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Mathematical Model for 3-Dimesional Circulation in Surf Zone
碎破帶 3차원 흐름에 대한 數學的 模型

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Abstract□An amended form of radiation stress is presented in the present model and the existence of the surface advection terms is verified through comparisons with wave energy equation. The model yields circulation patterns in both cross-shore and longshore directions on the plane beach slope. Comparison with laboratory experiments showed good agreements. Finally, a quasi-three dimensional model suitable for the entire nearshore zone is developed by linking the depth-integrated properties with vertical profiles.

要　旨：잉여응력의 수정된 형태가 현 모델에서 제시되며 자유수면 이류항의 존재가 파랑에너지식과의 비교를 통해서 입증되었다. 수식모델은 편평한 해빈경사에서의 3차원적인 흐름의 순환패턴을 산출하며 그 결과는 실험치와 비교하여 좋은 일치를 보였다. 마지막으로 수심적분된 특성과 그 수직분포를 연계하여 전 연안역에 적용가능한 준3차원적 모델이 개발되었다.

1. INTRODUCTION

A prominent feature in the nearshore zone is the wave-induced current circulation. It is commonly accepted that the primary driving force is the wave-induced radiation stress first introduced by Longuet-Higgins and Stewart (1961). Modelling this circulation has since advanced considerably from the earlier development by Noda et al. (1974) and Ebersole and Dalrymple (1979). Both of these earlier models were driven by a wave refraction model with no current feedback. In recent years, Yoo and O'Connor (1986b) developed a coupled wave-induced circulation model based upon what could be classified as a hyperbolic type wave equation; Yan (1987) and Winer (1988) developed their interaction models based up on parabolic approximation of the wave equation. All these models employed the depth-averaged formulations which have three major deficiencies 1) the surface effects due to wave-current interaction, which generally is very strong, are being neglected; 2) the bottom friction is expressed in terms of the mean velocity which makes the model unrealistic in areas where the current profile is strongly three dimensional but the mean current could be small such as in the surf zone and 3) the convective acceleration terms are also depth-averaged which has the same problem as (2). Recently, de Vriend and Stive (1987) improved the nearshore circulation model by employing a quasi-three dimensional technique. This technique is very attractive to accomodate the surf zone in which the depth-averaged model is no longer valid but the full three-dimensional modelling is currently not attainable. In this paper, this approach of quasi-three dimensional modelling is adopted for developing a circulation model in nearshore zone.

In Section 2, the fundamental conservation equations of mass and momentum time-averaged over turbulent scale are presented. In Section 3, the depth-integrated formulations serving as the basic equations for a quasi-three dimensional circulation model are derived. An amended form of radiation stress is presented and the existence of the surface

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advection terms is verified through comparisons with wave energy equation. Section 4 develops a new model prescribing turbulence-induced vertical current distributions in surf zone. This model employing the surface current boundary conditions given in Lee (1993) yields circulation patterns in both cross-shore and longshore directions. Finally, in Section 5, a quasi-three dimensional model suitable for the entire nearshore zone is developed by linking the depth-integrated properties with vertical profiles.

2. TURBULENCE-AVERAGED GOVERNING EQUATIONS

The strong presence of turbulence is a prominent feature in surf zone. Consequently, the fundamental equations governing the fluid motion should also include the turbulent effects. This is usually accomplished with the introduction of Reynolds stresses by time averaging over the turbulent fluctuations. Accordingly, the turbulence-averaged governing equations are presented here: the continuity equation for incompressible fluid,

$$\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y}+\frac{\partial \tilde{w}}{\partial z}=0 \tag{1}$$

the horizontal momentum equations,

$$\frac{\partial \tilde{u}}{\partial t}+\frac{\partial \tilde{u}\tilde{u}}{\partial x}+\frac{\partial \tilde{u}\tilde{v}}{\partial y}+\frac{\partial \tilde{u}\tilde{w}}{\partial z}=-\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tilde{\tau}_{xx}}{\partial x}+\frac{\partial \tilde{\tau}_{yx}}{\partial y}+\frac{\partial \tilde{\tau}_{zx}}{\partial z}\right) \tag{2}$$

$$\frac{\partial \tilde{v}}{\partial t}+\frac{\partial \tilde{v}\tilde{u}}{\partial x}+\frac{\partial \tilde{v}\tilde{v}}{\partial y}+\frac{\partial \tilde{v}\tilde{w}}{\partial z}=-\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tilde{\tau}_{xy}}{\partial x}+\frac{\partial \tilde{\tau}_{yy}}{\partial y}+\frac{\partial \tilde{\tau}_{zy}}{\partial z}\right) \tag{3}$$

and the vertical momentum equation,

$$\frac{\partial \tilde{w}}{\partial t}+\frac{\partial \tilde{w}\tilde{u}}{\partial x}+\frac{\partial \tilde{w}\tilde{v}}{\partial y}+\frac{\partial \tilde{w}\tilde{w}}{\partial z}=-\frac{1}{\rho}\frac{\partial(\tilde{p}+\rho g z)}{\partial z}+\frac{1}{\rho}\left(\frac{\partial \tilde{\tau}_{xz}}{\partial x}+\frac{\partial \tilde{\tau}_{yz}}{\partial y}+\frac{\partial \tilde{\tau}_{zz}}{\partial z}\right) \tag{4}$$

where the superscript $\sim$ is used to denote ensemble-averaging and $\tilde{\tau}$ includes the Reynolds stresses due to turbulence flow.

3. HORIZONTAL CIRCULATION MODEL

The governing equations for the horizontal circulation model are obtained after depth integration and wave-averaging. In order to protect from losing the Eulerian mean quantities at the mean water level, the depth integration is taken prior to wave-averaging them.

3.1 Depth-Integrated and Time-Averaged Equation of Mass

Integrating Eq. (1) over depth and employing the kinematic boundary conditions on the free surface and on the bottom, hereafter omitting the tildes, we get

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{\eta} u\,dz+\frac{\partial}{\partial y}\int_{-h}^{\eta} v\,dz=0 \tag{5}$$

Now let the turbulence-averaged velocity vector, $\mathbf{U}(u, v, w)$, be decomposed into mean velocity and wave fluctuation, which will be distinguished by the subscript c and w, respectively; thus,

$$\mathbf{U}=\mathbf{U}_c+\mathbf{U}_w \tag{6}$$

$$\eta=\eta_c+\eta_w \tag{7}$$

where $\mathbf{U}_w$ is the residual wave fluctuation which can be removed through the process of wave-averaging, and $\mathbf{U}_c$ is the time-averaged value of velocity which can be measured by a particle moving with the residual wave fluctuation. The velocity of a particular water particle with a mean position of $(\mathbf{x}_1, z_1)$ is $\mathbf{U}(\mathbf{x}_1+\zeta, z_1+\xi)$, where ζ and ξ are locations of the trajectory of the particle moving with the residual wave fluctuation. The trajectory is supposed to be closed. Then, we obtain the wave-averaged value of velocity, $\mathbf{U}_c$, as

$$\mathbf{U}_c(\mathbf{x}_1, z_1)=\frac{1}{T}\int_0^T \mathbf{U}(\mathbf{x}_1+\zeta, z_1+\xi)dt$$

Substituting Eqs. (6-7) and taking the wave-average after expanding in a Taylor series at $\eta=\eta_c$, Eq. (5) can be simplified as

$$\frac{\partial \eta_c}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c dz+\frac{\partial}{\partial x}\overline{(\eta_w u_w)}_{\eta_c}$$

$$+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} v_c dz+\frac{\partial}{\partial y}\overline{(\eta_w v_w)}_{\eta_c}=0 \tag{8}$$

The wave components are given by linear progressive wave theory as follow:

$$\frac{\partial \eta_c}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} v_c dz$$
$$+\frac{\partial M_x}{\partial x}+\frac{\partial M_y}{\partial y}=0 \tag{9}$$

where the x and y components of mass flux are defined as

$$M_x=\frac{E'k_x}{\rho\sigma},\ M_y=\frac{E'k_y}{\rho\sigma} \tag{10}$$

E' is defined as $\rho g H^2/8$. The 4th and 5th terms are considered the mass transport above the mean water level in the mass conservation.

