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Interrelationship Between the Drift–flux Model and the Two–fluid Model

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Abstract

For one–dimensional two–phase flow without phase change and without axially–temporally rapid change of pressure, the interrelationship between the drift–flux model and the two–fluid model is studied. It is derived on the basis of the fact that the vapor conservation equation is related to the momentum equation by the drift flux. Starting from the two–fluid model, we obtain the interfacial friction expressed in terms of drift–flux parameter. Also, by deriving the void propagation equation, the drift–flux is shown to have interrelationship with forces in the two–fluid model.

요 약

상 변화가 없고 압력 변화가 없는 일차원 이상 유동에 있어서 드리프트 플럭스 모델과 2-유체 모델 사이의 상관 관계가 연구되었다. 증기 보존식은 드리프트 플럭스를 통하여 운동량 보존식에 의존한다는 사실에 기초하여 관계가 얻어졌다. 2-유체 모델로부터 출발하여 드리프트 플럭스의 파라미터의 함수로 표현된 계면 마찰식을 얻었다. 또한 기공 전달식을 유도함으로써 드리프트 플럭스는 2-유체 모델의 마찰력과 상호 관계가 있음을 보였다.

1. Introduction

In order to eliminate the basic assumption of equal velocities between two phases, the drift–flux concept[1, 2] was introduced. In the basic assumption of the drift–flux model the relative velocity between two phases is specified in terms of other variables such as parameters related to drift-flux, void fraction, pressure, and geometry. In air–water flow we need three equations, two mass equations and one mixture equation, to obtain void fraction, pressure, vapor velocity and liquid velocity. The accuracy and usefulness of the drift-flux concept in flow regimes such as bubbly, slug, turbulent and churn flow has been verified and a number of data related to the drift–flux have been accumulated.

Recently, the two–fluid model[3, 4] was introduced to eliminate the drift–flux concept. In the model for the description of air–water flow we need a full set of conservation equations, two mass equations and two momentum equations. Additional information on the interfacial momentum exchange between two phases is needed in two–fluid model. Here, let us obtain the interrelationship between the drift–flux model and the two–fluid model.

2. Analytical Work

For one–dimensional two–phase flow without phase exchange and without axially–temporally rapid change of pressure, we have

mass :
$$\frac{\partial}{\partial t}\alpha_g + \frac{\partial}{\partial z}j_g = 0, \quad (1)$$
$$\frac{\partial}{\partial t}\alpha_f + \frac{\partial}{\partial z}j_f = 0, \quad (2)$$

momentum :
$$\frac{\partial}{\partial t}(\rho_g j_g) + \frac{\partial}{\partial z}(\rho_g j_g^2/\alpha_g) + \alpha_g\frac{\partial}{\partial z}P = -F_s - (\alpha\rho)_g g, \quad (3)$$
$$\frac{\partial}{\partial t}(\rho_f j_f) + \frac{\partial}{\partial z}(\rho_f j_f^2/\alpha_f) + \alpha_f\frac{\partial}{\partial z}P = F_s - F_w - (\alpha\rho)_f g, \quad (4)$$

where j_g and j_f : superficial vapor velocity and superficial liquid velocity, respectively,
F_s : interfacial momentum exchange,
F_w : wall friction force.

Let us replace j_g and j_f by the mixture superficial velocity j and the drift–flux j_{gf} using their following relationships :
$$j_g = \alpha_g j + j_{gf}, \quad (5)$$
$$j_f = \alpha_f j - j_{gf}. \quad (6)$$

Inserting Eqs.(5) and (6) into Eqs.(1) through (4) and adding two mass equations for the mixture mass equation yields

mass :
$$\frac{\partial}{\partial t}\alpha_g + \frac{\partial}{\partial z}j_{gf} + \frac{\partial}{\partial z}(\alpha_g j) = 0, \quad (7)$$
$$\frac{\partial}{\partial z}j = 0, \quad (8)$$

momentum :
$$\frac{\partial}{\partial t}\rho_g(\alpha_g j + j_{gf}) + \frac{\partial}{\partial z}\rho_g(\alpha_g j + j_{gf})^2/\alpha_g + \alpha_g\frac{\partial}{\partial z}P = -F_s - (\alpha\rho)_g g, \quad (9)$$
$$\frac{\partial}{\partial t}\rho_f(\alpha_f j - j_{gf}) + \frac{\partial}{\partial z}\rho_f(\alpha_f j - j_{gf})^2/\alpha_f + \alpha_f\frac{\partial}{\partial z}P = F_s - F_w - (\alpha\rho)_f g. \quad (10)$$

