⊙論 文

Application of Equivalent Nonlinear Method to Nonlinear Rolling Motoion of Ships

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등가 비선형화법 적용에 의한 선박의 비선형 횡동요 계산

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Key Words : White noise(백색 잡음), Non-white noise(비 백색 잡음), Nonlinear roll equation(비 선형 횡요 방정식), Equivalent linearization(등가선형화), Average method(평균화 법), Equivalent nonlinearization method(등가 비선형화 법)

요 약

불규칙 해상에서 선박의 큰 횡요각의 예측이 중요한 과제로 대두 되고 있다. 본 논문에서는 추계적 해석에 의한 이의 예측 방법을 제시한다. 즉, 주어진 비선형 횡요 운동 방정식으로 부터 배의 횡요각과 각속도의 확률 밀도 함수를 구하는 방법으로 평균화 법과 등가 비선형화 법을 적용하였고 각종 계수들의 값의 변화에 따른 예측 결과를 다른 논문에서 제시한 simulation결과와 비교하였다.

1. Introduction

The present study investigates a method of predicting the threshold crossing time by solving a nonlinear rolling equation of motion of a ship in irregular waves. Since the nonlinear nature of the rolling motion of a ship in waves is very complicated, a complete analytic solution of the problem has not been proposed so far.

In the past, those approaches to solve this problem can be illustrated as an equivalent linear equation,[1] a perturbation method,[2] and functional representation method.[3,4] But these yielded only limited information on the roll response statistics such as a mean square of the roll angle.

Another approach which is able to predict the form of the response distribution for nonlinear

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system response is Fokker–Plank–Kolmogorov (FPK) method. Dunne[5] has developed a new approximate method for dealing with a nonlinear systems which are disturbed by exitation that can not be adequately classed as wideband, and modeled as white noise. He combined the method of equivalent linearization and the FPK technique to obtain some useful results. The method has been applied estimating threshold crossing rates. The theoretical results were tested with extensive simulation results. That is, their method is based on the assumption that the crossing properties of the response can be approximately replaced by the excitation with a white noise process of suitable intensity. Then they reinstated the nonlinear restoring function from the equivalent linearized equation of motion.

The present study reinstates the full nonlinear damping and nonlinear restoring function with the equivalent white–noise intensity so that the non–linearity in the damping can be adequately modeled. This white–noise exited nonlinear equation of motion is solved by averaging method and equivalent nonlinearization method to obtain the needed joint probability density function.

2. Roll response model in random beam seas

If the influence of all other degrees of freedom can be neglected, the equation of rolling motion of a ship in random beam waves can be written in the following form

$$\ddot{\theta} + D(\dot{\theta}) + R(\theta) = n(t) \quad \cdots\cdots (1)$$

where θ is roll angle, $D(\dot{\theta})$ represents a damping function, and $R(\theta)$ represents a restoring function, and $n(t)$ is a Gaussian random process with zero mean and spectrum $S(\omega)$. The time scale is chosen so that the undamped natural roll frequency is unity. The damping and restoring function can be represented as

$$D(\dot{\theta}) = D_1\dot{\theta} + D_3\dot{\theta}^3 \quad \cdots\cdots (2)$$

$$R(\theta) = \theta + K_3\theta^3 \quad \cdots\cdots (3)$$

Solving equation (1) means the determination of a limited amount of information about the solution process. The threshold crossing probability can be considered as useful statistics. So the thresholds $\theta = \pm a$ considered in this study are high enough to be regarded as dangerous. The number of upcrossings per unit time of θ at threshold $a > 0$ can be written as[6]

$$\lambda(a) = \int_0^\infty \dot{\theta} f(a,\dot{\theta})\, d\dot{\theta} \quad \cdots\cdots (4)$$

where $f(\theta,\dot{\theta})$ is the joint probability density function, and the mean time $\mu(a)$ between upcrossings is the reciprocal of $\lambda(a)$

$$\mu(a) = \frac{1}{\lambda(a)} \quad \cdots\cdots (5)$$

3. Exact solution

When a dynamic system is subjected to white noise exitation, the exact solution can be obtained through the FPK equation. A few exact analytical solutions of FPK equation exist for random vibration problems. The most general is due to Caughey,[7] Consider the following equation of motion for a single degree of freedom system.

$$\ddot{\theta} + \dot{\theta} H(E) + g(\theta) = Z(t) \quad \cdots\cdots (6)$$

where

$$E = {}^1/_2\dot{\theta}^2 + V(\theta) \quad \cdots\cdots (7)$$

where E represents the total energy of the system, $V(\theta)$ is the potential energy of the system, $Z(t)$ is a white noise which has a constant spectral density J. Caughey has shown an exact solution for the probability density function $f(\theta,\dot{\theta})$ is as follows

$$f(\theta,\dot{\theta})=A\ exp\left[\frac{2}{J}\int_0^E H(\xi)d\xi\right] \cdots\cdots\cdots\cdots (8)$$

where A is a normalizing constant.