3.2 Depth-Integrated and Time-Averaged Equations of Momentum

Assuming that no horizontal viscous stress exist, Eq. (2) is integrated over depth to yield

$$\frac{\partial}{\partial t}\int_{-h}^{\eta} u dz+\frac{\partial}{\partial x}\int_{-h}^{\eta} uu dz+\frac{\partial}{\partial y}\int_{-h}^{\eta} uv dz$$
$$=\frac{1}{\rho}\left[-\frac{\partial}{\partial x}\int_{-h}^{\eta} p dz+p|_{\eta}\frac{\partial \eta}{\partial x}+p|_{-h}\frac{\partial h}{\partial x}+\tau_{Wx}-\tau_{Bx}\right] \tag{11}$$

where $\tau_{Wx}=\tau_{zx}|_{\eta}$ is a wind stress in the x direction and $\tau_{Bx}=\tau_{zx}|_{-h}$ is the bottom friction. Substituting U defined in Eq. (6), the time-averaged quantities is also obtained by expanding η in Taylor series at the mean water level, η_c,

$$\frac{\partial}{\partial t}\int_{-h}^{\eta_c} u_c dz+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c^2 dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} u_c v_c dz$$
$$+\frac{\partial}{\partial x}\int_{-h}^{\eta_c}\overline{u_w^2}dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c}\overline{u_w v_w}dz$$
$$+\frac{\partial}{\partial t}\overline{(\eta_w w_w)}|_{\eta_c}+\frac{\partial}{\partial x}\overline{(2\eta_w u_c u_w)}|_{\eta_c}$$
$$+\frac{\partial}{\partial y}\left[\overline{\eta_w(u_c v_w+v_c u_w)}\right]_{\eta_c}$$
$$=\frac{1}{\rho}\left[-\frac{\partial}{\partial x}\overline{\int_{-h}^{\eta} p dz}+\overline{p|_{-h}}\frac{\partial h}{\partial x}+\overline{\tau_{Wx}}-\overline{\tau_{Bx}}\right] \tag{12}$$

where the pressure at the free surface was assumed to be zero. The pressure term requires some special attention as it is composed of a time averaged pressure due to mean flow and a time averaged pressure due to the wave motion. The time averaged mean flow pressure can be obtained from the vertical momentum equation after some mathematical manipulations,

$$\bar{p}(z)=-\rho\overline{w_w^2(z)}+\rho g(\eta_c-z) \tag{13}$$

then, the total pressure is given as

$$p=\bar{p}+p_w=-\rho\overline{w_w^2(z)}+\rho g(\eta_c-z)+\rho g\eta_w K_p(z) \tag{14}$$

where K_p is the pressure response factor given by linear wave theory,

$$K_p=\frac{\cosh k(h+z)}{\cosh k(h+\eta_c)} \tag{15}$$

Substituting Eq. (14), finally, the depth-integrated and time-averaged momentum equation in the x direction is obtained;

$$\frac{\partial}{\partial t}\int_{-h}^{\eta_c} u_c dz+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c^2 dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} u_c v_c dz$$
$$+\frac{\partial}{\partial x}\int_{-h}^{\eta_c}\overline{u_w^2}dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c}\overline{u_w v_w}dz$$
$$+\frac{\partial}{\partial t}\overline{(\eta_w w_w)}|_{\eta_c}+\frac{\partial}{\partial x}\overline{(2\eta_w u_c u_w)}|_{\eta_c}$$
$$+\frac{\partial}{\partial y}\left[\overline{\eta_w(u_c v_w+v_c u_w)}\right]_{\eta_c}$$
$$=\frac{1}{\rho}\left[\frac{\partial}{\partial x}\int_{-h}^{\eta_c}\rho\overline{w_w^2(z)}dz-\rho g(\eta_c+h)\frac{\partial \eta_c}{\partial x}\right.$$
$$\left.-\frac{\rho g}{2}\frac{\partial\overline{\eta_w^2}}{\partial x}+\overline{\tau_{Wx}}-\overline{\tau_{Bx}}\right] \tag{16}$$

When the following definitions representing the excess momentum fluxes are introduced

$$S_{xx}=\rho\left[\int_{-h}^{\eta_c}(\overline{u_w^2}-\overline{w_w^2})dz+\frac{g}{2}\overline{\eta_w^2}+2u_c(\eta_c)\overline{\eta_w u_w(\eta_c)}\right] \tag{17}$$

$$S_{yx}=\rho\left[\int_{-h}^{\eta_c}\overline{u_w v_w}dz+u_c\overline{\eta_w v_{w\eta_c}}+v_c\overline{\eta_w u_{w\eta_c}}\right] \tag{18}$$

Equation (16) can be expressed as

$$\frac{\partial}{\partial t}\int_{-h}^{\eta_c} u_c dz+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c^2 dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} u_c v_c dz$$
$$+\frac{1}{\rho}\frac{\partial S_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial S_{yx}}{\partial y}+g(h+\eta_c)\frac{\partial \eta_c}{\partial x}$$

$$-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx}}}{\rho}=0 \tag{19}$$

The momentum equation in the y direction can be similarly obtained,

$$\frac{\partial}{\partial t}\int_{-h}^{\eta_c} v_c dz+\frac{\partial}{\partial x}\int_{-h}^{\eta_c} u_c v_c dz+\frac{\partial}{\partial y}\int_{-h}^{\eta_c} v_c^2 dz$$
$$+\frac{1}{\rho}\frac{\partial S_{xy}}{\partial x}+\frac{1}{\rho}\frac{\partial S_{yy}}{\partial y}+g(h+\eta_c)\frac{\partial \eta_c}{\partial y}$$
$$-\frac{\overline{\tau_{Wy}}}{\rho}+\frac{\overline{\tau_{By}}}{\rho}=0 \tag{20}$$

where

$$S_{yy}=\rho\left[\int_{-h}^{\eta_c}(\overline{v_w^2}-\overline{w_w^2})dz+\frac{g}{2}\overline{\eta_w^2}+2v_c(\eta_c)\overline{\eta_w v_w(\eta_c)}\right] \tag{21}$$

$$S_{xy}=\rho\left[\int_{-h}^{\eta_c}\overline{u_w v_w}dz+v_c\overline{\eta_w U_{w\eta_c}}+u_c\overline{\eta_w v_{w\eta_c}}\right] \tag{22}$$

For the case of linear progressive wave and mild slope, the radiation stress terms can be expressed in terms of wave charateristics as

$$S_{xx}=E'\left[n(\cos^2\theta+1)-\frac{1}{2}+2\cos\theta\frac{u_s}{C}\right] \tag{23}$$

$$S_{xy}=S_{yx}=E'\left[\sin\theta\left(n\cos\theta+\frac{u_s}{C}\right)+\cos\theta\frac{v_s}{C}\right] \tag{24}$$

$$S_{yy}=E'\left[n(\sin^2\theta+1)-\frac{1}{2}+2\sin\theta\frac{v_s}{C}\right] \tag{23}$$

where $n=Cg/C$ and u_s and v_s are the time-averaged current velocity at mean water level.

3.3 Roles of New Surface Advection Terms in Radiation Stress

The radiation stresses as derived above differ from classic definitions given by Longuet-Higgins and Stewart (1961) with the additional terms arising from the avection near the surface. These new terms have a number of important roles as will be elaborated here.

