

Redistribution of Passive Impurity by Long Waves in Coastal Zone 연안역에서의 장파에 의한 오염원 확산

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Abstract □ In this paper the effect of wave motion acting on the natural fields of dispersed material in the coastal zone is studied. After integrating the usual diffusion equation with respect to the depth using shallow-water approximation simpler equation for integrated concentration was obtained, which holds for long waves of arbitrary amplitude and for any arbitrary barotropic flows. Different situations of long wave action on impurity concentration in the frame of this equation are considered.

요 旨 : 연안역에서의 파동이 오염원의 확산에 미치는 영향에 관한 연구를 수행하였다. 천해근사를 사용하여 기존의 확산방정식을 수심에 관하여 적분함으로써, 임의의 진폭을 갖는 장파조건 및 임의의 흐름조건에 대하여 적용가능한 방정식을 유도하였다. 수립된 방정식을 사용하여 장파가 오염원의 농도에 영향을 미치는 여러 경우에 대하여 고찰하였다.

1. INTRODUCTION

The increasing antropogenic action in the coastal zone results in the change of the water composition which worsens the ecological stability of seas. The flows bring impurities over large distances influencing the ecosystems of the open sea. The methods for calculating the impurity diffusion, especially the passive one are rather well-developed; they are based on the solution of the diffusion equation or the method of marker (tracers) and are summarized in monographs (for example, Ozmidov, 1986; Eremeev and Ivanov, 1987; Voltsinger *et al.*, 1989). Here the main attention is paid to the calculation of the diffusion of impurity caused by localized sources which give the answers to the important questions on the zones of the influence of the impurity sources. Here another aspect of the problem-the effect of wave motions on natural impurity fields in the ocean is discussed. The space redistribution of the natural impurity concentration may work as tracer of wave motions; this problem has been discussed

already as applied to recording internal waves via the films of surface-active substance affecting the spectrum of ripples (Ermakov *et al.*, 1980; Koslov *et al.*, 1987b). Besides, the impurity redistribution in wave fields and their increase in separate zones may lead the bifurcations in the dynamics of ecological systems most of which are almost unstable. In the given paper the impurity redistribution under the action of some barotropic motions of the coastal zone (running-up waves, seiche oscillations) is considered.

2. DEFINING EQUATIONS

We take a three-dimensional diffusion equation of the semi-empirical theory of turbulence

$$\frac{\partial c}{\partial t} + \text{div}(c\vec{u}) + \frac{\partial}{\partial z}(cw) = \frac{\partial}{\partial z}(K_z \frac{\partial c}{\partial z}) + \text{div}(K_h * c) = 0 \quad (1)$$

as the initial equation, where \vec{u} and w are the horizontal and vertical components of the flow velocity.

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K_z and K_h are the coefficients of the vertical and horizontal turbulent exchange, which are supposed to be arbitrary coordinate functions, and c is the impurity concentration. The operations div and ∇ act in the horizontal plane. In the frames of this approach multicomponent impurities may be considered, but the dynamics of each component occurs independently in this approach.

Integrating the Eq. (1) over the depth from the bottom $z = -H(x, y)$ to the free surface $z = \eta(x, y, t)$ and taking into account kinematic boundary conditions:

$$w = \frac{\partial \eta}{\partial t} + \vec{u} \nabla \eta \quad (z = \eta) \quad (2)$$

$$w + \vec{u} \nabla H = 0 \quad (z = -H) \quad (3)$$

we obtain

$$\frac{\partial}{\partial t} \int_H^\eta c dz + div \int_H^\eta c \vec{u} dz = div \int_H^\eta K_h \nabla c dz + \Pi \quad (4)$$

where

$$\Pi = \left[K_z \frac{\partial c}{\partial z} - K_h \nabla c \nabla \eta \right]_{z=\eta} - \left[K_z \frac{\partial c}{\partial z} + K_h \nabla c \nabla H \right]_{z=-H} \quad (5)$$

is the impurity flow through the free surface and the ocean bottom taking into account their inclination. The Eq. (4) is exact. When the long waves effect on the impurity concentration, two approximations may be used:

– the flow homogeneity in depth is a typical approximation of barotropic long waves, it permits to transform the following integral

$$\int_H^\eta c \vec{u} dz = \vec{u} \int_{-H}^\eta c dz; \quad (6)$$

– the introduction of the averaged coefficient of the horizontal diffusion μ by formula:

$$\int_{-H}^\eta K_h \nabla c dz = \mu(x, y) \nabla \int_{-H}^\eta c dz \quad (7)$$

which is valid if $\nabla(K_h \nabla \eta)$ and $\nabla(K_h \nabla H)$ are small. Thus, introducing the impurity concentration integrated in depth

$$\Gamma(x, y, t) = \int_{-H}^\eta c dz \quad (8)$$

we obtain the final equation

$$\frac{\partial \Gamma}{\partial t} + div(\Gamma \vec{u}) = div(\mu \nabla \Gamma) + \Pi \quad (9)$$

which describes the impurity redistribution in long waves. This equation was already given in books (Voltsinger and Pyaskovsky, 1977; Voltsinger *et al.*, 1989), but it has been introduced on purpose to make the utilized approximations clear. We note also that the analogous equation (with somewhat different right part) is for describing the dynamics of surface-active films (without approximating the barotropy of the wave motion) (Ermakov *et al.*, 1980; Kozlov *et al.*, 1987a, b). This enables to use a part of the available solutions for studying the redistribution of the impurity integrated over depth in the field of long waves.

It should be noted that our approach is valid for the integral estimate of the impurity concentration. In this case we can say nothing about the behavior of the volume concentration. Usually it is distributed in depth extremely nonuniformly. Thus, surface-active films and oil spots gather accumulate together on the sea surface, plankton is located in a pycnocline, and hydrogen sulfide is located in deep layers. The volume concentration and the thickness of impurity layers depend on many factors, for example, on the relation of vertical hydrostatic forces and the vertical diffusion. These factors disappear if we use the integral characteristics of impurity. However, one should know the integral concentration to estimate the level of impurities in water medium. We note that as it follows from (9) the integral concentration is defined by a smaller number of factors which are hard to control and this is an undoubted merit which allows to use this theory in practice. Note that it was used by Bulgakov *et al.* (1988) to analyze the motion of field inhomogeneities of sound-scattering layers round Ampere Bank near Gibraltar (Atlantic) and proved to be effective.

3. THE APPROXIMATIONS AND EVALUATING FORMULAE

Estimating the wave motion effect in the coastal

zone on the impurity concentration one encounters a number of difficulties due to uncertainty of functions in the right-hand of Eq. (9). Many types of trapped waves on a shelf are generated by atmospheric perturbations, in particular, by the inhomogeneities of the wind field. Impurity is caused by the wind and this changes the impurity flowing through the ocean surface, i.e., the function Π . Besides, the intensification of turbulent and wave motions in separate regions changes the coefficient of horizontal diffusion μ . As the result the problem of calculating the variation of the impurity concentration can not be separated from the problems of calculating wave and turbulent motions, impurity comes both from atmosphere and from the bottom (sediment stirring-up by waves). The solution of similar self-consistent problems can not be found while many parts of these problems have no reliable justification. Thus, here we analyze only free wave motions in the coastal zone in which case we may neglect the atmosphere effect and suppose $\Pi=0$. We shall also consider the coefficient of the horizontal diffusion constant. As the result Eq. (9) is simplified:

$$\frac{\partial \Gamma}{\partial t} + \text{div}(\Gamma \vec{u}) = \mu \Delta \Gamma$$

This equation should be supplemented with initial boundary conditions. Being interested in the processes of the redistribution of impurities by waves, we write first:

$$\Gamma(x, y, t=0) = \Gamma_0 = \text{Const} \quad (11)$$

although there may be other initial distributions of impurity concentration (they will be discussed below). Boundary conditions arise from the wave character of perturbations: they are periodic along the direction of wave propagation (if a wave is periodical), bounded (or damping) at sea and earth boundaries or within the zone of wave perturbations.

