◉ 論 文

평면내 방향 기진력에 의한 평면밖 방향 운동의 예측

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Prediction of the Out-of-plane Motion due to the In-plane Excitation

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Key Words : Nonlinear Dynamics(비선형 동역학), Parametric Excitations(계수기진), Method of Multiple Scales(다척도 변수법), Nonlinear Roll Motion(비선형 횡동요), Dynamic Stability(동역학적 안정성)

Abstract

삼 자유도를 가진 부유물체의 동적 응답을 이론적으로 연구하였다. 평면내 방향 운동모우드에 대한 지배방정식을 선형화한 후, 그들의 조화해를 평명밖 방향 운동모우드의 방정식과 연성시켰다. 그렇게 해서 주어지는 방정식은 시간에 따라 변화하는 계수를 가진 형태로서, 평면밖 방향의 운동응답을 예측하는데 사용하였다. 그 결과, 평면내 방향 기진력을 받고서 같은 평면내 방향의 운동만을 보일 것으로 예측되는 부유물체가 평면밖 방향의 운동을 보일 수도 있음을 밝혔다. 동역학적 불안정성과 그 결과로 나타나는 평면밖 방향의 대진폭 운동을 보이고 있다. 본 결과는 주기적으로 동요하는 부유물체가 서로 연성된 운동을 하는 현상으로도 해석할 수 있다.

1. INTRODUCTION

It has been known to us for a long time that a vessel in a head or following waves may experience control problems such as broaching and the loss of course stability, especially in following regular waves. Wahab and Swaan[1)] used a linear formulation of the problem with constant coefficients and Froude-Krilov forces as the only wave forces and moments to consider the zero-frequency-of-encounter case. With this fairly simple model they were able to demonstrate the directional instability when a vessel with fixed controls

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is positioned anywhere over half of the wave cycle. This phenomenon is familiar to vessel operators.

In similar circumstances, a vessel runs a great risk of experiencing large-amplitude motions not only in in-plane modes, such as heave and pitch, but also in out-of-plane modes, such as roll and yaw. The latter phenomenon is caused by the nonlinear interactions among the modes of motion and is the subject of the present investingation.

Nayfeh, et al.[2] used model equations that account for the coupling between the pitch mode and the roll mode by including the dependence of the pitching moment on the roll displacement. Blocki[3] considered a model with only two degrees of freedom (heave and roll). The elimination of pitch implies that the model is symmetric with respect to the midship section (sometimes called fore-and-aft symmetry] and in a beam wave. Nayfeh and Sanchez[4] investigated the behavior of a vessel rolling in longitudinal waves. They used an analytical-numerical technique based on the method of multiple scales to predict the qualitative changes.

In this paper, we describe the real situation more accurately than Blocki[3] and Nayfeh and Sanchez.[4] Specifically, we lift the restriction of fore-and-aft symmetry, add a third degree of freedom (pitch), and consider head and following waves theoretically. The hydrodynamic viscous damping may be appropriately represented by a nonlinear from, especially when a large excitation amplitude is considered. In this case, however, the roll amplitude is regarded relatively larger than the pitch and heave amplitudes and the excitation amplitude is not necessarily large. Thus the heave and pitch motions are described by linear equations including a linear damping. Moreover, the heave and pitch motions are assumed to be independent of the roll motion, an assumption that is verified by the experiments.

2. EQUATIONS OF MOTION

We choose the right-handed coordinate systems shown in Figure 1. Moreover, we use Euler angles to define the rotations of the body-fixed coordinate system (whose origin is at the center of gravity G of the vessel) with respect to the inertial coordinate system. They are defined in the following sequence :

1) the yaw angle ψ is the rotation about the initial position of the z axis.

2) the pitch angle θ is the rotation about the new position of the y axis, and

3) the roll angle ϕ is the rotation about the final position of the x axis.

