demodulated and nearly returned to the initial wave form, showing the recurrence phenomenon.

Since a uniform wave train in deep water is modulated in its amplitude and frequency as the wave train propagates downstream, in the present study, we will investigate the instability of the nonlinear deep-water wave train not only by modulating wave amplitude at the start(initially amplitude-modualted wave train) but also by modulating wave frequency at the start(initially frequency-modulated wave train).

2. Experiment

2.1 Experimental Apparatus

Experiments were performed at Davidson Laboratory of Stevens Institute of Technology as performed in Part 1(refer Part 1. 'Experiment' for details).

2.2 Experimental parameters

2.2.1 Parameters of the initially amplitude-modulated wave train

Laboratory tests of the evolution of the initially amplitude-modulated wave train were conducted with a wave group composed of 7 waves with constant wave frequency, 1.1 Hz, and wave amplitude in a wave group was modulated by increasing and decreasing wave amplitude(Fig. 1). Basic wave amplitude from 1 inch to 3 inches were tested, and the wave amplitude modulation rates of 0.1, 0.2, and 0.3 inches were used. A wave group was generated repeatedly during the tests in following way :

$$\eta_k = ((k-1)a_i + a)\ \sin(2\pi ft) \qquad (1)$$

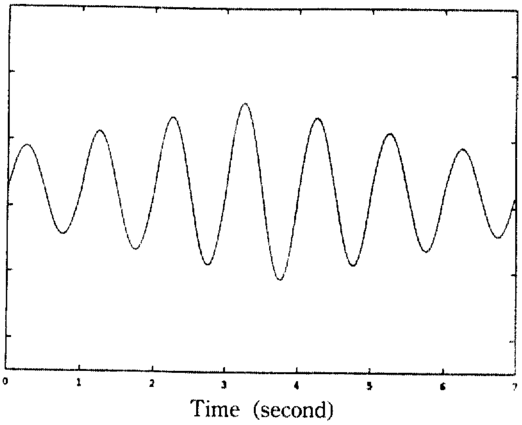

Fig. 1. Initially amplitude-modulated wave train.

$$k = 1,2,3,4$$

$$\eta_k = ((7-k)a_i + a)\ \sin(2\pi ft) \qquad (2)$$

$$k = 5,6,7$$

where η_k : wave surface elevation(inch)

a : wave amplitude(inch)

a_i : wave amplitude modulation rate(inch)

2.2.2 Parameters of the initially frequency-modulated wave train

A wave group of the initially frequency-modulated wave train was produced by modulating wave frequency from the first wave to the last wave at the same frequency modulation rate(Fig. 2). The wave frequency ranges, f=1.1-1.03 Hz, f=1.1-0.925 Hz, and f=1.1-0.75 Hz, were used with the wave frequency modulation rates, df=0.01 Hz, df=0.025 Hz, and df=0.05 Hz, respectively, and the wave amplitude was used from 1.0 inch to 3.0 inches. A wave group with modulating wave frequency is generated in following way :

$$\eta_k = a\ \sin(2\pi f_k t)$$

$$= a\ \sin[2\pi(f-(k-1)df)t] \qquad (3)$$

$$k = 1,2,3,\cdots,\ 8$$

3. Results

3.1 Evolution of the initially amplitude-modulated wave train

The average wave steepness is used for describing the wave steepness in a wave group in con-

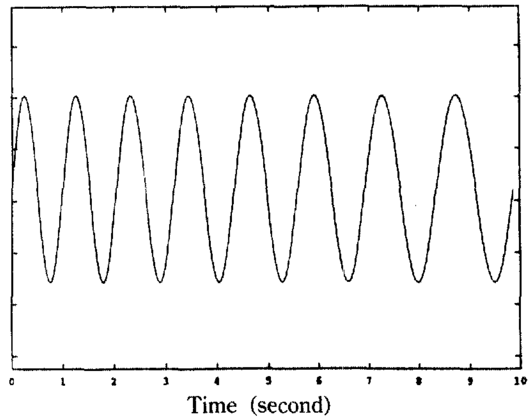

Fig. 2. Initially frequency-modulated wave train.

venient way. The average wave steepness is calculated by multiplying average wave amplitude by average wavenumber in a wave group.