We estimate the role of diffusion effects in comparison with inertial ones. Their relation is characterized by Reynolds number:

$$Re = \frac{\omega}{\mu k^2} \quad (12)$$

where ω and k are the frequency and wave number of long waves. When $Re \gg 1$ diffusion effects are small. Rough estimates with $\mu = (10-100) \text{ m}^2/\text{s}$ (typical values of coefficients of horizontal turbulent diffusion (Ozmidov, 1986) show that this condition is readily fulfilled for all types of long waves on the shelf. This means that diffusion may be neglected in the first approximation when calculating the variation of impurity concentration due to wave perturbation. As the result, Eq. (10) is rewritten in two equivalent forms:

$$\frac{\partial \Gamma}{\partial t} + \text{div}(\Gamma \vec{u}) = 0 \quad (13)$$

$$\frac{d\Gamma}{dt} + \Gamma \text{div} \vec{u} = 0 \quad (14)$$

On the other hand the long waves theory is based on the continuity equation (Voltsinger *et al.*, 1989), which we also give in two equivalent forms:

$$\frac{\partial D}{\partial t} + \text{div}(D \vec{u}) = 0 \quad (15)$$

$$\frac{dD}{dt} + D \text{div} \vec{u} = 0 \quad (16)$$

where $D = H + v$ is the total basin depth. Excluding $\text{div} \vec{u}$ from (14) and (16) we find the invariant relation:

$$\frac{\Gamma}{D} = \text{Const} \quad (17)$$

which is valid along the trajectory of the motion of liquid particles. The physical sense of this invariant is clear enough: in the absence of diffusion effects depth-averaged volume concentration of impurity Γ/D is preserved and the variation of the concentration Γ integrated over depth occurs due to freezing the passive impurity into the hydrodynamical field proportionally to the total basin depth. However, it is not very convenient to use invariant (17) in practice, because it is actually written in Lagrangian coordinates and the concentration is calculated in Euler coordinates. Therefore now we perform direct integration of Eq. (13). It is equivalent to the set of ordinary differential equations (a characteristic form):

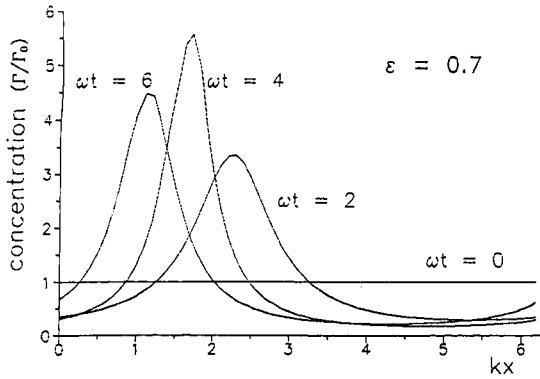


Fig. 1. The integrated concentration variations along a long wave ($\epsilon=0.7$) in a various time factor ωt : 1 $-\pi/3$, 2 $-2\pi/3$, 3 $-4\pi/3$, 4 -2π . 5-elevation of free surface.

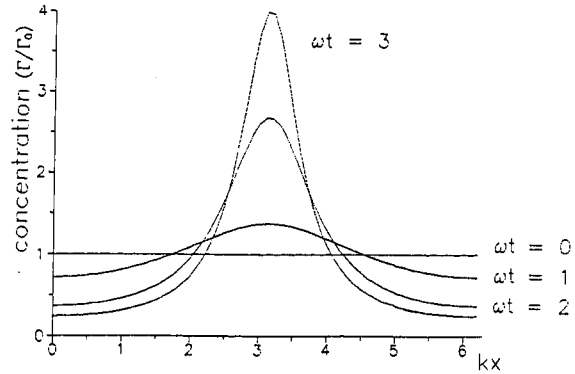


Fig. 2. Redistribution of impurity concentration by standing wave in a various time factor ωt : 1 -0 , 2 $-\pi$, 3-wave profile.

$$\frac{d\vec{r}}{dt} = \vec{u}, \quad \frac{d\Gamma}{dt} = \text{div} \vec{u} \quad (18)$$

and in a number of cases it may be solved in the explicit form.