The motion is described by the generalized coordinates

$$\{q\} = \{x,\ y,\ z,\ \phi,\ \theta,\ \psi\}' \cdots\cdots\cdots\cdots (1)$$

where x, y, and z are the components of a position vector R with respect to the inertial coordinate system. Motions of a general character are observed comprising one combination or another of all the six degrees of freedom of motion. However, it is known that the most important and fundamental motions that determine the safety of a vessel are the rolling, pitching, and heaving. Thus, Blocki[3] assumed the following order of

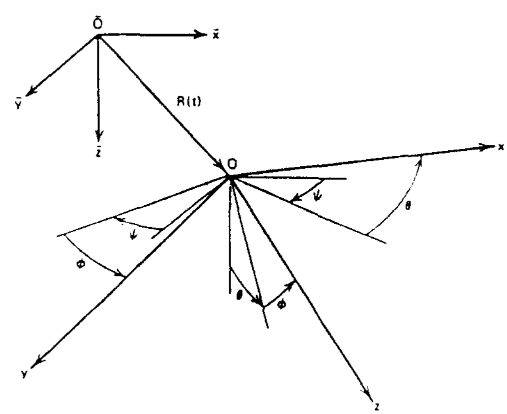

Figure 1. Coordinate Systems

magnitude of the different components of motion :

$$z=O(\varepsilon),\ \theta=O(\varepsilon),\ \phi=O(1),\ x=O(\varepsilon^2),\ y=O(\varepsilon^2),\ \text{and}\ \psi:\ O(\varepsilon^2) \quad \cdots\cdots (2)$$

where ε is a small dimensionless parameter, z is the heave, θ is the pitch, ϕ is the roll, x is the surge, y is the sway, and ψ is the yaw. Based on these assumptions, the latter three displacements and their velocities and accelerations are set equal to zero. Then the equations of motion to O (ε^2) become written in the following form :

$$\ddot{z}+2\zeta_z\dot{z}+\omega_z^2 z=\bar{Z}(t) \quad \cdots\cdots (3)$$

$$\ddot{\theta}+2\zeta_\theta\dot{\theta}+\omega_\theta^2\theta=\bar{M}(t) \quad \cdots\cdots (4)$$

$$\ddot{\phi}+\omega_\phi^2\phi+2\mu_1\dot{\phi}+2\mu_3\dot{\phi}^3-\alpha_3\phi^3-\frac{1}{2}(K_{\phi z}\phi z+K_{\phi\theta}\phi\theta+K_{\dot{\phi}\dot{z}}\dot{\phi}\dot{z}+K_{\dot{\phi}\dot{\theta}}\dot{\phi}\dot{\theta})=\bar{K}(t) \quad \cdots (5)$$

where ζ_z and ζ_θ are damping coefficients : ω_z, ω_θ, and ω_ϕ are the natural frequencies ; μ_1 and μ_3 are linear and cubic roll damping coefficients ; α_3 is the constant cubic stiffness coefficient ; and $K_{\phi z}$, $K_{\phi\theta}$, $K_{\dot{\phi}\dot{z}}$, and $K_{\dot{\phi}\dot{\theta}}$ are the constant coefficients of the quadratic coupling terms. Here we use arbitrary values for the coefficients to understand the possibility of a nonlinear dynamic behavior for any floating bodies.

Blocki[3] considered only heave and roll modes by assuming that there is no pitch and ignored the kinematic-kinematic coupling ; thus, he had only one (static-static) quadratic term : $K_{\phi z}\phi Z$. Here, we include additional static-static couplings between the pitch and roll modes as well as kinematic-kinematic couplings among all three modes.

Assuming simple harmonic excitation waves approaching the vessel from head or following directions, we write

$$\bar{Z}(t)=\bar{Z}_o\ \cos\Omega t \quad \cdots\cdots (6)$$

$$\bar{M}(t)=\bar{M}_o\ \cos(\Omega t+\tau_d) \quad \cdots\cdots (7)$$

where Ω is the frequency of the exciting waves, τ_d is the phase delay of the pitch moment relative to the heave froce, $\bar{Z}_0$ is a measure of the amplitude of heave excitaton force, and $\bar{M}_0$ is a measure of the amplitude of the pitch excitation moment. Here, $\bar{Z}_0$ and $\bar{M}_0$ are functions of the wave height as well as the position of the mass center in the wave.

Since the sets (3) and (6) and (4) and (7) are uncoupled linear equations, their steady-state solutions can be expressed as

$$z=a_z\cos(\Omega t+\tau_z) \quad \cdots\cdots (8)$$

$$\theta=a_\theta\cos(\Omega t+\tau_\theta) \quad \cdots\cdots (9)$$

where a_z and a_θ are the amplitudes of heave and pitch respectively, τ_z are the phase lags of heave and pitch relative to the excitation wave, and τ_θ is a function of ζ_θ and τ_d.