For small average wave steepness, $ka_{ave}=0.14$ (wave frequency, $f=1.1$ Hz, wave amplitude, $a=1.0$ inches, and wave amplitude modulation rate, $a_i=0.1$ inches), wave amplitude modulations were developed with relatively large growth rate and then two-dimensional breaking occurred in the middle of evolution with weak breaking intensity. However, this breaking disappeared soon after short breaking distance and waves were gradually demodulated to the nearly original wave form at the final stage far after breaking region(Fig. 3). The fundamental wave frequency is dominant in the wave field, however, due to fast growing wave instability with the effect of the initial amplitude modulation, the upper side-band frequency at $f_{upp}=1.24$ Hz and the lower side-band frequency at $f_{low}=0.96$ Hz grow from the initial stage in symmetric form(Fig. 4). As the wave train propagates breaking region, the upper and the lower side-band frequencies grow rapidly, and during beaking process, the magnitude of the fundamental frequency is decreased abruptly. The lower side-band frequency grows larger than the upper side-band frequency after breaking at downstream, while the fundamental frequency begins to recover its energy and returns to the almost original form at the end of evolution as the wave train becomes demodulated, exhibiting the recurrence phenomenon. This phenomenon was first observed by Lake(1977) in his laboratory experiment for investigation of the long-time wave train evolution. There are shown another new small upper and lower side-band frequency components at $f_{upp}=1.38$ Hz and $f_{low}=0.82$ Hz, respectively.

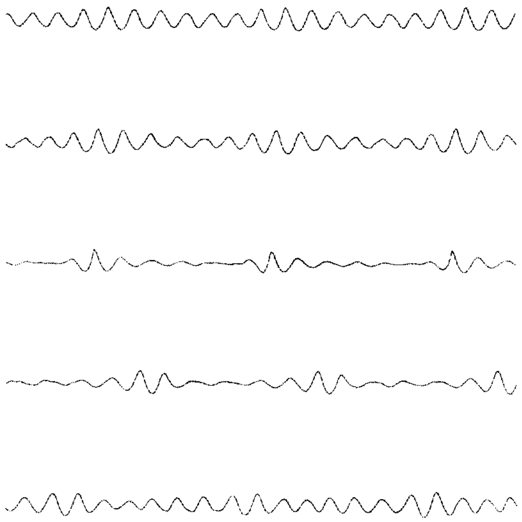

Fig. 3. Wave profiles of $ka_{ave}=0.14$. Measuring stations at 20, 70, 120, 170, and 220 ft from the wavemaker.

At the average wave steepness of 0.23($f=1.1$ Hz, $a=1.5$ inches, and $a_i=0.3$ inches), due to intensified nonlinear wave interaction, two-dimensional breaking was observed from upstream and continued to midstream with relatively longer breaking distance. The frequency modulations, the upper side-band frequency at $f_{upp}=1.24$ Hz and the lower side-band frequency at $f_{low}=0.96$ Hz, begin from the early stage with the large growth rate of the lower side-band frequency(Fig. 5). The lower side-band frequency dominates in the wave field as the wave train propagates midstream and develops more in its magnitude at downstream. At the final stage of evolution, however, the lower side-band frequency decreases rapidly but the fundamental frequency recovers its energy with small magnitude. There is also shown new lower side-band frequency at $f_{low}=0.82$ Hz with small magnitude. Three-dimensional instability and breaking were observed at large average wave