Eliminating the pressure gradient from Eqs.(9) and (10) gives
$$\alpha_f\rho_g\frac{\partial}{\partial t}(\alpha_g j + j_{gf}) - \alpha_g\rho_f\frac{\partial}{\partial t}(\alpha_f j - j_{gf}) + \alpha_f\frac{\partial}{\partial z}\rho_g(\alpha_g j + j_{gf})^2/\alpha_g - \alpha_g\frac{\partial}{\partial z}\rho_f(\alpha_f j - j_{gf})^2/\alpha_f = -F_s + \alpha_g F_w + \alpha_g\alpha_f\Delta\rho\, g, \quad (11)$$

where $\Delta\rho$: density difference between liquid and gas.

Through use of the mixture equation, Eq.(8), Eqs.(7), (8) and (11) become, respectively,

mass :
$$\frac{\partial}{\partial t}\alpha_g + j\frac{\partial}{\partial z}\alpha_g + \frac{\partial}{\partial z}j_{gf} = 0, \quad (12)$$
$$\frac{\partial}{\partial z}j = 0, \quad (13)$$

momentum :
$$a\frac{\partial}{\partial t}\alpha_g + b\frac{\partial}{\partial z}\alpha_g + c\frac{\partial}{\partial t}j + d\frac{\partial}{\partial t}j_{gf} + e\frac{\partial}{\partial z}j_{gf} = f, \quad (14)$$

where $a = (\alpha_f\rho_g + \alpha_g\rho_f)\, j,$
$$b = \alpha_f\rho_g\,\{2j(\alpha_g j + j_{gf}) - (\alpha_g j + j_{gf})^2/\alpha_g\}/\alpha_g + \alpha_g\rho_f\,\{2j(\alpha_f j - j_{gf}) - (\alpha_f j - j_{gf})^2/\alpha_f\}/\alpha_f,$$
$$c = -\alpha_g\alpha_f\Delta\rho,$$
$$d = \alpha_f\rho_g + \alpha_g\rho_f,$$
$$e = \alpha_f\rho_g\, 2(\alpha_g j + j_{gf})/\alpha_g + \alpha_g\rho_f\, 2(\alpha_f j - j_{gf})/\alpha_f,$$
$$f = -F_s + \alpha_g F_w + \alpha_g\alpha_f\Delta\rho\, g.$$

In the drift–flux model two equations among the above three conservation equations are independent. Note that Eq.(13) is an independent equation. In the transient conditions there is no way in which Eqs.(12) and (14) are dependent on each other.

At steady state Eqs.(12) and (14) become, respectively, mass: $$j\frac{\partial}{\partial z}\alpha_g + \frac{\partial}{\partial z}j_{gf} = 0, \quad (15)$$

momentum: $$b\frac{\partial}{\partial z}\alpha_g + e\frac{\partial}{\partial z}j_{gf} = f. \quad (16)$$

For the dependence between Eqs.(15) and (16) the following condition should be satisfied :

$$\frac{j}{b} = \frac{1}{e} = \frac{0}{f}. \quad (17)$$

From Eq.(17) we can generate the following two conditions for the validity of the drift-flux concept

$$b = ej, \quad (18)$$

$$f = 0. \quad (19)$$

Using the definitions, b and e, the condition (18) becomes

$$\alpha_f\rho_g(\alpha_g j + j_{gf})^2/\alpha_g^2 + \alpha_g\rho_f(\alpha_f j - j_{gf})^2/\alpha_f^2 = 0. \quad (20)$$

Because of the impossibility of the satisfaction of Eq.(20), we need the following requirements :

$$\frac{\partial}{\partial z}\alpha_g = 0, \quad (21)$$

$$\frac{\partial}{\partial z}j_{gf} = 0. \quad (22)$$

The requirements, Eqs.(21) and (22), represent steady-state fully-developed flow without phase change. In other words, the strict validity of the drift-flux model is limited to steady-state fully-developed flow.

If wall friction force, F_w is much smaller than gravity force, $\alpha_f\Delta\rho$ g, the interfacial friction factor, C_D, can be expressed in terms of the drift flux :

$$F_S = C_D V_r |V_r|,$$

$$C_D = (\alpha_f\alpha_g)^3 \Delta\rho\, g/(j_{gf}|j_{gf}|),$$

where V_r : relative velocity between gas and liquid.