4. Nonlinear damping, non−white excitation

The present method is based on the assumption that the crossing properties of the response to an non−white excitation can be approximated by replacing the excitation with a white noise process of suitable intensity J.

4. 1 Equivalent linearization

First the system is linearized in a conventional way. The equivalent linearization technique applied to equation (1) replaces the system by an equivalent linear system

$$\ddot{\theta}+C_{eq}\dot{\theta}+K_{eq}\theta=n(t) \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

The equivalent linear coefficients in (9) are determined by minimizing the mean square of the linearization error.

For the linearized system, the mean upcrossing times $\mu_{S(w)}(a)$ and $\mu_J(a)$ can be readily obtained. $\mu_{S(w)}(a)$ and $\mu_J(a)$ represent the mean upcrossing times which are calculated from non−white process and white noise respectively. The non−white process describes the exciting moment due to wave excitation.

The value of J is chosen to minimize the square error

$$\int_0^{a_{max}} [\mu_{S(w)}(a)-\mu_J(a)]^2\ da \cdots\cdots\cdots\cdots (10)$$

where a_{max} is a suitable high threshold. The newly determined J is the best fit for the mean crossing time functions over the range of thresthold of interest. If one reinstate the nonlinear damping and the nonlinear restoring, the equation (1) can be rewitten as

$$\ddot{\theta}+D(\dot{\theta})+R(\theta)=\sqrt{J}\quad Z(t) \cdots\cdots\cdots\cdots\cdots (11)$$

where Z(t) represents the white−noise excitation. If we can get the probability density function of equation (11), then the expected values which are needed to calculated the equivalent linear coefficients in equation (9) can be obtained. Thus the whole process can be an iteration scheme.

4. 2 Average method

The average method for the random vibration studies described by Roberts and Spanos[8] is adopted to calculate the approximate value of $f(\theta,\dot{\theta})$ for the equation (11).

The basic concept of the method for randomly excited oscillators is as follows when the energy dissipated per cycle is due to light damping, the total energy can be treated as approximately constant over one cycle of oscillation. The period of free oscillation is found to be given by

$$T(E)=4\int_0^{\theta_c}\frac{d\theta}{\sqrt{2(E-V)}} \cdots\cdots\cdots\cdots (12)$$

where θ_c is such that

$$V(\theta_c)=E \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

The error integral

$$I=\int_0^{T(E)}\varepsilon^2\ dt \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

can be minimized with respect to H(E). This yields the following expression for H(E) for use in equation (11)

$$H(E)=\frac{\int_0^{\theta_c} D([2(E-V(\theta)]^{1/2})\ d\theta}{\int_0^{\theta_c}[2(E-V(\theta)]^{1/2})\ d\theta} \cdots (15)$$

A combination of equation (8) and equation (15) now gives an approximate expression by $f(\theta,\dot{\theta})$.

4. 3 Equivalent nonlinear equation method (ENLEM)

Equation (1) which represents the equation of the rolling motion in random sea can not be expressed as indicated in equation (6). At this stage, an ENLEM is adopted. The basic idea of the method is to replace a given nonlinear stochastic system by an equivalent non-linear equation (ENLE) whose exact steady state solution is available. Following the description of Roberts and Spanos,[8] equation (1) can be replaced with an ENLE such that the error term

$$\varepsilon = c_1\dot{\theta} + c_3\dot{\theta}^3 - \dot{\theta}H(E) \qquad (16)$$

can be minimized. If it is assumed that

$$H(E) = \sum_r \sum_s d_{rs} H_{rs}(E) \qquad (17)$$

then the mean square of this error can be minimized with respect to all the coefficients d_{rs}, that is

$$\frac{\partial E\{\varepsilon^2\}}{\partial d_{rs}} = 0 \qquad (18)$$

where $E\{\varepsilon^2\}$ represents the expectation of ε^2, Equation (18) yields a set of algebraic equations from which d_{rs} can be found. Considering the order of the damping term, it is possible to choose,

