예방보수와 3가지 형태의 고장을 갖는 두 요소로 구성된 병렬 시스템의 확률분석

- Stochastic analysis of a two-unit parallel system with three types of failure & preventive maintenance -

Che-Soong Kim*

요 약

본 논문에서는 여러가지 형태의 고장을 갖는 동일한 두 요소로 구성된 병렬 시스템의 신뢰도를 평가하는 마코프 모형을 제시 하였다. 여기서 고려하는 고장형태는 인간의 오류에 의한 고장, 하드웨어에 의한 고장, 하나의 원인에 의해서 여러개의 구성요소가 동시에 고장나는 Common cause 고장형태로 나누었다. 시스템은 임의의 시점에서 예방보수를 받을수 있고, 고장률과 예방 보수률은 일정하다고 가정했다. 또한 수리률이 임의의 분포를 따를 경우 시스템 신뢰도및 평균 고장시간을 구했다.

1. Introduction

There are numerous systems which are interconnected by human links. In an earlier reliability analysis, the prediction of the system reliability was directed only at the equipment portion. The reliability of the human element was neglected. However, Wiliams(1958) recognized this need in the late 1950s. He point out that the true equipment or system reliability analysis must also include the human aspect of the reliability analysis. Since the beginning of the last decade, there has been considerable interest in human initiated equipment failures and their effect on the overall system reliability.

According to Meister(1962), about 20-30 percent of total equipement - related failures are due to human errors. Furthermore, according to Hagen(1976), about 10 - 15 percent of total failures are directly related to human errors.

According to Meister (1966), human reliability is defined as the probability that a job or a task will be successfully completed by personnel at any required stage in system operation within a required minimum time. Similarly, human error is defined as a failure to perform a prescribed task which could results in damage to equipment and property or disruption of scheduled operations.

The auther of reference(8) has categorized the human errors as follws;

- Maintenance error
- Fabrication error
- Design error
- Operator error
- Inspection error
- Handling error

* Dept. Industrial Engineering, Seoul National University

접수 : 1993년 4월 10일
확정 : 1993년 4월 21일

Some of the cause of human errors are inadequate maintenance or operating procedures for the operating personnel, poor job environments, poor or inadequate tools, wrong interpretations of instruments, poor training or skill of the operating personnel and so on.

The human element is always present in industrial installations, whether it be in design or operation. We can therefore speaks of real man - machine systems in which man is a system component. Reliability analysis cannot provide the reliability of the complete man-machine system unless it takes the human factors into account. Therefore, it may be said that wherever people are involved, errors will be made. These may occur regardless of their training, skill or experience. Similar models can be found in (3.8)

The aim of this paper is concerned with determining system reliability, MTFF for parallel system with consideration of human failure. Therefore, this paper presents two Markov models of well known redundant system.

2. Assumptions

There following assumptions are common for all two models.

1. Failure are statistically independent.
2. The repair unit is as good as new and repair rate is general.
3. Failed system is never repaired
4. Common cause failure rate is constant
5. When both the units are in normal operaion, the system goes for PM at random epochs. The maintenance rate is constant.
6. Both units may fail-simultaneously due to the occurrence of a common cause failure.
7. The system comprises two identical unit.
8. Each unit's human and hardware failure can be separated.
9. Unit hardware failure and human error rates are constant.

3. Notations

There following notations are common for all two models

S_0 = Both units operating normally

S_1 = One unit failed due to human errors, other operating

S_2 = One unit failed due to hardware failures, other operating

S_3 = Both unit failed(at least one due to human errors)

S_4 = Both unit failed(at least one due to hardware failures)

S_5 = Both unit failed due to common cause failures

S_6 = System in the preventive maintenance

$P_i(t)$ = Probability that system is in state i at time t ; for i = 1 to 6

$P_j(t,x)$ = Probability density(w.r.t repair time) that system is in state j and has an elapsed repair time of x ; for j = 1,2

$U_j(x)$ = Repair rate when system is in state j and has an elapsed repair time of x ; for j = 1,2

$g_j(x)$ = Probability density function of repair time when system is in state j and has an elapsed repair time of x; for j = 1,2

λ = Hardware failure rate
λ_c = Common-cause failure rate
λ_h = Human error rate
λ_{PM} = Constant rate of reaching preventive maintenance state
υ = Preventive maintenance rate
s = Laplace transform variable

4. Analysis

In this section equations for Markov models I,II are developed.