We consider, for example, in the linear approximation a progressive long wave in the basin with constant depth- H :

$$\eta = a \sin(kx - \omega t), \quad \omega = ck, \quad c = \sqrt{gH} \quad (19)$$

The velocity field in the long wave is also monochromatic

$$u = a \sqrt{g/H} \sin(kx - \omega t) \quad (20)$$

The solution of Eq. (13) in the field (20) is found by Kozlov *et al.* (1987b) for another problem and here it is reproduced in view of its importance:

$$\frac{\Gamma}{\Gamma_0} = \frac{1 - \epsilon^2}{1 - \epsilon(1 - \cos\beta\omega t) \sin(kx - \omega t) + \epsilon\beta \cos(kx - \omega t) \sin\beta\omega t - \epsilon^2 \cos\beta\omega t} \quad (21)$$

where $\epsilon = a/H$ and $\beta = \sqrt{1 - \epsilon^2}$. It is seen from Eq. (21) that the variation of impurity concentration for each fixed wave phase occurs periodically, the period of the concentration variation is $1/\beta$ being greater than the long wave period. Fig. 1 gives the distribution of impurity concentration along the long wave. It is unsymmetrical with the profound maximum variation of concentration achieves

$$\frac{\delta\Gamma}{\Gamma_0} = 4\epsilon\beta^{-2} \quad (22)$$

and may be rather considerable in waves with large amplitude, for example, in surf zone when $\epsilon \rightarrow 1$ and $\beta \rightarrow 0$. It should be noted that waves with large amplitude are nonsinusoidal therefore we should use formula (21) very carefully.

For the case of waves with small amplitude the formula (21) essentially simplifies to:

$$\frac{\Gamma}{\Gamma_0} = 1 + \epsilon[\sin(kx - \omega t) - \sin kx] \quad (23)$$

It follows from this formula that the variation of impurity concentration becomes evident half-period after the long wave emergence and the maximum value of its variation may be twice that of the relative wave height.

In the coastal zone where long waves intensively reflect from the shelf, a standing wave of the form

$$\eta = a \cos(kx) \cos(\omega t) \quad (24)$$

may appear. The exact solution Eq. (13) for impurity concentration in this case has the form

$$\frac{\Gamma}{\Gamma_0} = \frac{1}{\cosh[2\epsilon \sin^2(\omega t/2)] + \cos kx \sinh[2\epsilon \sin^2(\omega t/2)]} \quad (25)$$

where again $\epsilon = a/H$. The concentration achieves its maximum value in the wave amplitude at time mo-

ments $\omega t = \pi, 2\pi, 3\pi$, etc. (see Fig. 2):

$$\frac{\Gamma_{max}}{\Gamma_0} = \exp(2\varepsilon) \quad (26)$$

Comparing (22) and (26) we see that the maximum of the concentration variation in the standing wave is smaller than that in the travelling wave, especially in waves with large amplitude.

Further to perform estimating calculations we consider only linear wave fields. In this case Eq. (13) with taking into account the initial condition (11) may be also simplified and integrated in the general form:

$$Q = \frac{\Gamma - \Gamma_0}{\Gamma_0} = - \int_0^t \text{div} \vec{u}(x, y, t) dt \quad (27)$$

(we neglect the component $Q\vec{u}$ in (13)). Eq. (27) defines the variation of the impurity concentration via scalar value $\text{div} \vec{u}$, and it is the only defining parameter which characterizes the wave motion.

If the basin has a constant depth, $\text{div} \vec{u}$ unambiguously depends on the water level. Indeed, the linearized continuity Eq. (15) has the form:

$$\frac{\partial v}{\partial t} + \text{div}(H\vec{u}) = 0 \quad (28)$$

and at $H = \text{Const}$ permits to calculate $\text{div} \vec{u}$, thus allowing for integrating Eq. (27):

$$Q = \frac{\Gamma - \Gamma_0}{\Gamma_0} = \frac{\eta(x, y, t) - \eta(x, y, 0)}{H} \quad (29)$$

Thus, in the linear approximation the variations of impurity concentration are proportional to the water level, and in fact this follows from the invariant relation (17), since in the linear approximation there is no difference between Lagrangian and Eulerian descriptions if the basin's depth is constant. We have actually already obtained formula (29) in particular example (23).

The Eq. (29) also helps to understand the nature of nonstationary variations of impurity concentration caused by a steady-state wave, which we have obtained in the frames of Eq. (21). It depends on the influence of the initial conditions in the field of concentration. Since we considered the wave perturbation unlimited in space, the transient process

occurs for unlimited long time if there are no diffusion effects. If the origin (initial perturbation) occupies the bounded region in the space, then after the wave leaves the origin, the variation of impurity concentration will follow the wave profile, and residual strain remains in the origin. To analyze it one has to take into account diffusion effects.