The first is related to the physical role. It is generally accepted that longshore current inside the surf zone is induced by excess radiation stresses. According to the classic definition, the calculated longshore current distribution within the surf zone is triangular increasing linearly with distance from the shoreline up to the breaking point and then ending there abruptly (Longuet-Higgins, 1970). In order to smooth the distribution to more realistically represent the actual observation an artifical lateral mixing term is introduced. Unrealistically large mixing coefficients are often required fit the actual data (Bowen, 1969; Svendsen and Putrevu, 1990). The new definition completely eliminates the need of the mixing term and yields good agreement with the observations as shown in Lee (1993). The second and more fundamental importance is the fact that the new definition gives the correct form of wave energy transformation in the presence of current.

Consider waves approaching a straight coastline at an angle, and being absorbed by breaking in the surf zone. Since energy E and momentum M are conserved, the following relation is exact regardless of the theory of waves.

$$\frac{dE}{dt}=C_a'\frac{dM}{dt} \tag{26}$$

where C_a' is the y-directional velocity of a fluid body which is $C_a/\sin\theta$. Here C_a is defined by ω/k and yet C has been defined by σ/k. dE/dt and dM/dt are clearly equal to $\partial F_x/\partial x$ and $\partial S_{xy}/\partial x$, respectively. The x-axis is directed onshore, and y-axis alongshore in accordance with a right-handed system. Hence,

$$\frac{\partial F_x}{\partial x}=\frac{C_a}{\sin\theta}\frac{\partial S_{xy}}{\partial x} \tag{27}$$

Substituting Eq. (24), we have

$$\frac{\partial F_x}{\partial x}=\frac{C_a}{\sin\theta}\frac{\partial}{\partial x}\left(E'[\sin\theta(\cos\theta n+\frac{u_s}{C})+\cos\theta\frac{v_s}{C}]\right) \tag{28}$$

The energy flux was expressed by

$$F_x=(u_s+Cg\cos\theta)\frac{w}{\sigma}E' \tag{29}$$

then Eq. (28) gives the relation

$$\frac{\partial}{\partial x}(\cos\theta\frac{v_s}{C_a}E')=0 \tag{30}$$

which results in the expression of the longshore current as follows.

$$\cos\theta\frac{v_s}{C}H^2=Const \tag{31}$$

$$\rightarrow\ v_s=Const\frac{C}{\cos\theta H^2} \tag{32}$$

This expression is consistent with that from the surf zone model given in Lee (1993). Therefore, this relation between energy and momentum conservations demonstrates the validity of the surf zone model. In addition, it also presents that the surface advection terms have to be included in the radiation stresses of second order if currents exist.

4. VERTICAL CIRCULATION MODEL

The local imbalance between the depth-varying momentum flux and the depth uniform set-up force is known to be the driving force for causing vertical circulation. This driving mechanism was originally postulated by Nielsen and Sorensen (1970) and was first analytically treated in a thesis by Dally (1980). Svendsen (1984) developed a theoretical model using the first order approximation technique in describing breaking waves, and Hansen and Svendsen (1984) further incorperated the effect of the bottom boundary layer in the solution. More recently, several other ideas have been suggested. Okayasu *et al.* (1988) estimated the undertow profiles based on the assumed mean shear stress and eddy viscosity, and Yamashit a *et al.* (1990) developed a numerical model which consists of surface and inner layers. Here, the theoretical undertow and longshore current models are carried further over straight parallel contours as described below.

4.1 Theoretical Undertow Model

The vertical circulation model of a steady, two-dimensional motion in the x-z plane begins with the turbulent-averaged equations of momentum conservation and the wave-and turbulent-averaged equation of mass conservation. Assuming a steady, two-dimensional motion in the x-z plane, Eq. (9) becomes,

$$\frac{\partial}{\partial x}(Q_x+M_x)=0 \tag{33}$$

where

$$Q_x=\int_{-h}^{\eta_c}u_c dz \tag{34}$$

The depth-integrated discharge of x *component*, Q_x, can be expressed by the mass flux since the depth-integrated total mass flux has to be zero in the steady. That is,

$$Q_x=-M_x \tag{35}$$

Integrating Eq. (2) up to the free surface and applying a free surface kinematic boundary condition with help of Leibnitz's rule yields

$$\frac{\partial}{\partial x}\int_z^{\eta}uudz-uw|_z=\frac{1}{\rho}\Big[-\frac{\partial}{\partial x}\int_z^{\eta}pdz+p(\eta)\frac{\partial\eta}{\partial x}+\tau_{zx}(\eta)-\tau_{zx}(z)\Big] \tag{36}$$

where any horizontal viscous stresses are assumed not to exist and the vertical shear stress is assumed to be expressed in the form

$$\tau_{zx}=\rho\nu_t\frac{\partial u}{\partial z} \tag{37}$$

where ν_t is total kinematic viscosity, which is composed of both eddy and molecular viscosities. The shear stress at surface, $\tau(\eta)$, is assumed only due to wind stress τ_W. Separating the velocity into the current and wave components and taking time-average, Eq. (36) becomes

$$\frac{\partial}{\partial x}\int_z^{\eta_c}(\overline{u_w^2}-\overline{w_w^2})dz=-\frac{g}{2}\frac{\partial\overline{\eta_w^2}}{\partial x}+\frac{\partial}{\partial x}(2\overline{\eta_w u_c u_w})|_{\eta_c}-g(\eta_c-z)\frac{\partial\eta_c}{\partial x}+\frac{\tau_{Wx}}{\rho}-\nu_t\frac{\partial u_c}{\partial z}|_z \tag{38}$$

The effect of squared mean current was assumed to be small enough compared to the rest. In shallow water, the first term becomes

$$\frac{\partial}{\partial x}\int_z^{\eta_c}(\overline{u_w^2}-\overline{w_w^2})dz=g\frac{(\eta_c-z)}{(\eta_c+h)}\frac{\partial}{\partial x}\Big[\cos^2\theta\frac{H^2}{8}\Big] \tag{39}$$

and the second term on the right hand side reduces to,

$$\frac{g}{2}\frac{\partial\overline{\eta_w^2}}{\partial x}=\frac{g}{16}\frac{\partial H^2}{\partial x} \tag{40}$$

The first term on the right hand side can be determined by the depth-integrated equation of momentum, Eq. (19), under the following assumptions; 1) the flow is in steady state, and 2) the effect of squared mean current are negligible, Eq. (39), then, becomes

$$\frac{\partial \eta_c}{\partial x}=-\frac{1}{\rho g(h+\eta_c)}\left[\frac{\partial S_{xx}}{\partial x}-\overline{\tau_{Wx}}+\overline{\tau_{Bx}}\right] \tag{41}$$

Substituting Eqs. (39-41) into Eq. (38) and nondimensionalizing by $z'=(\eta_c-z)/(\eta_c+h)$ results in,

$$-\frac{\nu_t}{\eta_c+h}\frac{\partial u_c}{\partial z'}\Big|_{z'=z'}\Big\{\frac{g}{16}\frac{\partial H^2}{\partial x}-2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx}}}{\rho}\Big\}-\frac{g}{16}\frac{\partial H^2}{\partial x}+2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)+\frac{\overline{\tau_{Wx}}}{\rho} \tag{42}$$

According to the above equation, we can estimate the shear stress at the mean water level,

$$-\frac{\nu_t}{\eta_c+h}\frac{\partial u_c}{\partial z'}\Big|_{\eta_c}=-\frac{g}{16}\frac{\partial H^2}{\partial x}+2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)+\frac{\overline{\tau_{Wx}}}{\rho} \tag{43}$$

and the shear stress at the bottom,

$$-\frac{\nu_t}{\eta_c+h}\frac{\partial u_c}{\partial z'}\Big|_{-h}=\frac{\overline{\tau_{Bx}}}{\rho} \tag{44}$$