Model I

Model I represents a repairable two identical unit parallel system with single repair facility. Each unit can fail in two separate failure modes(i.e. due to human errors or hardware failures) In addition, both units may fail simultaneously due to common cause failures. As soon as any one of the operating unit fails, the repair is made immediately. The parallel system fails when both units are in the failed state. At least one unit must operate normally for the system success.

We assume that the unit failure rate is constant and the repair rate is not constant. Thus, the repair time is distributed by arbitrary distribution. The fully repaired unit is put back into its normal operation. And, the system under goes preventive maintenance randomly in time when all the units are in normal operation. The system transition diagram is shown in Fig.I

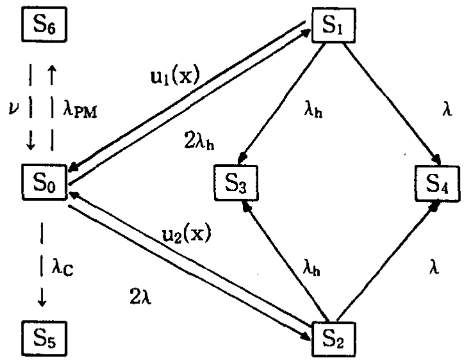

Fig.I System diagram for two unit parallel system with repair

Let us consider the procedure of analysis by using Laplace transforms for the model. The following system of differential equations is associated with Fig.I

$$\dot{P}_0(t) + (\lambda_c + \lambda_{PM} + 2\lambda_h + 2\lambda)P_0(t) = \int_0^t P_1(t,x)U_1(x)dx + \int_0^t P_2(t,x)U_2(x)dx + \nu P_6(t) \tag{1}$$

$$\frac{\partial P_1(t,x)}{\partial t} + \frac{\partial P_1(t,x)}{\partial x} = -(U_1(x) + \lambda + \lambda_h)P_1(t,x) \tag{2}$$

$$\frac{\partial P_2(t,x)}{\partial t} + \frac{\partial P_2(t,x)}{\partial x} = -(U_2(x) + \lambda + \lambda_h)P_2(t,x) \tag{3}$$

$$\dot{P}_3(t) = \lambda_h \int_0^t P_1(t,x)dx + \lambda_h \int_0^t P_2(t,x)dx \quad (4)$$

$$\dot{P}_4(t) = \lambda \int_0^t P_1(t,x)dx + \lambda \int_0^t P_2(t,x)dx \quad (5)$$

$$\dot{P}_5(t) = \lambda_c P_0(t) \quad (6)$$

$$\dot{P}_6(t) = -\nu P_6(t) + \lambda_{PM} P_0(t) \quad (7)$$

Where the prime on P(t) denotes the differentiation with respect to time t

The initial conditions are as follows ;

$P_1(0) = 1,\ P_i(0) = 0,\ i = 1$ to 7 and

$P_1(t,0) = 2\lambda_h P_0(t),\ P_2(t,0) = 2\lambda P_0(t)$ (8)

Using the Laplace transforms to solve equation (1)-(7) under the initial conditions (8), they become respectively;

$$(s + \lambda_c + \lambda_{PM} + 2\lambda_h + 2\lambda)\overline{P_0}(s) - \int_0^\infty \overline{P_1}(s,x)U_1(x)dx - \int_0^\infty P_2(s,x)U_2(x)dx - \nu P_6(s) = 1 \quad (9)$$

$$s\overline{P_1}(s,x) + \frac{\partial \overline{P_1}(s,x)}{\partial x} = -(U_1(x) + \lambda + \lambda_h)\overline{P_1}(s,x) \quad (10)$$

$$s\overline{P_2}(s,x) + \frac{\partial \overline{P_2}(s,x)}{\partial x} = -(U_2(x) + \lambda + \lambda_h)P_2(s,x) \quad (11)$$

$$s\overline{P_3}(s) = \lambda_h \int_0^\infty \overline{P_1}(s,x)dx + \lambda_h \int_0^\infty \overline{P_2}(s,x)dx \quad (12)$$

$$s\overline{P_4}(s) = \lambda \int_0^\infty \overline{P_1}(s,x)dx + \lambda \int_0^\infty \overline{P_2}(s,x)dx \quad (13)$$

$$s\overline{P_5}(s) = \lambda_c \overline{P_0}(s) \quad (14)$$

$$(s + \nu)\overline{P_6}(s) = \lambda_{PM} \overline{P_0}(s) \quad (15)$$

$$\overline{P_1}(s,0) = 2\lambda_h \overline{P_0}(t),\quad \overline{P_2}(s,0) = 2\lambda \overline{P_0}(s) \quad (16)$$