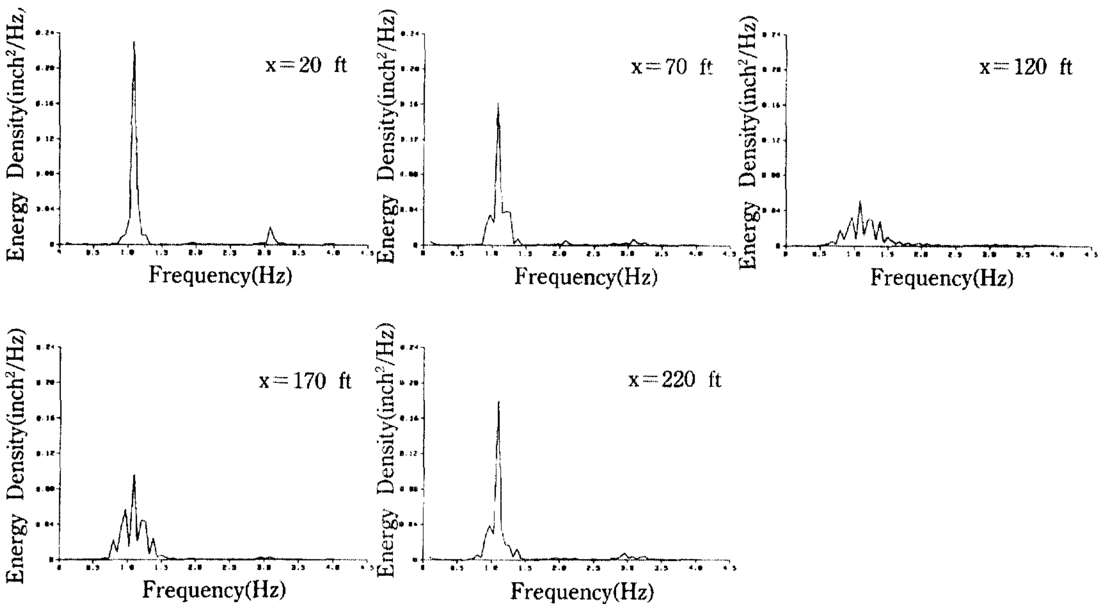

Fig. 4. Evolution of energy density spectrum at $ka_{ave}=0.14$.

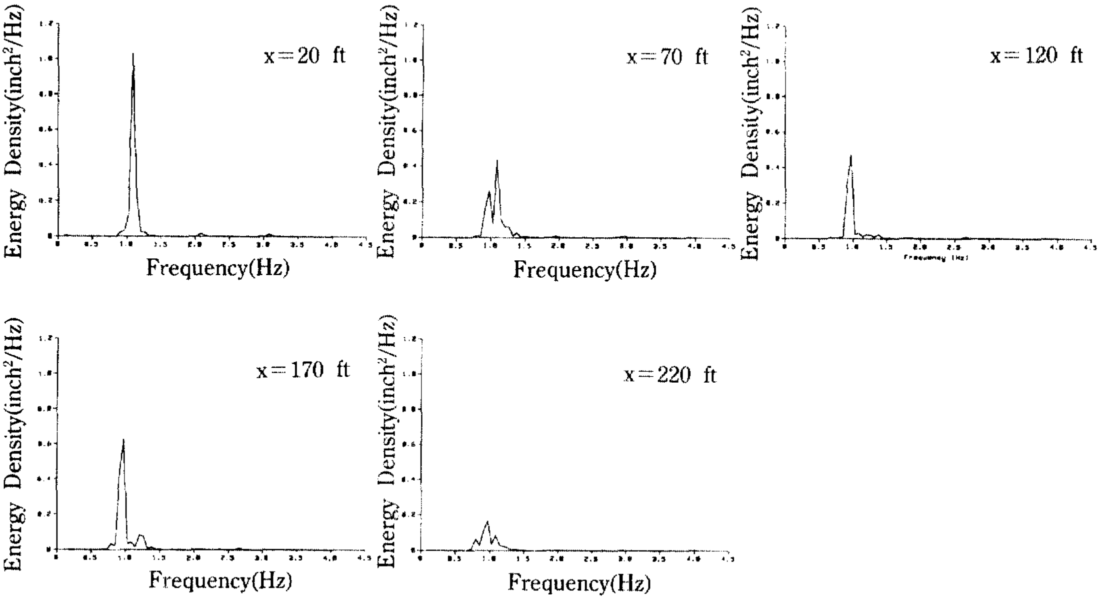

Fig. 5. Evolution of energy density spectrum at $ka_{ave}=0.23$.