Integrating Eq. (42) with respect to z' after replacing ν_t by an eddy viscosity ε_z assumed to be constant, we get

$$u_c(z')=C_{x1}z'^2+C_{x2}z'+C_{x3} \tag{45}$$

where

$$C_{x1}=-\frac{\eta_c+h}{2\varepsilon_z}\Big\{\frac{g}{16}\frac{\partial H^2}{\partial x}-2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx,tb}}}{\rho}\Big\} \tag{46}$$

$$C_{x2}=-\frac{\eta_c+h}{\varepsilon_z}\Big\{-\frac{g}{16}\frac{\partial H^2}{\partial x}+2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)+\frac{\overline{\tau_{Wx}}}{\rho}\Big\} \tag{47}$$

$$C_{x3}=u_s \tag{48}$$

where the vertical eddy viscosity, ε_z, will be estimated in Section 4.3. Substituting Eq. (45) into Eq. (34), the $\overline{\tau_{Bx,tb}}$ term appeared in Eq. (46) can be expressed by

$$\frac{\overline{\tau_{Bx,tb}}}{\rho}=-\frac{6\varepsilon_z}{\eta_c+h}(\bar{u}-u_s)+2\Big\{\frac{g}{16}\frac{\partial H^2}{\partial x}-2g\frac{\partial}{\partial x}\left(\cos\theta\frac{u_s}{C}\frac{H^2}{8}\right)-\frac{\overline{\tau_{Wx}}}{\rho}\Big\} \tag{49}$$

where $\bar{u}=Q_x/(\eta_c+h)$. The $\overline{\tau_{Bx,tb}}$ denotes the shear stress due to turbulence. A brief discussion on the bottom velocity is given below. Since a bottom streaming velocity exists in a thin bottom layer, the bottom velocity contains two components,

$$\mathbf{U}_B=\mathbf{U}_{B,tb}+\mathbf{U}_{strm} \tag{50}$$

where the first component represents the bottom velocity due to the wave breaking induced turbulent flow and the second term is the stream velocity in the oscillatory boundary. These two components generally are opposite to each other. The bottom shear stress is also assumed to consist of two terms,

$$\overline{\tau_B}=\overline{\tau_{B,tb}}+\overline{\tau_{B,bf}} \tag{51}$$

This decomposition, which simplifies the formulation of the vertical circulation model considerably, is proven to be appropriate for determining the wave-induced currents as long as the bottom boundary layer remains thin and is not much disturbed by the turbulent motions originating from the surface wave breaking (Dong and Anastasiou, 1991). The first component representing shear stress induced by eddy viscosity and is given by

$$\frac{\overline{\tau_{B,tb}}}{\rho}=-\frac{\varepsilon_z}{\eta_c+h}\frac{\partial \mathbf{U}_{tb}}{\partial z'}\Big|_{-h} \tag{52}$$

where $\mathbf{U}_{tb}$ is a vectorial expression of the turbulent-induced currents as given in Eq. (45). The second term is the shear stress due to bottom drag given by,

$$\overline{\tau_{B,bf}}=\rho F_c|u_{orb}|(\mathbf{U}_{B,tb}+\mathbf{U}_{strm}) \tag{53}$$

under the assumption of weak current. The vertical eddy viscosity will be estimated in Section 4.3.

4.2 Longshore Current Model

Integrating Eq. (3) as done in the undertow model yields

$$\frac{\partial}{\partial x}\int_{z}^{\eta} vudz - vw|_{z} = \frac{1}{\rho}[\tau_{zy}(\eta) - \tau_{zy}(z)] \quad (54)$$

where the assumption has been made that the lateral mixing is negligible under small current. The shear stress is assumed to be in the form of

$$\tau_{zy} = \rho \nu t \frac{\partial v}{\partial z} \quad (55)$$

Again, the shear stress at surface, $\tau(\eta)$, is assumed to be due to wind stress ρ_W only. Separating the velocity into the current and wave components and taking time-average, Eq. (54) becomes

$$\frac{\partial}{\partial x}\int_{z}^{\eta_c} (v_c u_c + \overline{v_w u_w})dz + \frac{\partial}{\partial x}(\overline{\eta_w v_c u_w})|\eta_c + \frac{\partial}{\partial x}(\overline{\eta_w u_c v_w})|_{\eta_c} = \frac{1}{\rho}\left[\overline{\tau_{Wy}} - \rho \nu_t \frac{\partial v_c}{\partial z}|_z\right] \quad (56)$$

The second and third terms become zero as described in Section 3.3. Assuming the effect of squared mean current is small enough, Eq. (56) is reduced to

$$\frac{\partial}{\partial x}\int_{z}^{\eta_c} \overline{v_w u_w} dz = \frac{\overline{\tau_{Wy}}}{\rho} - \nu_t \frac{\partial v_c}{\partial z}|_z \quad (57)$$

In shallow water, the LHS becomes

$$\frac{\partial}{\partial x}\int_{z}^{\eta_c} \overline{v_w u_w} dz = \frac{(\eta_c - z)}{(\eta_c + h)}\frac{\partial}{\partial x}\left[g \cos\theta \sin\theta \frac{H^2}{8}\right] \quad (58)$$

by linear wave theory. Substituting Eq. (58) into Eq. (57) and nondimensionalizing by $z' = (\eta_c - z)/(\eta_c + h)$ gives the following,

$$-\frac{\nu_t}{\eta_c + h}\frac{\partial v_c}{\partial z'}|_{z'} = -z'\frac{\partial}{\partial x}\left[g \cos\theta \sin\theta \frac{H^2}{8}\right] + \frac{\overline{\tau_{Wy}}}{\rho} \quad (59)$$

The shear stress at the mean water level can then be obtained,

$$-\frac{\nu_t}{\eta_c + h}\frac{\partial v_c}{\partial z'}|_{\eta_c} = \frac{\overline{\tau_{Wy}}}{\rho} \quad (60)$$

as well as the shear stress at the bottom,

$$-\frac{\nu_t}{\eta_c + h}\frac{\partial v_c}{\partial z'}|_{-h} = \frac{\overline{\tau_{By}}}{\rho} = -g\frac{\partial}{\partial x}\left[\cos\theta \sin\theta \frac{H^2}{8}\right] + \frac{\overline{\tau_{Wy}}}{\rho} \quad (61)$$

Integrating Eq. (59) over z' after replacing ν_t by eddy viscosity, ε_z, assumed to be constant, we get

$$v_c(z') = C_{y1}z'^2 + C_{y2}z' + C_{y3} \quad (62)$$

where

$$C_{y1} = -\frac{\eta_c + h}{2\varepsilon_z}\left\{-\frac{\overline{\tau_{Wy}}}{\rho} + \frac{\overline{\tau_{By,tb}}}{\rho}\right\} \quad (63)$$

$$C_{y2} = -\frac{\eta_c + h}{\varepsilon_z}\frac{\overline{\tau_{Wy}}}{\rho} \quad (64)$$

$$C_{y3} = v_s \quad (65)$$

The depth-averaged mean longshore current is then obtained by

$$\bar{v} = v_s + \frac{C_{y1}}{3} + \frac{C_{y2}}{2} \quad (66)$$

4.3 Estimation of Eddy Viscosity

The vertical eddy viscosity for both cross-shore and longshore components can estimated here by using the same approach proposed by Peregrine and Svendsen (1978). Their appoach draws upon the similarities between breaking zone and turbulent wake flows. The fomula for eddy viscosity assumes the followin form (de Vriend and Stive, 1987)

$$\varepsilon_z = N_z |u_{orb}|(\eta_c + h) \quad (67)$$

where N_z is an unknown coefficient.