Equations (10)-(11) can be rearranged as;

$$\frac{1}{\overline{P_1}(s,x)} \frac{\partial \overline{P_1}(s,x)}{\partial x} = -(U_1(x) + \lambda + \lambda_h + s)$$

$$\frac{1}{\overline{P_2}(s,x)} \frac{\partial \overline{P_2}(s,x)}{\partial x} = -(U_2(x) + \lambda + \lambda_h + s)$$

Hence,

$$\overline{P_1}(s,x) = \overline{P_1}(s,0)\exp(-(\lambda+\lambda_h+s)x - \int_0^x U_1(u)du) \quad (17)$$

$$\overline{P_2}(s,x) = \overline{P_2}(s,0)\exp(-(\lambda+\lambda_h+s)x - \int_0^x U_2(u)du) \quad (18)$$

and

$$U_1(x) = \frac{g_1(x)}{\exp(-\int_0^x g_1(u)du)} \quad (19)$$

$$U_2(x) = \frac{g_2(x)}{\exp(-\int_0^x g_2(u)du)} \quad (20)$$

Taking account of equations (15)-(20), equation(9) can then be written as;

$$(s+\lambda_c+\lambda_{PM}+2\lambda_h+2\lambda)\overline{P_0}(s) - 2\lambda_h P_0(s)\int_0^\infty g_1(x)\exp(-(\lambda+\lambda_h+s)x)dx$$
$$- 2\lambda P_0(s)\int_0^\infty g_2(x)\exp(-(\lambda+\lambda_h+s)x)dx - \nu\overline{P_0}(s) = 1$$

Hence,

$$\overline{P_0}(s) = (s+\nu)/((s+\nu)(s+a_1-2\lambda_h\overline{g_1}(a_2+s)-2\lambda\overline{g_2}(a_2+s))-\lambda_{PM}) \quad (21)$$

$$\overline{P_1}(s) = \frac{2\lambda_h\overline{P_0}(s)}{s+a_2}(1-\overline{g_1}(s+a_2)) \quad (22)$$

$$\overline{P_2}(s) = \frac{2\lambda\overline{P_0}(s)}{s+a_2}(1-\overline{g_2}(s+a_2)) \quad (23)$$

$$\overline{P_3}(s) = \frac{2\lambda_h\overline{P_0}(s)}{s(s+a_2)}(\lambda+\lambda_h-\lambda_h\overline{g_1}(s+a_2)-\lambda\overline{g_2}(s+a_2)) \quad (24)$$

$$\overline{P_4}(s) = \frac{2\lambda\overline{P_0}(s)}{s(s+a_2)}(\lambda+\lambda_h-\lambda_h\overline{g_1}(s+a_2)-\lambda\overline{g_2}(s+a_2)) \quad (25)$$

$$\overline{P_5}(s) = \lambda_c/s\overline{P_0}(s) \quad (26)$$

$$\overline{P_6}(s) = \lambda_{PM} \;/\; (s+\nu)\overline{P_0}(s) \quad (27)$$

Laplace transform of the system reliability are

$$\overline{R}(s) = \overline{P_0}(s) + \overline{p_1}(s) + \overline{P_2}(s) \quad (28)$$

The mean time to failure(MTTF) of a system is given by

$$MTTF = \lim_{S\to 0}\overline{R}(s) \quad (29)$$

By substituting equation (28) into equation (29) and letting $s \to 0$, we get

$$MTTF = \overline{P_0}(s)\left(1 + \frac{2\lambda_h(1-\overline{g_1}(a_2)) + 2\lambda(1-\overline{g_2}(a_2))}{a_2}\right) \quad (30)$$

where, $a_1 = \lambda_c + \lambda_{PM} + 2\lambda + 2\lambda_h, \ a_2 = \lambda + \lambda_h$

From equation (28) the system reliability is given by

$$R(t) = L^{-1}(\overline{R}(s)) \quad (31)$$

Model II.

This model is a special case of Model I. In this model no repair is considered. In other words no failed unit is repaired.(i.e., $U_1(x) = U_2(x) = 0$) The system fails due to human error, hardware failures or common cause failures. This model separates only the human errors from the handware failures. In other words only those human errors due to which both units fail on would have failed. More clearly, due to a human action the system fails when both units are functioning normally.