Now we consider the diffusion effect on the redistribution of impurity concentration under the action of wave perturbations. Taking into account the linearity of wave motions we may again simplify Eq. (10) neglecting $Q\vec{u}$:

$$\frac{\partial Q}{\partial t} - \mu \Delta Q = -\text{div} \vec{u} \quad (30)$$

Its solution can be readily found

$$Q(r, t) = - \int \text{div} \vec{u}(\rho, \tau) G(r - \rho, t - \tau) d\rho d\tau \quad (31)$$

where r and ρ are one-dimensional or two-dimensional vector depending on the considered geometry of wave motion, G is the Green function of the linear diffusion equation, particularly for unbounded basin it is known to be equal to:

(one-dimensional problem)

$$G = \frac{1}{\sqrt{4\pi\mu t}} \exp(-r^2/4\mu t) \quad (32)$$

(two-dimensional problem)

$$G = \frac{1}{4\pi\mu t} \exp(-r^2/4\mu t) \quad (33)$$

We consider again the effect of a plane progressive long wave (19). In this case the integral in Eq. (31) is calculated explicitly:

$$Q = \frac{\varepsilon}{1 + Re^{-2}} \left\{ e^{-\mu k^2 t} \left[\frac{1}{Re} \cos kx - \sin kx \right] - \frac{1}{Re} \cos(kx - \omega t) \right\} \quad (34)$$

At long time periods the impurity concentration becomes stationary (in the system of coordinates connected with the wave)

$$Q = \frac{\varepsilon}{\sqrt{1 + Re^{-2}}} \sin(kx - \omega t - \theta), \quad \theta = \arctan(1/Re) \quad (35)$$

It is vividly seen that the variation of impurity concentration decreases when diffusion is taken into account and shifts from the wave crest to the front slope. Since the characteristic Reynolds number is considerable for long-wave motions in the coastal zone, these effects can be neglected. Nevertheless if time periods are long the diffusion influences the stabilization of the concentration field and the attenuation of the effect of initial conditions. Therefore, to perform estimating calculations we shall use diffusionless approximation (Eqs. (27) and (29)) but without taking into account the transient process, bearing in mind, however, that it could reinforce perturbations approximately twice and cause additional space and time variability of the picture obtained.

Above we have considered the redistribution of the impurity which is initially homogeneous. It is evident that Eq. (10) admits inhomogeneous in the horizontal plane, equilibrium distributions of concentration satisfying the equation:

$$\Delta\Gamma_0=0 \quad (36)$$

In this case to estimate the impurity variation due to waves, Eq. (13) may be simplified once again if we introduce

$$\Gamma=\Gamma_0+\Gamma' \quad (37)$$

and neglect the component $\Gamma'u$, then we obtain

$$\Gamma'-\int_0^t \text{div}(\Gamma_0 \bar{u}) dt \quad (38)$$

In particular, if $\Gamma_0=qH$, where $q=\text{Const}$ and H also satisfies Eq. (36), using Eq. (28) we obtain from Eq. (38) an elementary relation:

$$\Gamma'=q\eta \quad (39)$$

and the concentration variations in this case follow the wave profile.

As an example we consider once again the motion of a progressive long wave in a basin with constant depth along axis x . If Γ_0 depends only on the coordinate y , the concentration variation follows the water level:

$$\Gamma=\Gamma_0(y)(1+\eta(x,t)/H) \quad (40)$$

If $\Gamma_0=qx$ with constant q , then

$$\Gamma=qx[1+\varepsilon \sin(kx-\omega t)]-\frac{q\varepsilon}{k} \cos(kx-\omega t) \quad (41)$$

It seems that in the region of considerable concentration its variations follow the surface level and in the region of small concentration it is proportional to the integral of the level.

The simple examples given show the main factors of the long waves which affect on the variation of impurity concentration. Now we consider more realistic situations of two-dimensional wave field.

4. THE EFFECT OF SEICHE OSCILLATIONS

Long-waves in the coastal zone are characterized by a great variety of wave motions such as seiche oscillations, edge, topographic and shelf waves, Poincare and Kelvin waves. We investigate for example the effect of seiche oscillations on the variation of the impurity concentration in the coastal zone.