In this study, a coefficient, N_z, for both cross-shore and longshore components is estimated by assuming that 1) the depth-averaged longshore current is proportional to the surface longshore current, namely,

$$\bar{v} = \gamma v_s$$

and 2) the turbulent shear stress can be expressed in terms of the depth-averaged longshore current so that;

$$\frac{\overline{\tau_{By,tb}}}{\rho} = F_w |u_{orb}| \gamma v_s$$

where γ is a constant coefficient. From Eq. (66) we have

$$\gamma v_s = v_s - \frac{\eta_c + h}{6\varepsilon_z}F_w |u_{orb}| \gamma v_s \quad (68)$$

which yields

$$\frac{\overline{\tau_{By,tb}}}{\rho}=\frac{6\varepsilon_z}{\eta_c+h}(1-\gamma)v_s \tag{69}$$

If the turbulent-induced shear stress dominanates the bottom friction, the folllowing relation can be obtained:

$$\frac{6\varepsilon_z}{\eta_c+h}(1-\gamma)v_s=\frac{1}{\rho}\frac{\sin\theta}{C_a}D \tag{70}$$

where D is the local rate of dissipation. Substituting the expression of longshore current at the surface level given in Lee (1993) into Eq. (70), we obtain the expression of eddy viscosity,

$$\frac{\varepsilon_z}{\eta_c+h}=\frac{D}{6\rho(1-\gamma)\beta_L C_a(\beta Cg_b-Cg)} \tag{71}$$

where the local rate of dissipation, D, and the absolute phase speed, C_a, are given by

$$D=-\nabla\cdot\left[\beta Cg_b\frac{\mathbf{K}}{k}\frac{\omega}{\sigma}E\right]\quad C_a=\frac{\omega}{k}$$

Based on the concept described in the surf zone model of Lee (1993), Eq. (71) can be simplified as

$$\frac{\varepsilon_z}{\eta_c+h}=-\frac{1}{24(1-\gamma)\beta_L}\frac{\nabla\cdot(\mathbf{K}H/k)}{(1-Cg/(\beta Cg_b))}|u_{orb}| \tag{72}$$

Therefore, the coefficient, N_z, in Eq. (67) becomes

$$N_z=-\frac{1}{24(1-\gamma)\beta_L}\frac{\nabla\cdot(\mathbf{K}H/k)}{(1-Cg/(\beta Cg_b))} \tag{73}$$

4.4 Theoretical Solutions

In this subsection, theoretical solutions for the vertical velocity distributions are presented in surf zone of a plane beach. Wind stress effect is omitted in the solutions.

4.4.1 Undertow Model

According to Eq. (72), Eqs. (46-48) are rewritten here as

$$C_{x1}=3(\bar{u}-u_s)-\frac{3}{2}\frac{\eta_c+h}{\varepsilon_z}P \tag{74}$$

$$C_{x2}=\frac{\eta_c+h}{\varepsilon_z}P. \tag{75}$$

$$C_{x3}=u_s \tag{76}$$

where

$$P\cong\frac{gH}{8}\frac{\partial H}{\partial x}-\frac{g^2m\beta\beta_H\beta_O(1-\beta_O)H_b}{8CC_a^2}$$

$$\bar{u}=-\frac{g}{8}\frac{\cos\theta H^2}{C(\eta_c+h)}$$

$$u_s=\beta_O\cos\theta(\beta Cg_b-Cg) \tag{77}$$

where m is a beach slope, P and u_s are based on the concept described in the surf zone model of Lee (1993), and the wave angle is assumed to be nearly normal in order to approximate P. The velocity profile can then be calculated by Eq. (45). Four parameters are to be designated; they are, γ: the ratio of mean current to surface current; β: the dissipation coefficient; β_O: the onshore current magnitude coefficient; and β_L: the longshore current magnitude coeffcient.

Figure 1 shows the comparisions of the computed vertical profiles of the cross-shore current with those measured by Hansen and Svendsen (1984). The test conditions were: slope=1:34.25; H_o=0.12m and T= 1 sec. The parameters used in the computations are: β=0.17; β_O=0.07; β_L=5.0 and γ=0.982. The limiting wave height at breaking point is determined by the Miche's criterion with κ=0.78. Fig. 2 plots the profile changes across the untire surf zone using the same paramters as given above. In order to examine the effect of the advection term (the second term) given in Eq. (46), the results when the term is neglected are also represented as dotted lines in Figure 3. The effect seems to show the significant deviation from the measurements as it is close to

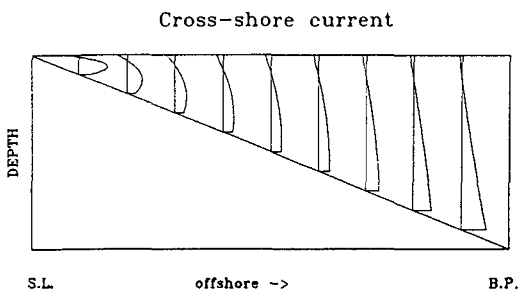

Fig. 1. Vertical profiles of cross-shore current.

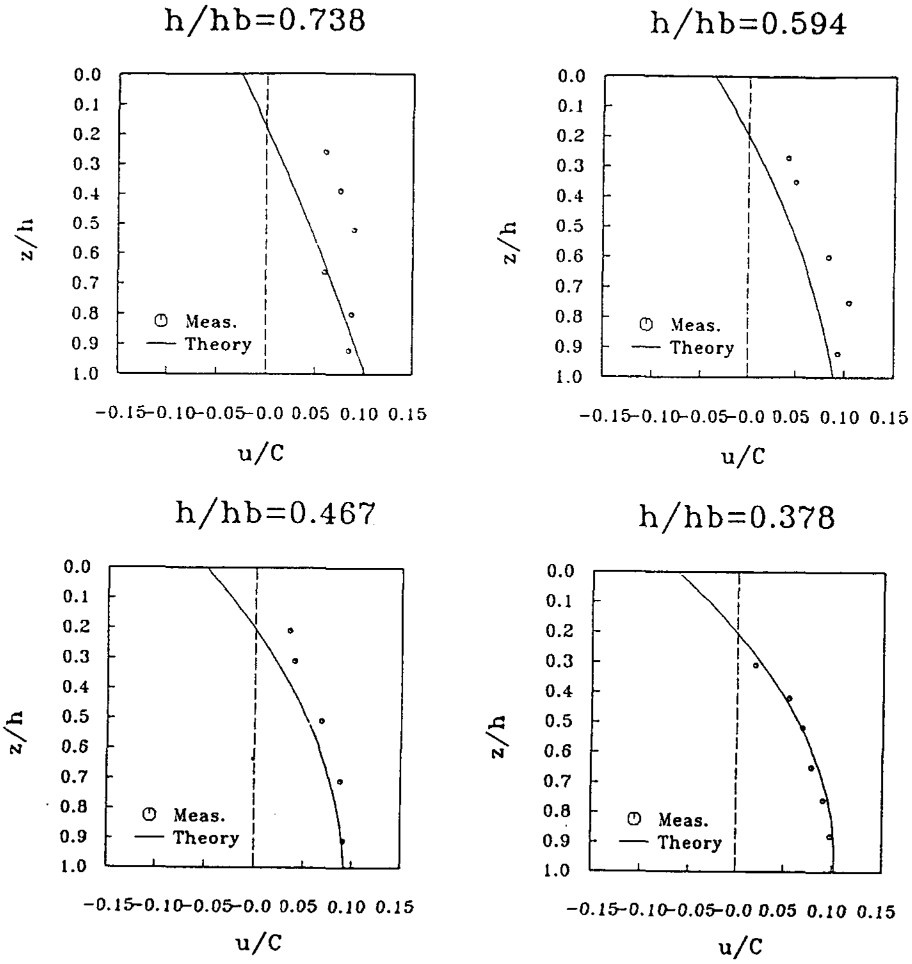

Fig. 2. Comparison with experiments presented by Hansen and Svendsen (1984).

the shoreline under the same input condition. However, the difference also seems to show the overall agreement with the experiments by small reduction of the γ value as shown in Fig. 3.