We point out here the results predicted by the traditional linear analysis : The longitudinal waves such as head or following waves will cause only the heave and pitch motions which are the modes of motion in the same plane(i. e., in-plane) as the direction of the plane progressive waves, and the roll motion is not generated by any means.

We consider the case in which the vessel is in longitudinal waves so that $\bar{K}(t)=0$ in equation (5). Substituting (8) and (9) into (5), we obtain

$$\ddot{\phi}+\omega_\phi^2\phi+2\mu_1\dot{\phi}+2\mu_3\dot{\phi}^3-\alpha_3\phi^3+[f_1\ \cos(\Omega t+\tau_z)+f_3\ \cos(\Omega t+\tau_\theta)]\,\phi+[f_2\ \sin(\Omega t+\tau_z)+f_4\ \sin(\Omega t+\tau_\theta)]\,\dot{\phi}=0 \quad \cdots (10)$$

where

$$f_1=-\frac{1}{2}a_zK_{\phi z}\quad f_2=\frac{1}{2}\Omega a_zK_{\dot{\phi}\dot{z}},$$

$$f_3=-\frac{1}{2}a_\theta K_{\phi\theta}\quad f_4=\frac{1}{2}\Omega a_\theta K_{\dot{\phi}\dot{\theta}} \quad \cdots\cdots (11)$$

Equation (10) is a nonlinear equation with time-varying coefficients that includes the effect of the pitch and heave motions on the roll motion and thus describes reality more closely than the equations analyzed by Blocki[3] and Nayfeh and Sanchez[4].

3. ANALYSIS

An approximate analytical solution of equation (10) is obtained for small but finite amplitudes for the case of lightly damped vessels. The straightforward expansion shows that resonances occur when $\Omega/\omega_\phi \approx 1, 2, 4$, etc. The first two cases are known as the fundamental and principal parametric resonances, respectively. It was concluded by Blocki[3] that the most dangerous case is $\Omega/\omega_\phi \approx 2$. Nayfeh and Sanchez[4] presented a bifurcation diagram in terms of the frequency and amplitude of the excitation and showed that the principal resonance occurs at the smallest excitation amplitude.

We use the method of multiple scales to determine a first-order approximation to the solution of equation (10). We begin by assuming that an approximation to ϕ can be written in the following form :

$$\phi(t\,;\,\varepsilon) \approx \varepsilon\phi_1(T_0, T_1) + \varepsilon^3\phi_3(T_0, T_1) \quad \cdots\cdots (12)$$

where $T_0 = t$ is a fast time scale, characterizing motions occurring at the frequency Ω and ω_ϕ ; $T_1 = \varepsilon^2 t$ is a slow scale, characterizing the modulation of the amplitude and phase due to the nonlinearity, damping, and resonances ; and ε is a dimensionless amplitude of the motion, which is used solely as a bookkeeping device.

The time derivatives are transformed into

$$\frac{d}{dt} \approx D_0 + \varepsilon^2 D_1 \quad \cdots\cdots (13)$$

$$\frac{d^2}{dt^2} \approx D^2_0 + 2\varepsilon^2 D_0 D_1 \quad \cdots\cdots (14)$$

where

$$D_0 = \frac{\partial}{\partial T_0} \text{ and } D_1 = \frac{\partial}{\partial T_1} \quad \cdots\cdots (15)$$

and terms of $O(\varepsilon^3)$ have been neglected. Next we must scale the linear damping and forcing so that all damping and forcing as well as the static restoring moment interact at the same order. We put $\mu_1 = \varepsilon^2\hat{\mu}_1$ and $f_i = \varepsilon^2\hat{f}_i$ for i = 1, 2, 3, and 4. The implication of the latter is that small-ampitude pitch and heave motions can produce large-amplitude rolling.