steepness, $ka_{ave}=0.36$($a=2.5$ inches and $a_i=0.3$ inches). Moderate three-dimensional breaking initiated from upstream and turned to two-dimensional breaking as moving to midstream after several three-dimensional breakings. The upper side-band frequency at $f_{upp}=1.24$ Hz and the lower side-band frequency at $f_{low}=0.96$ Hz are developed from the initial stage again(Fig. 6). The lower side-band frequency is developed very rapidly and almost reaches the fundamental frequency from upst-

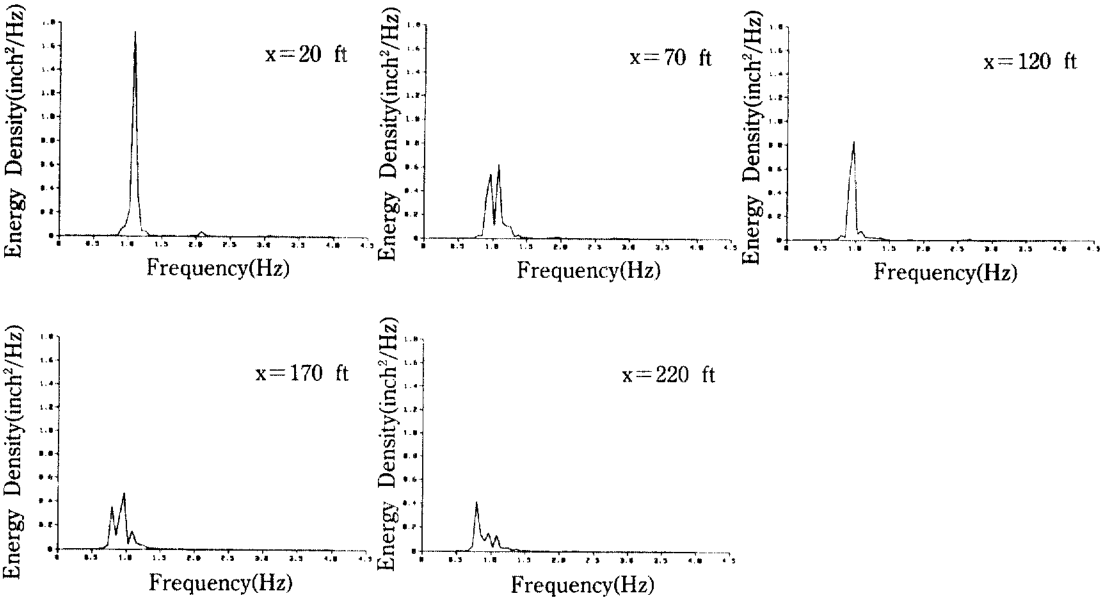

Fig. 6. Evolution of energy density spectrum at $ka_{ave}=0.36$.

ream. However, after breaking region at downstream, the dominant lower side-band frequency at $f_{low}=0.96$ Hz diminishes its magnitude, while there appears another new lower side-band frequency at $f_{low}=0.82$ Hz with relatively large growth rate and becomes dominant in the wave evolution at the final stage.

Compared to the evolution of the uniform wave train, due to strong nonlinear wave interaction, more rapid growth of the side-band frequencies (especially in the lower side-band frequency), accompanied by earlier wave amplitude modulations and breaking, were observed in the evolution of the initially amplitude-modulated wave train. The breaking process repeated every 7 waves of one wave group periodicity. Wave amplitude modulation rate may be irrelavant in deciding the type of the wave instability and breaking since almost similar breaking process occurred at the different wave amplitude modulation rate with the same basic wave amplitude, but dependent on the frequency modulations, showing rapid growth of the lower side-band frequency from the early stage at large wave amplitude modulation rate. One notable characteristic is that the upper and the lower side-band frequency components always grow at $f_{upp}=1.24$ Hz and at $f_{low}=0.96$ Hz. There is also shown large growth of new lower sied-band frequency after breaking process at $f_{low}=0.82$ Hz, which becomes dominant in the wave evolution at the final stage at large average wave steepness. No second breaking was observed through the tests.