4.4.2. Longshore Current Model

According to Eq. (69), the coefficients in the longshore current model given in Eqs. (63-65) are rewritten here as

$$C_{y1}=-3(1-\gamma)v_s \tag{78}$$

$$C_{y2}=0 \tag{79}$$

$$C_{y3}=v_s \tag{80}$$

where $v_s=\beta_L\sin\theta(\beta Cg_b-Cg)$ as given in Lee (1993). Fig. 4 shows the comparasions between computed profiles and the laboratory data measured by Visser (1991). The test conditions were: slope=1:10, H_o=9.6 cm, T=1 sec and θ_o=16.4°. %(give wave height, length and bottom slope also). The values of parameters are as follows; β=0.2, β_L=5.0, and γ=0.96. It is seen that γ plays an important role. A maximum value 1 results in a uniform longshore current profile whereas a limiting minimum value 2/3 results in a no-slip bottom velocity. From the comparisons with experimental data, a value near 0.95 is suggested. Fig. 5 plots the longshore current profile variations across the surfzone.

Fig. 6 illustrates the three-dimensional current profiles inside the surf zone using the same conditions as Figure 3 with the exception that the input

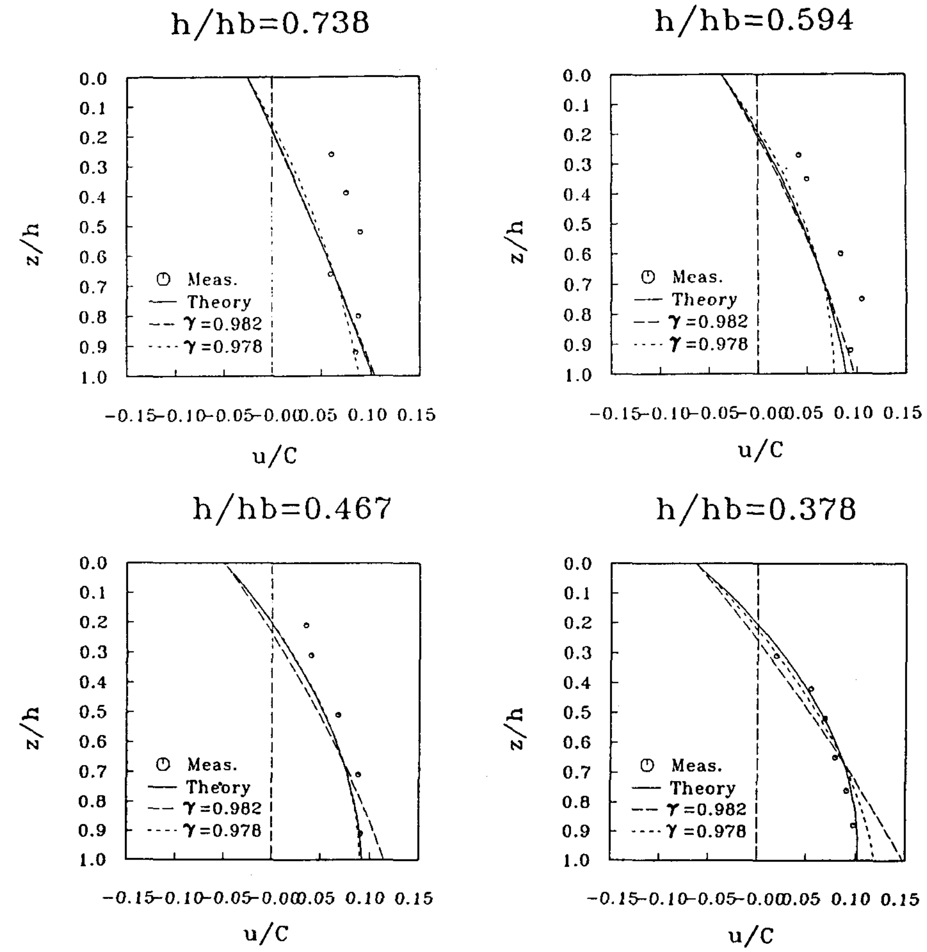

Fig. 3. Effect of the advection term in the undertow model.

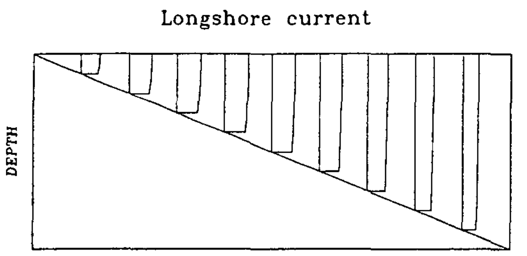

Fig. 4. Vertical profiles of longshore current.

wave is oblique at 10° in deep water clockwise to the shoreline normal. The three-dimensional current forms a clockwise spiral from top to bottom.

4.5 Model Adoption for General Three-Dimensional Topography

The theoretical models so far developed are for parallel contours. For irregular bathymetries, boundary conditions the equation is required to get the surface velocity as the surface boundary condition. Since solving another equation might be ineffective for modelling, the bottom shear stress in terms of depth-averaged current is considered instead of the surface velocity. For the prediction of a longshore current this alternative way gives the exactly same result. For that of the undertow, however, this will give the different result. The bottom shear stress suggested by Longuet-Higgins (1970) is nowmodified

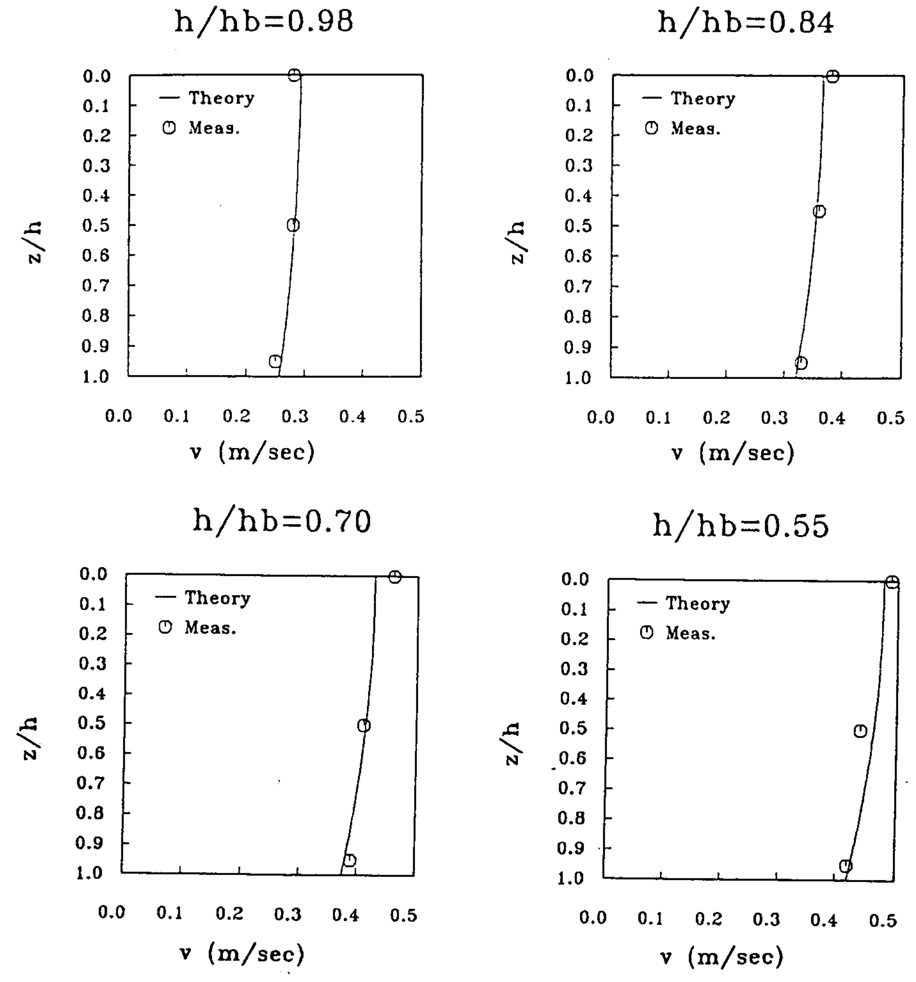

Fig. 5. Comparison with experiments presented by Visser (1991).