We consider a rectangular basin with constant depth, when there is no rotation. The wave field may be presented by a number of seiche oscillations of the following form

$$\eta=a \cos(k_x x) \cos(k_y y) \sin(\omega t) \quad (42)$$

where

$$\omega^2=gH(k_x^2+k_y^2), \quad k_x=n\pi/L_x, \quad k_y=m\pi/L_y$$

and L_x and L_y are the basin dimensions, n and m are the integers characterizing the numbers of the oscillation mode. If unperturbed impurity concentration is homogeneous in the basin, its variation replicate the level variation in the basin according to Eq. (29) (see Fig. 3). If the concentration changes along one of the directions, for example, $\Gamma_0=qx$, the concentration variation is described as follows:

$$\Gamma'=q\varepsilon(x \cos(k_x x) + \frac{k_x}{k_x^2+k_y^2} \sin(k_x x)) \cos(k_y y) \sin(\omega t) \quad (43)$$

and in this case the variation of field concentration becomes asymmetric along the axis x (Fig. 4).

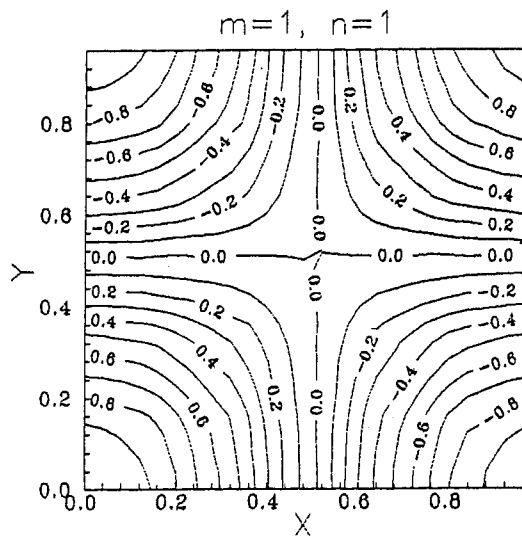


Fig. 3. The variation of impurity concentration isolines qH/a for the first mode of seiche oscillations with $\Gamma_0 = \text{Const}$.

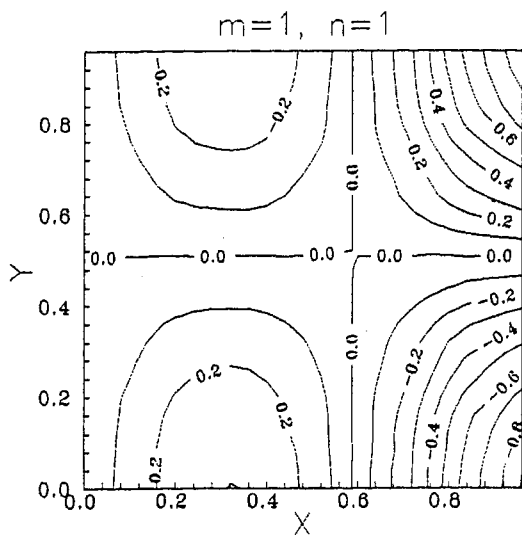


Fig. 4. The variation of impurity concentration isolines $\Gamma H/qaL$ for the first mode of seiche oscillations with inhomogeneous distribution of undisturbed concentration ($\Gamma_0 = qx$, $L = L_x = L_y$).

We made some estimates. The amplitude of seiche barotropic oscillations in the Black and Mediterranean Seas does not exceed 1 m in the open sea (according to the data of observations and the results of numerical calculations described by Blatov

et al. (1984)). At an average depth not less than 1500 m the relative heights of waves η/H do not exceed 10^{-3} . According to the formulae given above the wave variations of impurity concentration will be of the same order, and consequently, they may be neglected. On the contrary, in the coastal zone: bays and estuaries the depth is small (10–40 m) and the wave amplitude may be equal to 10 m. Thus concentration variations achieve 50 and taking into account the wave variability and nonlinearity they achieve 100 impurity redistribution in the coastal zone becomes rather essential. In particular, we should like to mention the region near the edge, where in some cases the flows in running-up waves may be the only source of impurity. The analogous estimates with the aid of the proposed formulae may be done also for other types of wave motions in the coastal zone. However, the corresponding expressions appear to be rather cumbersome and require the use of computers for describing the detailed space structure of variations of the impurity concentration.

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