Furthermore, when only one unit is operating normally, due to the same human action the operating unit failed. In other words, if both the units had been operating normally instead of only one, the entire system would have failed due to the same action. The system transition diagram is shown in Fig. II

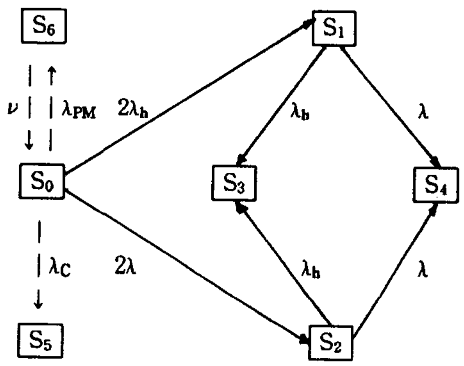

Fig. II Transiton diagram for two unit parallel system without repair

$$P_0(t) + (\lambda_c + \lambda_{PM} + 2\lambda + 2\lambda_h)P_0(t) = \nu P_6(t) \quad (32)$$

$$\dot{P}_1(t) + (\lambda + \lambda_h)P_1(t) = 2\lambda_h P_0(t) \quad (33)$$

$$\dot{P}_2(t) + (\lambda + \lambda_h)P_2(t) = 2\lambda P_0(t) \quad (34)$$

$$\dot{P}_3(t) = \lambda_h P_1(t) + \lambda_h P_2(t) \quad (35)$$

$$\dot{P}_4(t) = \lambda P_1(t) + \lambda P_2(t) \quad (36)$$

$$\dot{P}_5(t) = \lambda_c P_0(t) \tag{37}$$

$$\dot{P}_6(t) + \nu P_0(t) = \lambda_{PM} P_0(t) \tag{38}$$

Where the prime on P(t) denots the differentiation with respect to time t.

At time $t = 0$, $P_0(0) = 1$, $P_i(0) = 0$, $i = 1$ to 6

Using the Laplace transform to solve equations (32)-(38) under the initial conditions, they become respectively;

$$(s + \lambda_c + \lambda_{PM} + 2\lambda + 2\lambda_h)\, \overline{P_0}(s) - \nu\, \overline{P_6}(s) = 1 \tag{39}$$

$$(s + \lambda + \lambda_h)\, \overline{P_1}(s) = 2\lambda_h\, \overline{P_0}(s) \tag{40}$$

$$(s + \lambda + \lambda_h)\, \overline{P_2}(s) = 2\lambda\, \overline{P_0}(s) \tag{41}$$

$$s\, \overline{P_3}(s) = \lambda_h\, \overline{P_1}(s) + \lambda_h\, \overline{P_2}(s) \tag{42}$$

$$s\, \overline{P_4}(s) = \lambda\, \overline{P_1}(s) + \lambda\, \overline{P_2}(s) \tag{43}$$

$$s\, \overline{P_5}(s) = \lambda_c\, \overline{P_0}(s) \tag{44}$$

$$s\, \overline{P_6}(s) + \nu\, \overline{P_0}(s) = \lambda_{PM}\, \overline{P_0}(s) \tag{45}$$

Therefore,

$$\overline{P_0}(s) = \frac{s + \nu}{(s + a_1)(s + \nu) - \lambda_{PM}} \tag{46}$$

$$\overline{P_1}(s) = \frac{2\lambda_h(s + \nu)}{(s + a_2)((s + a_1)(s + \nu) - \lambda_{PM})} \tag{47}$$

$$\overline{P_2}(s) = \frac{2\lambda(s + \nu)}{(s + a_2)((s + a_1)(s + \nu) - \lambda_{PM})} \tag{48}$$

$$\overline{P_3}(s) = \frac{2\lambda_h(s + \nu)(\lambda + \lambda_h)}{(s + a_2)((s + a_1)(s + \nu) - \lambda_{PM})} \tag{49}$$

$$\overline{P_4}(s) = \frac{2\lambda(s + \nu)(\lambda + \lambda_h)}{(s + a_2)((s + a_1)(s + \nu) - \lambda_{PM})} \tag{50}$$

$$\overline{P_5}(s) = \frac{\lambda_c(s + \nu)}{s\,((s + a_1)(s + \nu) - \lambda_{PM})} \tag{51}$$

$$\overline{P_6}(s) = \frac{\lambda_{PM}}{(s + a_1)(s + \nu) - \lambda_{PM}} \tag{52}$$