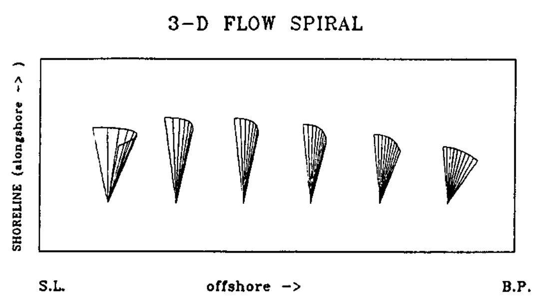

Fig. 6. Combined three-dimensional profiles.

for both cross-shore and longshore directions by

$$\overline{\tau_{B,tb}} = \rho F_w |u_{orb}| \gamma \bar{U} \tag{81}$$

where F_w can be estimated in terms of N_z introduced in Section 4.3,

$$F_w = 6N_z \frac{\gamma}{1-\gamma}$$

and in terms of wave characteristics,

$$F_w = \frac{\beta}{4\gamma\beta_L} \frac{\nabla\cdot(\mathbf{K}H/k)}{(\beta - Cg/Cg_b)} \tag{82}$$

When the bottom shear stress given in Eq. (49) is applied as a boundary condition instead of the surface velocity in the undertow model, three coefficients of the undertow model, C_{x1}, C_{x2} and C_{x3}, are written by

$$C_{x1} = -\frac{\eta_c + h}{2\varepsilon_z}(P + \overline{\tau_{Bx,tb}}) \tag{83}$$

$$C_{x2}=\frac{\eta_c+h}{\varepsilon_z}P \tag{84}$$

$$C_{x3}=\bar{u}-\frac{C_{x1}}{3}-\frac{C_{x2}}{2} \tag{85}$$

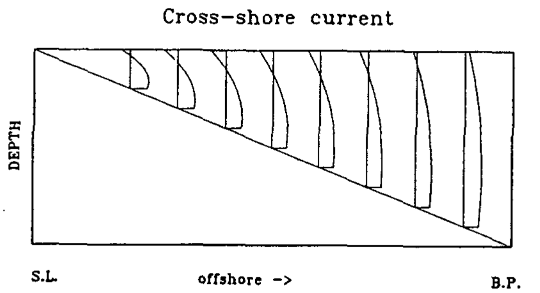

Fig. 7. Vertical profiles of cross-shore current by using bottom shear stress.

The result is shown in Figs. 7-8 for the same experimental conditions as used by Hansen ans Svendsen (1984) given in Section 3.4. The 'S.B.C.' indicates the full theory obtained by the surface boundary condition, and the 'B.B.C.' indicates the approximate theory obtained by the bottom shear stress with neglecting the advection term. The full theory was obtained by $\gamma=0.982$, the approximate theory by $\gamma=0.978$. The comparison with experiments is still in agreement. Therefore, instead of the boundary condition given by surface currents, the bottom shear stress is used in the practical model for the complicated bathymetry, and the advection terms are omitted.

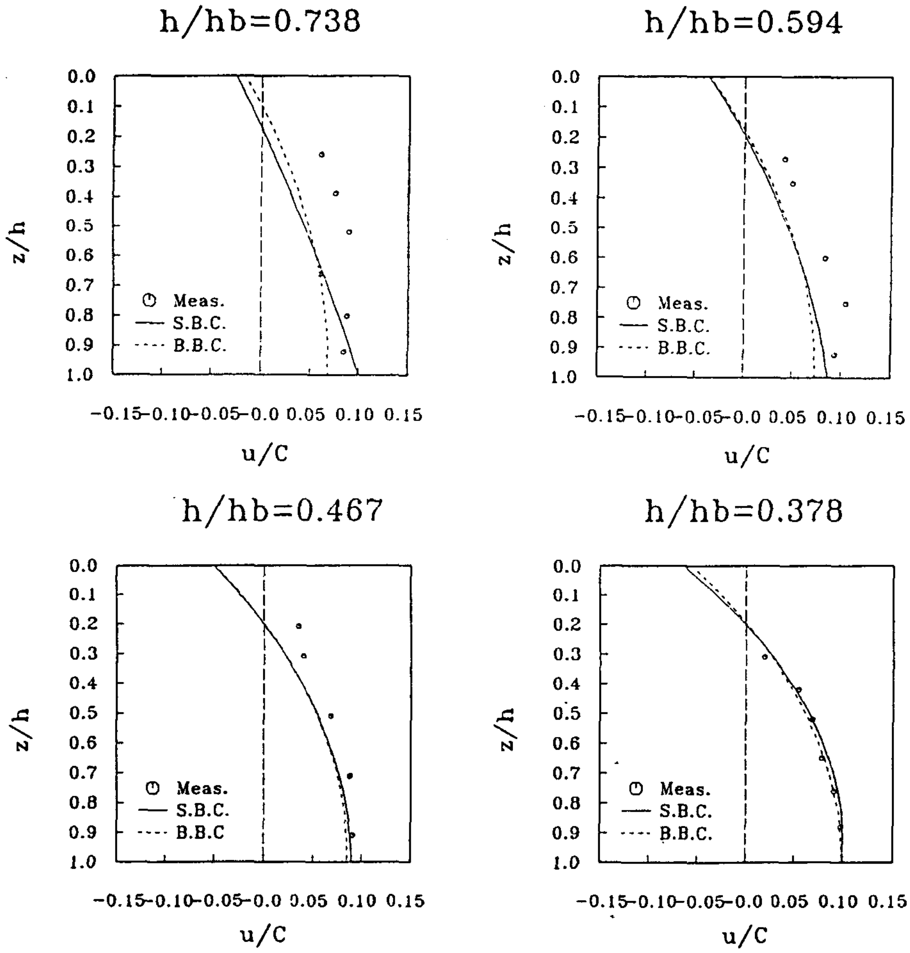

Fig. 8. Comparison with experiments presented by Hansen and Svendsen (1984).

5. QUASI-3D MODEL

The depth-integrated horizontal model is now combined with the vertical theoretical model to a quasi-3D model. This quasi-3D model looks promising since it provides three-dimensional information at almost the same cost of a two-dimensional horizontal model although it produces the relatively simple variation of vertical profile.

5.1 Modification of 2D Depth-Integrated Equations

Both from the theoretical solution in simple cases as well as laboratory measurements, velocity variation with respect to depth may be approximated as the function of parabola of 2nd order.

$$u_c=C_{x1}z'^2+C_{x2}z'+C_{x3} \tag{86}$$

$$v_c=C_{y1}z'^2+C_{y2}z'+C_{y3} \tag{87}$$

where C_1, C_2 and C_3 will be determined in the next subsection, and expressed in terms of Q, H, $h+\eta_c$, and $\overline{\tau_W}$. Substituting Eqs. (86) and (87) into the convective acceleration terms yields

$$\int_{-h}^{\eta_c} u_c^2 dz=\left[\frac{Q_x^2}{(h+\eta_c)^2}+T_{xx}\right](h+\eta_c) \tag{88}$$

$$\int_{-h}^{\eta_c} u_c v_c dz=\left[\frac{Q_x Q_y}{(h+\eta_c)^2}+T_{xy}\right](h+\eta_c) \tag{89}$$

$$\int_{-h}^{\eta_c} v_c^2 dz=\left[\frac{Q_y^2}{(h+\eta_c)^2}+T_{yy}\right](h+\eta_c) \tag{90}$$

where

$$Q_x=\int_{-h}^{\eta_c} u_c dz \tag{91}$$

$$Q_y=\int_{-h}^{\eta_c} v_c dz \tag{92}$$

$$T_{xx}=\left[\frac{4C_{x1}^2}{45}+\frac{C_{x2}^2}{12}+\frac{C_{x1}C_{x2}}{6}\right] \tag{93}$$

$$T_{xy}=T_{yx}=\left[\frac{4C_{x1}C_{y1}}{45}+\frac{C_{x2}C_{y2}}{12}+\frac{C_{x1}C_{y2}}{12}+\frac{C_{x2}C_{y1}}{12}\right] \tag{94}$$

$$T_{yy}=\left[\frac{4C_{y1}^2}{45}+\frac{C_{y2}^2}{12}+\frac{C_{y1}C_{y2}}{6}\right] \tag{95}$$