3.2 Evolution of the initially frequency-modulated wave train

We have watched rapidly growing wave amplitude and frequency modulations in the wave evolution of the amplitude-modulated wave train. The large growth rate of the frequency and amplitude modulations is also observed when a wave group is initially modulated in its frequency. Since the relatively strong wave-wave interaction started at the generation of the wave train at the wave paddle(the longer wave with the lower wave frequency overtaking the shorter wave with the high frequency), the low fundamental wave frequency was dominant in the wave field through the evolution.

At the low average wave steepness of 0.12(wave amplitude, a=1.0 inch, wave frequency range, f=1.1-1.03 Hz, and wave frequency modulation rate, df=0.01 Hz), small amplitude and frequency modulations with small growth rate of the upper and the lower side-band frequency components were observed in the wave evolution, but no breaking occurred. As the average wave steepness was increased 0.17(a=1.5 inches, f=1.1-1.03 Hz, and df=0.01 Hz), the wave train began to be modulated from the early stage and wave amplitude modulations became active as approaching midstream with two-dimensional breaking. The frequency

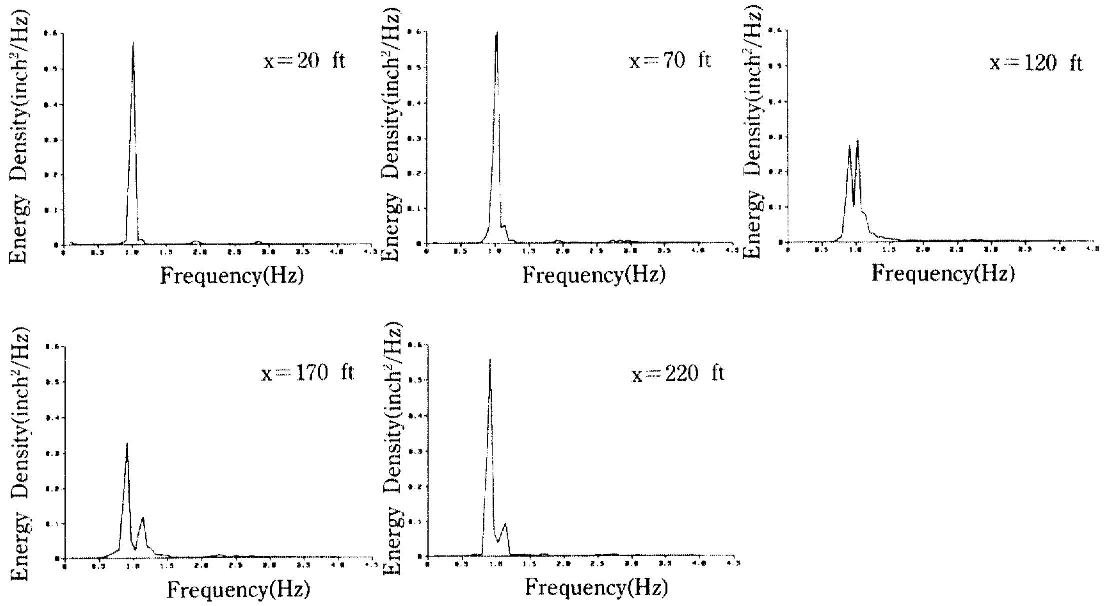

Fig. 7. Evolution of energy density spectrum at $ka_{ave}=0.17$.