The most conspicuous defect of the drift-flux model is the absence of any damping at all frequencies. Therefore, Boure[5] introduces the following first-order partial differential equation expressing the void-drift closure :

$$\zeta\frac{\partial}{\partial t}j - (\Sigma - j)\frac{\partial}{\partial t}\alpha - \Pi\frac{\partial}{\partial z}\alpha + \frac{\partial}{\partial t}j_{gf} = \frac{1}{\Theta}(f - j_{gf0}), \quad (23)$$

where j_{gf0} : j_{gf} in steady-state fully-developed flow,

$\zeta, \Sigma, \Pi, \Theta$: parameters,

f : j_{gf} in transient developed flow.

Equation (23) proposed for the void-drift closure by Boure is compared to Eq.(14) which comes from the momentum conservation equation. An important practical problem is the response of quasi fully-developed flow to small disturbances. Assuming small perturbation of α, j, and j_{gf} from the fully-developed flow, we have

$$\alpha_g = \alpha_{g0} + \alpha_g',$$

$$j = j_0 + j',$$

$$j_{gf} = j_{gf0} + j_{gf}',$$

and neglecting the second order perturbation, Eqs.(12) through (14) become

mass : $$\frac{\partial}{\partial t}\alpha_g' + j_0\frac{\partial}{\partial z}\alpha_g' + \frac{\partial}{\partial z}j_{gf}' = 0, \quad (24)$$

$$\frac{\partial}{\partial z}j' = 0, \quad (25)$$

momentum :

$$a_0\frac{\partial}{\partial t}\alpha_g' + b_0\frac{\partial}{\partial z}\alpha_g' + c_0\frac{\partial}{\partial t}j' + d_0\frac{\partial}{\partial t}j_{gf}' + e_0\frac{\partial}{\partial z}j_{gf}' = g, \quad (26)$$

where $g = \sum_{x=\alpha,j,j_{gf}} \frac{\partial f}{\partial x})_0\, x'$.

For simplicity let us eliminate the superscript′ and subscripts, g and 0. Because of the linearity of Eqs.(24) through (26), we can obtain the void propagation equation through the elimination of derivative terms related to j and j_{gf}. From Eqs.(25) and (26) we have

$$c\frac{\partial^2}{\partial z\partial t}j = 0$$
$$= -a\frac{\partial^2}{\partial z\partial t}\alpha - b\frac{\partial^2}{\partial z^2}\alpha - d\frac{\partial^2}{\partial z\partial t}j_{gf} - e\frac{\partial^2}{\partial z^2}j_{gf} + \frac{\partial}{\partial z}g. \quad (27)$$

The third, fourth, and fifth terms in the right-hand side of Eq.(27) can be expressed in terms the derivative terms of a using Eq.(24). Then, we have the following void propagation equation :

$$\frac{\partial}{\partial t}\alpha + (j - \frac{f_\alpha}{f_{j_{gf}}})\frac{\partial}{\partial z}\alpha - \frac{d}{f_{j_{gf}}}\{\frac{\partial^2}{\partial t^2}\alpha + \frac{e}{d}\frac{\partial^2}{\partial z \partial t}\alpha - \frac{1}{d}(b - ej)\frac{\partial^2}{\partial z^2}\alpha\} = 0, \quad (28)$$

where $f_x = \frac{\partial f}{\partial x}$, $x = \alpha, j_{gf}$.

Equation (28) is exactly the same form as the propagation equation suggested by Boure except its parameters are treated as unknowns.

When the ordinary drift–flux model is compared to Eq.(28), we have

$$\frac{\partial}{\partial \alpha} j_{gf} = -\frac{f_\alpha}{f_{j_{gf}}}$$

3. Conclusions

–Basically the drift–flux model is the same as the two fluid model for the steady–state fully–developed flow. Therefore data for the correlation of drift–flux should be taken in the steady– state fully–developed region.

–In the steady–state fully–developed region the following relationship is satisfied :

$$F_S = \alpha_g F_W + \alpha_g \alpha_f \Delta\rho\, g.$$

–Boure's void–drift closure equation is another expression of the additional momentum equation.

–The void propagation equation with void dispersion can be obtained considering the full momentum equations.

–Through comparison of the void propagation equation between the drift–flux model and the two–fluid model, we obtain the following relationship :

$$\frac{\partial}{\partial \alpha} j_{gf} = -\frac{f_\alpha}{f_{j_{gf}}}$$

Nomenclature

English

C_D : interfacial friction factor
F_S : interfacial momentum exchange
F_W : wall friction force
g : gravitational constant
j : superficial velocity
j_{gf} : drift flux
P : pressure
V_r : relative velocity between gas and liquid

Greek

α_g : vapor fraction
α_f : liquid fraction
ρ_g : vapor density
ρ_f : liquid density

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