Substituting these definitons and equations (12) − (14) into equation (10), and then equating coefficients of like powers of ε, we obtain

$$0(\varepsilon) : D^2_0\phi_1 + \omega^2_\phi\phi_1 = 0 \quad \cdots\cdots (16)$$

$$\begin{aligned} 0(\varepsilon^3) : & D^2_0\phi_3 + \omega^2_\phi\phi_3 \\ &= -2D_0D_1\phi_1 - 2\hat{\mu}_1 D_0\phi_1 - 2\mu_3(D_0\phi_1)^3 + \alpha_3\phi^3_1 \\ &\quad -\hat{f}_1 \cos(\Omega t + \tau_z)\,\phi_1 - \hat{f}_2 \sin(\Omega t + \tau_z)\,D_0\phi_1 \\ &\quad -\hat{f}_3 \cos(\Omega t + \tau_\theta)\,\phi_1 - \hat{f}_4 \sin(\Omega t + \tau_\theta)\,D_0\phi_1 \end{aligned} \quad \cdots\cdots (17)$$

We can write the solution of equation (16) as

$$\phi_1(T_0, T_1) = A(T_1)e^{i\omega_\phi T_0} + cc \quad \cdots\cdots (18)$$

where cc stands for the complex conjugate of the preceding term. The function $A(T_1)$ is an arbitrary complex function of T_1 at this level of approximation. It is determined by imposing the solvability condition at the next level of approximation.

Substituting equation (18) into equation (17) yields

$$\begin{aligned} & D^2_0\phi_3 + \omega^2_\phi\phi_3 \\ &= -2i\omega_\phi[A' + \hat{\mu}_1 A]\,e^{i\omega_\phi T_0} \\ &\quad -2\mu_3(i\omega_\phi)^3[A^3 e^{3i\omega_\phi T_0} - 3A^2\bar{A}e^{i\omega_\phi T_0}] \\ &\quad +\alpha_3[A^3 e^{3i\omega_\phi T_0} + 3A^2\bar{A}e^{i\omega_\phi T_0}] \\ &\quad -\frac{1}{2}\hat{f}_1[Ae^{i(\Omega T_0 + \omega_\phi T_0 + \tau_z)} + \bar{A}e^{i(\Omega T_0 - \omega_\phi T_0 + \tau_z)}] \\ &\quad +\frac{1}{2}i\hat{f}_2[i\omega_\phi A e^{i(\Omega T_0 + \omega_\phi T_0 + \tau_z)} \end{aligned}$$

$$-i\omega_\phi\bar{A}e^{i(\Omega T_0-\omega_\phi T_0+\tau_z)}]$$

$$-\frac{1}{2}\hat{f}_3[Ae^{i(\Omega T_0+\omega_\phi T_0+\tau_\theta)}+\bar{A}e^{i(\Omega T_0-\omega_\phi T_0+\tau_\theta)}]$$

$$+\frac{1}{2}i\hat{f}_4[i\omega_\phi Ae^{i(\Omega T_0-\omega_\phi T_0+\tau_\theta)}$$

$$-i\omega_\phi\bar{A}e^{i(\Omega T_0-\omega_\phi T_0+\tau_\theta)}]+cc \cdots\cdots\cdots\cdots (19)$$

Because we are considering the principal parametric resonance corresponding to $\Omega\approx 2\omega_\phi$, we introduce a detuning parameter $\hat{\sigma}$ according to

$$\Omega=2\omega_\phi+\varepsilon^2\hat{\sigma} \cdots\cdots\cdots\cdots (20)$$

Then we substitute equation (20) into equation (19) and find that secular terms are eliminated from ϕ_3 if

$$2i(\dot{B}+\mu_1 B)+(6i\mu_3\omega^2_\phi-\frac{3\alpha_3}{\omega_\phi})B^2\bar{B}$$

$$+\frac{f}{\omega_\phi}\bar{B}e^{i\sigma t+i\tau_f}=0 \cdots\cdots\cdots\cdots (21)$$

where the overdot denotes the derivative with respect to the original time t, $\sigma=\Omega-2\omega_\phi$, $B=\varepsilon A$, the unscaled amplitude (here we have eliminated ε by rewriting all the variables in their original form ; ε is no longer needed), and

$$f=\frac{1}{2}(f_1e^{i\tau_z}-f_2\omega_\phi e^{i\tau_z}+f_3e^{i\tau_\theta}-f_4\omega_\phi e^{i\tau_\theta})$$

$$=|f|\,e^{i\tau_f} \cdots\cdots\cdots\cdots (22)$$

Here f is an effective amplitude, due to the combined influence of heave and pitch, and is a complex function of $K_{\phi z}$, $K_{\phi\theta}$, $K_{\dot{\phi}\dot{z}}$, $K_{\dot{\phi}\dot{\theta}}$, a_z, a_θ, Ω, ω_ϕ, τ_z, and τ_θ [see eq.(11)].