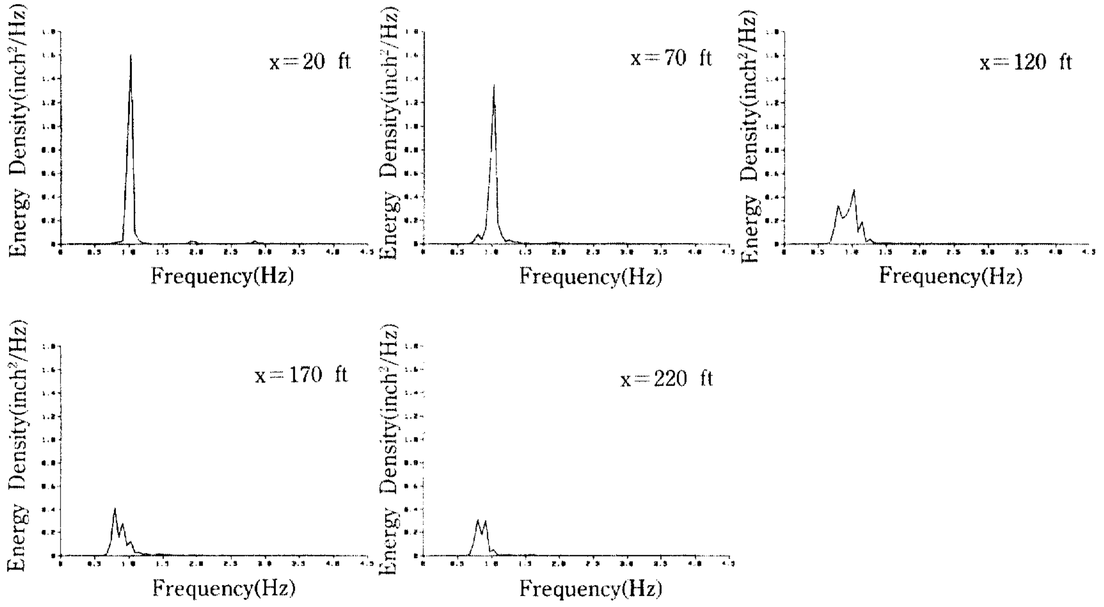

Fig. 8. Evolution of energy density spectrum at $ka_{ave}=0.29$.

modulations start from the early stage, spreading small energy to the upper and the lower side-band frequencies(Fig. 7). The magnitude of the low fundamental wave frequency is significantly reduced as the lower side-band frequency is rapidly developed in the middle of the evolution, and the lower side-band frequency becomes dominant in the wave evolution as approaching downstream. Almost two-dimensional but sometimes weak three-dimensional instability appeared at the early stage at the average wave steepness of 0.29 ($a=2.5$ inches, $f=1.0$-0.93 Hz, and $df=0.01$ Hz) and breaking proceeded through long distance from upstream to downstream. Two-dimensional weak breaking initiated from upstream and deformed to three-dimensional breaking with increasing breaking intensity due to strong nonlinear wave interaction as propagating midstream, and then turned to two-dimensional breaking again at downstream as dissipating wave energy by breaking process. Wave frequency modulations are developed from the early stage with small growth of the upper and the lower sied-band frequencies (Fig. 8). The side-band frequencies grow during breaking process and the lower side-band frequency becomes dominant frequency as moving to downstream.

At the average wave steepness of 0.25($a=3.0$ inches, $f=1.1$-0.75 Hz, and $df=0.05$ Hz), owing to strong wave-wave interaction, wave amplitude

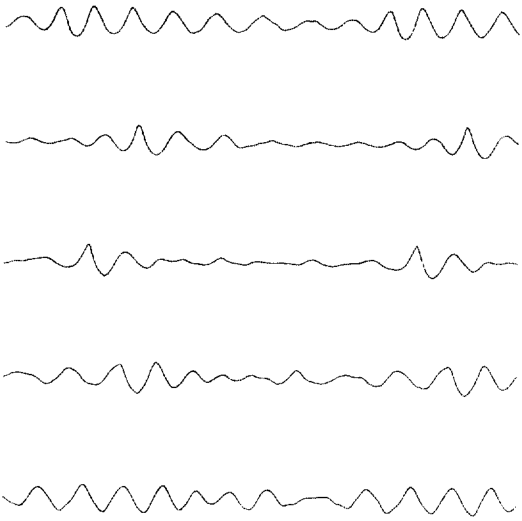

Fig. 9. Wave profiles of $ka_{ave}=0.25$. Measuring stations at 20, 70, 120, 170, and 220 ft from the wavemaker.