$$H_{rs}(E) = (2E)^{(r+s-1)/2} \qquad (19)$$

This yields

$$H(E) = d_{01} + 2d_{03}E \qquad (20)$$

5. Numerical Results

To apply the proposed scheme to non-white excitation, the following test spectrum was adopted following Dunne[5] for the mutual comparison purposes

$$S(\omega) = \begin{cases} \dfrac{PJ}{1+(P-1)\,|\,\omega-\omega_0\,|} & \omega<\omega_c \\ 0 & \omega>\omega_c \end{cases} \qquad (21)$$

where J, ω_0 and P are scale, location and shape parameters respectively, with P=1 represents the white-noise process and ω_c is cut-off frequency. Fig. 1 shows the shape of the test spectrum, and the frequency response function.

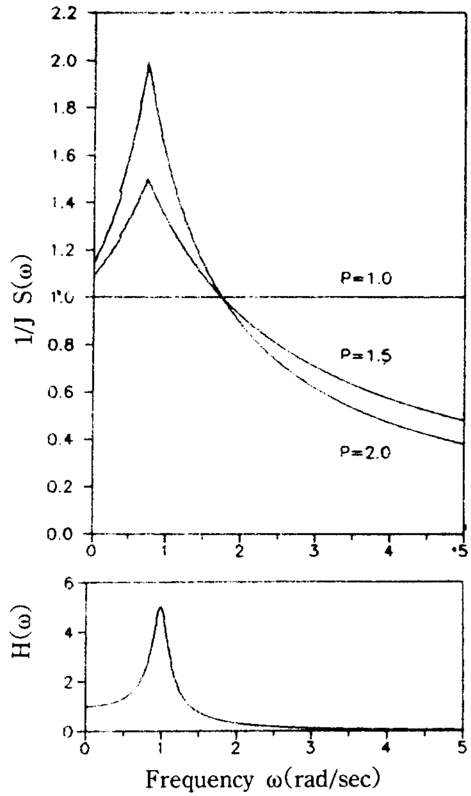

Fig. 1 Test Spectrum and Frequency Response Function

Fig. 2 shows mean upcrossing time against threshold for white noise intensity J=0.07, nonlinear coefficient $K_3=-0.5$, linear damping coefficient $D_1=0.2$ and three values of nonlinear damping coefficient D_3. The square represents the simulation points which are given by Dunne for comparison purposes. The short dotted line represents the results obtained by Dunne. Dunne obtained by reinstating the nonlinear restoring function, but with linear damping. The long dotted line and solid line represent the average method and equivalent nonlinearization method respectively. The higher the damping, the larger the mean crossing time as expected. The restoring function is plotted against the roll angle to show the non−linear region of restoring. The proposed schemes show better results over the Dunne's scheme. Fig. 3 is similar except those J=0.05 and D_1=0.02. Even though the nonlinear damping coefficient is much larger than the linear coefficient, the proposed scheme agrees well with the simulation results.

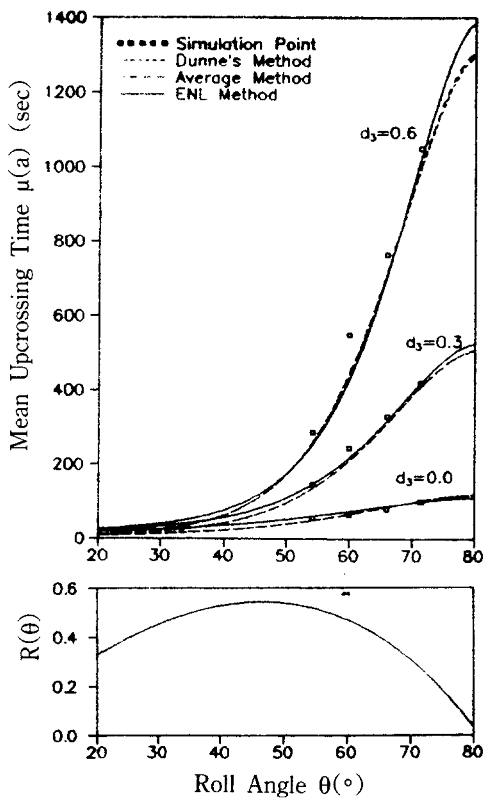

Fig. 2 Mean Upcrossing Time VS. Threshold(J=0.07, C_1=0.2, K_3=−0.5)