Next, we express the function B in the following polar form :

$$B=\frac{1}{2}ae^{i\beta} \cdots\cdots\cdots\cdots (23)$$

where a and β are the amplitude and phase of the response. It follows that

$$\dot{a}=-\mu_1 a\frac{3}{4}\mu_3\omega_\phi^2 a^3-\frac{1}{2}\frac{|f|}{\omega_\phi}a\sin\gamma \cdots (24)$$

$$a\dot{\gamma}=a\sigma+\frac{3}{4}\frac{\alpha_3}{\omega_\phi}a^3-\frac{|f|}{\omega_\phi}a\cos\gamma \cdots\cdots (25)$$

where $\gamma=\sigma t-2\beta+\tau_f$.

Periodic responses correspond to constant a and γ or fixed-point solutions of equations (24) and (25). To determine the fixed points, we let $\dot{a}=\dot{\gamma}=0$. There exist two sets of fixed-point solutions : First, a=0 is always a solution, which corresponds to the trivial solution of equation (5) ; that is,

$$\phi(t)\approx 0 \cdots\cdots\cdots\cdots (26)$$

Second, when $a\neq 0$, manipulating equations (24) and (25) yields a set of algebraic equations which can be solved numerically to determine a and γ. The equation for a is

$$c_4a^4+2c_2a^2+c_0=0 \cdots\cdots\cdots\cdots (27)$$

where

$$c_4=\frac{9}{4}\mu_3^2\omega_\phi^4+\frac{9}{16}\frac{\alpha_3^2}{\omega_\phi^2}$$

$$c_2=3\mu_1\mu_3\omega_\phi^2+\frac{3}{4}\frac{\alpha_3}{\omega_\phi}\sigma \cdots\cdots\cdots\cdots (28)$$

$$c_0=4\mu_1^2+\sigma^2-\frac{|f|^2}{\omega_\phi^2}$$

In this case, it follows from equations (23), (18), and (12) and from the relationship between A and B that

$$\phi(t)\approx a\cos[\frac{1}{2}(\Omega t-\gamma+\tau_f)] \cdots\cdots\cdots\cdots (29)$$

where a and γ are given by equations (24) and (25), and we have used eq.(20).

The fixed-point solutions are also verified by numerically integrating the modulation equations (24) and (25) using a 5th- and 6th-order Runge-Kutta-Verner's scheme with double precision arithmetics. The investigation of the stability of various fixed-point solutions, which has been determined by Oh[9], required an extensive work and thus is omitted from the present paper.

4. RESULTS AND DISCUSSION

Equations (24)−(25) and (26)−(30) yield two types of force-response curves depending on the values of the parameters.

Figure 2 shows a typical supercritical-type force-response curve when $\mu_1=\mu_3=0.04$, $\alpha_3=1.0$, and $\sigma=0.20$. As the resultant forcing amplitude $|f|$ is increased from 0, only the trivial solution exists and it is stable until the bifurcation point $|f|=0.2155$. As $|f|$ increases beyond the bifurcation point, the trivial solution becomes unstable and a nontrivial solution appears, which is stable. As $|f|$ is increased further, the roll amplitude grows nonlinearly and monotonically.

Figure 3 shows a typical subcritical-type force-response curve ; it displays some interesting features. The values of the parameters are the same as those in Figure 2 except that the sign of σ has been changed. As the effective amplitude $|f|$ is increased from 0, only one solution is possible ; the trivial solution, which is stable. As $|f|$ passes 0.0957 (ζ_1) approximately, three solutions are possible, the trivial solution, which is stable, and two nontrivial solutions, the larger of which is