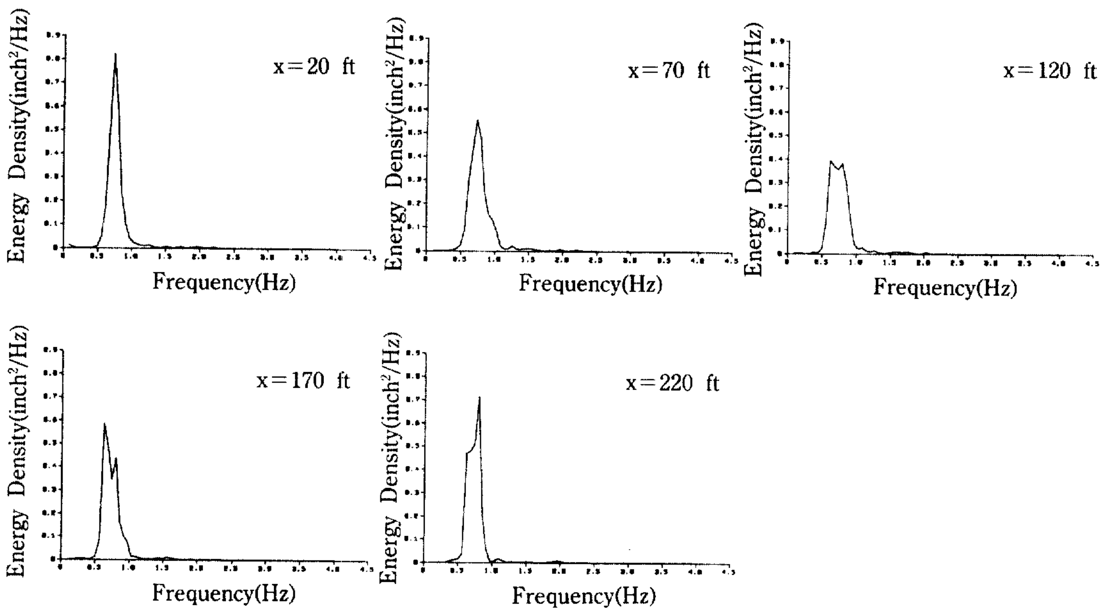

Fig. 10. Evolution of energy density spectrum at $ka_{ave}=0.25$.

modulations were developed very rapidly and a steep wave profile was observed before breaking, and then couple of spilling-type breaking waves were generated in the middle of evolution(Fig. 9). However, at the final stage far after breaking region, the wave train was demodulated and returned to the almost original wave form. The upper and the lower side-band frequency becomes dominant in the wave frequency range in the middle of evolution after spilling breaking(Fig. 10). However, as moving to downstream after breaking events, the low fundamental frequency recovers its energy as waves are demodulated and becomes dominant again, but the lower side-band frequency still remains in large magnitude.

4. Conclusion

Since the uniform deep-water wave train is modulated in its amplitude and frequency in the middle of evolution, initially amplitude and frequency-modulated wave trains are also examined to see the effects of the initial wave modulations. Compared with the wave evolution of the uniform wave train, in the evolution of the initially amplitude-modulated wave train, more rapidly growing wave amplitude and frequency modulations are observed from the early stage at large average wave steepness, and from the middle of the evolution at smaller average wave steepness. The lower side-band frequency is rapidly developed from the early stage and becomes dominant in the middle of evolution. The lower side-band frequency is always developed at $f_{low}=0.96$ Hz, however, at large average wave steepness, there appears another growth of new lower side-band frequency at $f_{low}=0.82$ Hz. One remarkable feature is observed that at small average wave steepness of 0.14($f=1.1$ Hz, $a=1.0$ inch, and $ai=0.1$ inches), the dominant fundamental wave frequency diminishes its magnitude greatly as wave train propagates downstream. However, at the final stage after breaking, the fundamental wave frequency recovers and reaches the original magnitude as the wave train is demodulated, showing a recurrence phenomenon.

Wave amplitude and frequency modulations are also developed earlier than the evolution of the uniform wave train in the initially frequency-modulated wave train. At the largest wave frequency modulation rate, $df=0.05$ Hz, due to strong wave-wave interaction, several large spilling breaking waves occur in the middle of evolution. The lower side-band frequency is developed rapidly from the early stage and becomes domonant in the wave evolution at the final stage after breaking. From the tests of the initially frequency-modulated wave train, it is inferred that the onset of wave instability and breaking of the deep-water wave train does not depend on the initial wave frequency modulation rate, but on the initial wave steepness. However, once the initial wave steepness exceeds the critical point, the characteristic of breaking varies according to the initial frequency modulation rate.

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