Substituting into Eq. (19) leads to the following x-directional modified momentum equation:

$$\frac{\partial Q_x}{\partial t}+\frac{\partial}{\partial x}\left[\frac{Q_x^2}{h+\eta_c}+(h+\eta_c)T_{xx}\right]+\frac{\partial}{\partial y}\left[\frac{Q_x Q_y}{h+\eta_c}+(h+\eta_c)T_{xy}\right]+\frac{1}{\rho}\frac{\partial S_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial S_{yx}}{\partial y}+g(h+\eta_c)\frac{\partial \eta_c}{\partial x}-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx}}}{\rho}=0 \tag{96}$$

The modified momentum equation of y direction from Eq. (20)

$$\frac{\partial Q_y}{\partial t}+\frac{\partial}{\partial x}\left[\frac{Q_x Q_y}{h+\eta_c}+(h+\eta_c)T_{yx}\right]+\frac{\partial}{\partial y}\left[\frac{Q_y^2}{h+\eta_c}+(h+\eta_c)T_{yy}\right]+\frac{1}{\rho}\frac{\partial S_{xy}}{\partial x}+\frac{1}{\rho}\frac{\partial S_{yy}}{\partial y}+g(h+\eta_c)\frac{\partial \eta_c}{\partial y}-\frac{\overline{\tau_{Wy}}}{\rho}+\frac{\overline{\tau_{By}}}{\rho}=0 \tag{97}$$

As noted below, the bottom friction consists of turbulent shear stress and bottom frictions due to viscous and streaming flows, which can be expressed as

$$\overline{\tau_B}=F_w|u_{orb}|\bar{\mathrm{U}}+F_c|u_{orb}|(\mathrm{U}_{B,tb}+\mathrm{U}_{strm}) \tag{98}$$

where the bottom velocity is determined by Eqs. (86) and (87) according to the coefficients obtained in the next subsection.

The continuity equation results in the same equation as before.

$$\frac{\partial \eta_c}{\partial t}+\frac{\partial}{\partial x}(Q_x+M_x)+\frac{\partial}{\partial y}(Q_y+M_y)=0 \tag{99}$$

5.2 Estimation of Integral Coefficients for Vertical Velocity Profile

Eq. (2) is elaborated to obtain the time-averaged equation,

$$\frac{\partial}{\partial x}\left[\int_{z^c}^{\eta}\overline{(u_w^2-w_w^2)}dz+\frac{g}{2}\overline{\eta_w^2}\right]+\frac{\partial}{\partial y}\left[\int_{z^c}^{\eta}\overline{u_w v_w}dz\right]=-g(\eta_c-z)\frac{\partial \eta_c}{\partial x}+\frac{\overline{\tau_{Wx}}}{\rho}-\nu_t\frac{\partial u_c}{\partial z}\Big|_z$$

The shear stress enforced at surface was assumed to be only wave-and wind-induced, and the effect of advection terms was neglected. Substituting $\partial \eta_c$

$/\partial x$ given in Eq. (19), the following nondimensional form can be obtained:

$$-\frac{\nu_t}{\eta_c+h}\frac{\partial u_c}{\partial z'}|_{z'}=z'\left\{\frac{g}{16}\frac{\partial H^2}{\partial x}-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx}}}{\rho}\right\} -\frac{g}{16}\frac{\partial H^2}{\partial x}+\frac{\overline{\tau^{Wx}}}{\rho} \tag{100}$$

We replace ν_t by a constant eddy viscosity, ε_z, assuming that the turbulent motion originating from the surface wave breaking is uniform over water depth. Integrating with respect to z' yields

$$u_c(z')=C_{x1}z'^2+C_{x2}z'+C_{x3} \tag{101}$$

where the coefficients are approximated as

$$C_{x1}=-\frac{\eta_c+h}{2\varepsilon_z}\left\{\frac{g}{16}\frac{\partial H^2}{\partial z}-\frac{\overline{\tau_{Wx}}}{\rho}+\frac{\overline{\tau_{Bx,tb}}}{\rho}\right\} \tag{102}$$

$$C_{x2}=-\frac{\eta_c+h}{\varepsilon_z}\left\{-\frac{g}{16}\frac{\partial H^2}{\partial x}+\frac{\overline{\tau_{Wx}}}{\rho}\right\} \tag{103}$$

$$C_{x3}=\frac{Q_x}{\eta_c+h}-\frac{C_{x1}}{3}-\frac{C_{x2}}{2} \tag{104}$$

Eq. (3) is also elaborated to obtain the y\$-directional time-averaged equation of integrated form,

$$\frac{\partial}{\partial y}\left[\int_z^{\eta_c}\overline{(v_w^2-w_w^2)}dz+\frac{g}{2}\overline{\eta_w^2}\right]+\frac{\partial}{\partial y}\left[\int_z^{\eta_c}\overline{u_w v_w}dz\right] =-g(\eta_c-z)\frac{\partial\eta_c}{\partial y}+\frac{\overline{\tau_{Wy}}}{\rho}-\nu_t\frac{\partial v_c}{\partial z}|_z$$

When the gradient of mean water level given in Eq. (20) is substituted, the following simple form can be obtained in terms of the bottom shear stress the same for x direction:

$$-\frac{\nu_t}{\eta_c+h}\frac{\partial v_c}{\partial z'}|_{z'}=z'\left\{\frac{g}{16}\frac{\partial H^2}{\partial y}-\frac{\overline{\tau_{Wy}}}{\rho}+\frac{\overline{\tau_{By}}}{\rho}\right\} -\frac{g}{16}\frac{\partial H^2}{\partial y}+\frac{\overline{\tau_{Wy}}}{\rho} \tag{105}$$

Integrating with respect to z yields

$$v_c(z')=C_{y1}z'^2+C_{y2}z'+C_{y3} \tag{106}$$

where the coefficients are expressed in terms of wave characteristics and current quantities resulted from the depth-integrated wave and circulation models;

$$C_{y1}=-\frac{\eta_c+h}{2\varepsilon_z}\left\{\frac{g}{16}\frac{\partial H^2}{\partial z}-\frac{\overline{\tau_{Wy}}}{\rho}+\frac{\overline{\tau_{By,tb}}}{\rho}\right\} \tag{107}$$

$$C_{y2}=-\frac{\eta_c+h}{\varepsilon_z}\left\{-\frac{g}{16}\frac{\partial H^2}{\partial y}+\frac{\overline{\tau_{Wy}}}{\rho}\right\} \tag{108}$$

$$C_{y3}=\frac{Q_y}{\eta_c+h}-\frac{C_{y1}}{3}-\frac{C_{y2}}{2} \tag{109}$$

6. CONCLUSION

The surface advective terms were added to the conventional radiation stress by taking Taylor series expansion at the mean water level. The resulting radiation stress was proven to be consistent with the wave energy flux in the wave-current coexisting field.

The surface properties obtained from the surf zone model enabled us to develop the theory for the vertical circulation model which had suffered obscurity of boundary conditions. In addition, the friction coefficient and eddy viscosity applicable to the turbulent flow in a surf zone have been estimated in terms of energy dissipation. The developed model yielded the theoretical results comparable with laboratory experiments.

Based on the examination of the theoretical model for vertical profiles of currents in steady state, a quasi-three dimensional circulation model suitable for the entire nearshore zone is developed by linking the depth-integrated properties with the vertical profiles.

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