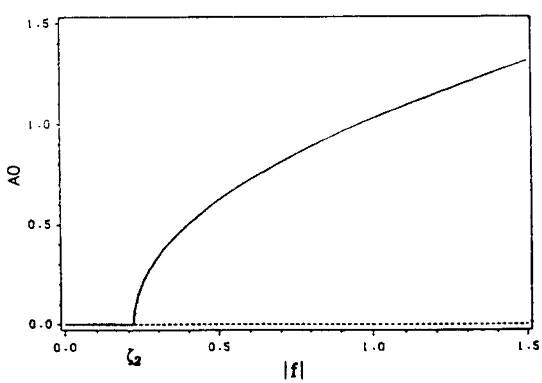

Figure 2. Force-response Curves(supercritical type)

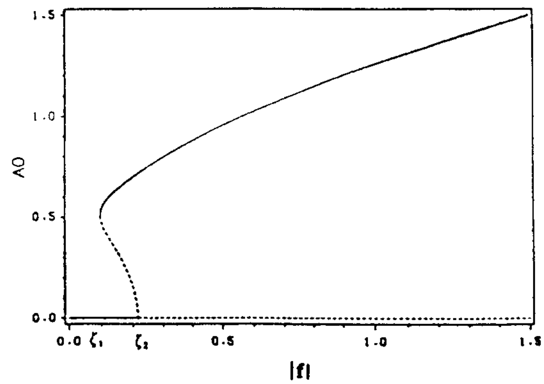

Figure 3. Force-response Curves(subcritical type)

stable and the smaller one is unstable. As $|f|$ passes 0.2155 (ζ_2) approximately, only two solutions are possible, the trivial solution, which is unstable, and a rather large-amplitude nontrivial solution, which is stable.

In an experiment, one would never see a motion corresponding to the lower-amplitude nontrivial solution in the range $0.0957<|f|<0.2155$. For $|f|>0.2155$, one would always see a rolling motion. In the range $0.0957<|f|<0.2155$, one could expect to see one of two possible motions : either no roll at all(corresponding to the trivial solution), or a rather large-amplitude roll motion. The initial conditions, or external disturbances, determine which motion will develop. The existence of two stable responses to the same excitation is a characteristic of subcritical instability and has been observed in many other mechanical and structural systems. Some of such results can be found in the works of Sethna[5] and Hatwal, Mallik, and Ghosh[6].

As the effective amplitude $|f|$ is slowly increased from 0, the rolling motion will not be excited until the bifurcation point $|f|=0.2155$ (ζ_2) is reached. As $|f|$ is increased further, the trivial solution becomes unstable and the roll motion suddenly occurs ; that is, the roll amplitude suddenly jumps to a large value. From there, the "rolling" solution is the only stable response, and its amplitude increases nonlinearly and monotonically as $|f|$ is increased further.

As $|f|$ is decreased slowly from a large enough value, the magnitude of the large-amplitude roll motion decreases nonlinearly, and rolling continues even after the iump-up bifurcation point (ζ_2) is passed. When $|f|$ reaches the second bifurcation point $|f|=0.0957$ (ζ_1), the large-amplitude roll motion suddenly disappears and the respoonse is roll free ; that is, the jump-down phenomenon occurs.

Figures 4 (a) and (b) show two frequency-response curves for different values of the effective amplitudes $|f|$ of excitation. The values of the parameters are $\mu_1=\mu_3=0.04$, $\omega_\Phi=\alpha_3=1.0$ for both cases and $|f|=0.20$ for (a) and $|f|=0.15$ for (b). Figure 4 (a) shows that the trivial fixed-point solution is unstable when $-0.1834=-|\sigma_c| \leq\sigma\leq |\sigma_c|=0.1834$, where σ_c is the critical value of the detuning parameter σ. The same is true in Figure 4 (b) in the range $-0.1269=-|\sigma_c| \leq\sigma\leq |\sigma_c=0.1269$. Thus, we note that for values of σ between the two bifurcation values $(-|\sigma_c|$ and $+|\sigma_c|)$, the large-amplitude roll motion is the only stable response. Because the trivial solution is unstable in this region, the model will display violent rolling spontaneously even when the model is given zero initial conditions.