Laplace transforms of the system reliability are

$$\bar{R}(s) = \bar{P_0}(s) + \bar{P_1}(s) + \bar{P_0}(s) \tag{53}$$

The mean time to failure(MTTF) of a system is given by

$$MTTF = \lim_{s \to 0} \bar{R}(s) \tag{54}$$

By substituting equation (53) into equation (54) and letting $s \to 0$, we get

$$MTTF = \frac{\nu(a_2 + 2\lambda + 2\lambda_h)}{a_2(a_1\nu - \lambda_{PM})} \tag{55}$$

where, $a_1 = \lambda_c + \lambda_{PM} + 2\lambda + 2\lambda_h$, $a_2 = \lambda + \lambda_h$

From equation (53) the system reliability is given by

$$R(t) = L^{-1}(\bar{R}(s)) \tag{56}$$

Numerical values of system reliability and meam time to failure (MTTF) were obtained using the above developed equations with different values of λ, λ_h, λ_c, λ_{PM}, ν and u_1, u_2. Table I gives the values of system reliability vs time for all the two models. System reliability values under row A correspond to $\lambda = 0.01$, $\lambda_h = \lambda_c = \lambda_{PM} = \nu = 0$ and $u_1 = u_2 = 0.02$ whereas system reliability values under row B correspond to $\lambda = 0.01$, $\lambda_h = 0.02$, $\lambda_c = 0.05$, $\lambda_{PM} = 0.0002$, $\nu = 0.02$ and $u_1 = u_2 = 0.02$. Thus, it shows that the system reliability decreases as the time increases. Table II represents the values of MTTF for different values of human error rate (λ_h). Values of λ and λ_c for the values of Table II are equal to 0.01 and 0.005 respectively. The value of $\nu = 0.02$, $\lambda_{PM} = 0.0002$ and $u_1 = u_2 = 0.02$ are considered for Table II. The mean time to failure (MTTF) decreases with increase in human error rate.

Table I. System Reliability for Model I, II.

Time(hours)		0	10	20	30	40	50	60	70	80	90	100
Model I	A	1	0.989	0.971	0.942	0.911	0.889	0.844	0.792	0.776	0.745	0.713
	B	1	0.912	0.896	0.835	0.776	0.723	0.667	0.613	0.568	0.527	0.485
Model II	A	1	0.981	0.967	0.925	0.891	0.843	0.796	0.746	0.697	0.649	0.601
	B	1	0.908	0.894	0.829	0.764	0.698	0.639	0.580	0.526	0.473	0.429

Table II. Mean time to failure (MTTF)

Human Error Rate		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
MTTF (hours)	Model I	650	430	260.47	150.36	116.39	90.68	71.27	56.92	46.38	38.96	31.24
	Model II	300	100	60	42.86	33.33	27.27	23.08	20	17.65	15.79	14.29

5. Conclusions

The models presented in this paper are typical examples of man-machine system. The analysis presented explains the effect of critical human error rate on system reliability. The analysis will be very useful to the design engineers to optimize their designs to achieve reliability goals.

References

1. B.S., Dhillon and Chanan Singh, "ENGINEERING RELIABILITY", *John Willey and Sons*, New York.

2. B.S., Dhillon, "On Human Reliability Bibliography", *Microelectron Reliability*, vol. 19, 1979.

3. B.S., Dhillon, "Stochastic Models for Predicting Human Reliability", *Microelectron Reliability*, vol. 21, 1981.

4. B.S. Dhillon and R.B., Misra, "Reliability Evaluation of Systems with Critical Human Error", *Microelectron Reliability*, vol. 24, No.4, 1984.

5. Hagen, E.W., "Human Reliability Analysis", *Nucl. Safety*, vol.17, 1976.

6. H. Kragt, "Human Reliability Engineering", *IEEE Trans. Reliability*, vol.27, 1978.

7. L.R. Goel and Praven Gupta, "Analysis of a Two-Engine Aeroplane Models with Two Types of Failure and Preventive Maintenance", *Microelectron Reliability*, vol.24, No.4,1984.

8. D.Meister, "The problem of Human-Initiated Failures", *Eighth National Symposium on Reliability and Quality Control*,1962.

9. D.Meister,"Subjective Data in Human Reliability Estimates", *Annual Rel- iability and Maintainability Symposium*,1978.

10. Subramanyam N. Rayapati, "Reliability and Availability Analysis of on Surface Transit Systems", *Microelectron Reliability*, vol.24,1984.

11. Williams, H.L.,"Reliability Evaluation of the Human Component in Man-Machine Systems", *Electrical Manufacturing*, 1958.

12. Swainm, A.D., "Shortcuts in Human Reliability Analysis; Generis Techniques in Systems Reliability Assessment",*Noordhoff, Leyden*, 1974.