When $-1.0\leq\sigma\leq-0.1834$ in Figure 4 (a) and $-0.8809\leq\sigma\leq-0.1269$ in Figure 4 (b), there exist three fixed-point solutions : a stable large-amplitude roll, and unstable small-amplitude roll, and a stable no-roll motion. The initial conditions determine which motion will be exhibited by the vessel. When proper external disturbances are imposed on the vessel, the response will exhibit the jump-up or jump-down phenomena between the two stable large-amplituede and trivial solutions. The unstable fixed-point solutions are not

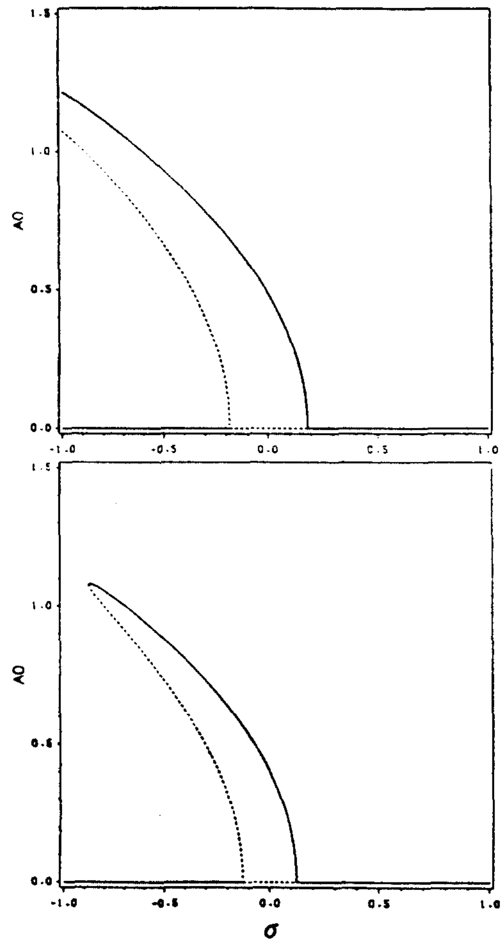

Figure 4. Frequency-response Curves

achievable in experiments, irrespective of whether they are trivial or nontrivial solutions.

When $\sigma>0.1834$ in Figure 4 (a) and $\sigma>0.1269$ and $\sigma<-0.8809$ in Figure 4 (b), the vessel does not display any roll motion because the trivial no-roll motion is the only stable fixed-point solution in these regions.

5. CONCLUSIONS AND REMARKS

To design more comfortable and safe vessels, one must understand the complicated dynamics of a ship moving in a general irregular seaway. Included among the important dynamic parameters are the ratios of natural frequencies and the nonlinear interactions among the hydrostatic and hydrodynamic forces and moments. The goal of the present effort was to contribute some basic insight toward an eventual entire understanding.

It has been demonstrated theoretically that a vessel in head or following waves can spontaneously develop severe rolling motion. The energy put into the longitudinal modes of motion (pitch and heave) by the longitudinal excitations may be fed into the transverse mode of motion (roll) by means of nonlinear coupling among those modes. The results of this paper applied to the behavior of a vessel is an address of the fact that a dynamical system subject to the in-plane excitations may exhibit the out-of-plane mode of responses.

To investigate the loss of dynamic stability and the development of large-amplitude rolling motions of a vessel, we began with a dynamic system of theree degrees of freedom. The present model to describe the parametrically excited rolling response of a vessel is an improvement over the one used in the previous work of Blocki[3] and Nayfeh and Sanchez[4] : In the real situation, the vessel will necessarily experience pitch motion and thus the pitch mode as well as heave should be included in investigating the parametric resonance of the roll mode. Both the pitch and heave modes are used to determine the effective amplitude of the parametric excitation of the roll mode. In the equation for the roll, the kinematic-kinematic nonlinear coupling terms among the three modes are included as well as the static-static terms. Thus, the present approach is closer to reality when the longitudinal asymmetry of a vessel with respect to the midship section is taken into account. Further, to verify this by the following experiments, the model should be placed longitudinally in the towing basin to eliminate the possibility of any external excitation in roll and hence to produce the pure parametic excitation by the regular plane waves.

The principal parametric resonance was considered ; the frequency of the wave excitation is approximately twice the natural frequency of the roll mode. The method of multiple scales was used to determine a first-order approximation to the solution. Supercritical- and subcritical-type force-response curves were obtained. The latter shows the coexistence of multiple stable solutions and the jump phenomena between the multiple fixed-point solutions ; a frequent feature of nonlinear dynamics. Frequency-response curves were also obtained.

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