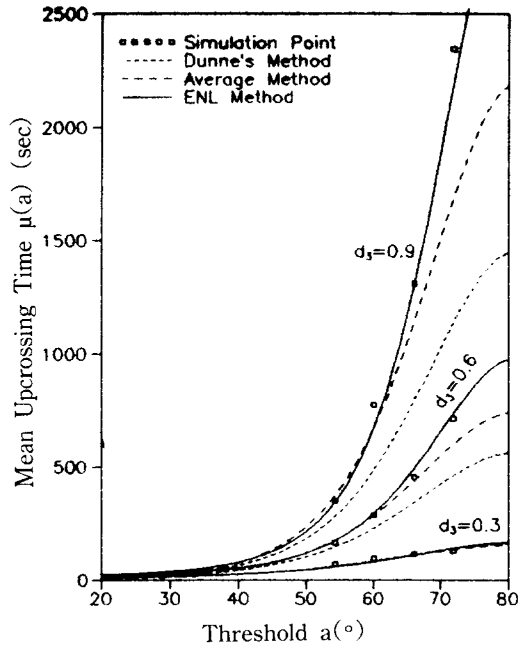

Fig. 3 Mean Upcrossing Time VS. Threshold(J=0.05, C_1=0.02, K_3=−0.5)

The linear damping is considered next. The results in Fig. 4 shows that the proposed scheme and the Dunne's scheme give similar results with each other as expected.

Fig. 5 and Fig. 6 are similar to Figs. 2 and 3, except that D_3=0.6 and the parameter P varies. The proposed schemes show better results than those proposed by Dunne[5] when the nonlinear damping dominates. Generally speaking, the equivalent nonlinear methods show better results than thore calculated by the average method. This can be explained by the fact that the average method is based on the slow varying energy concept.

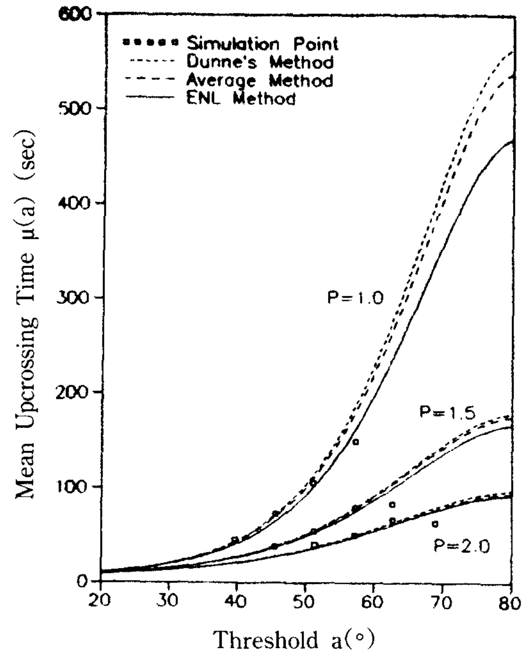

Fig. 4 Mean Upcrossing Time VS. Threshold($J=0.09$, $C_1=0.4$, $C_3=0$, $K_3=-0.5$)

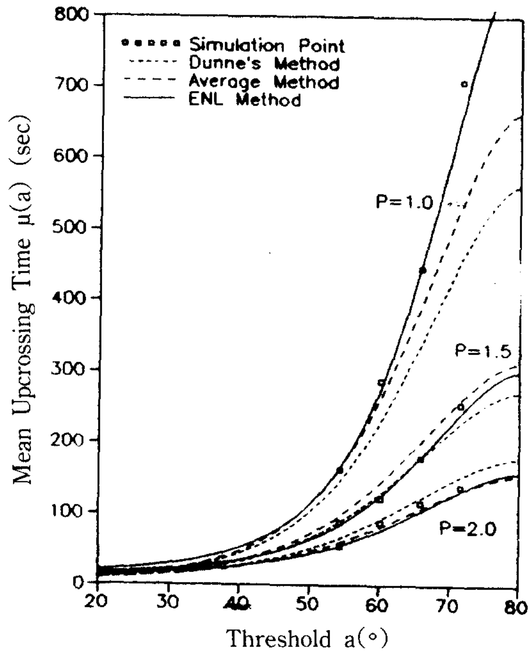

Fig. 6 Mean Upcrossing Time VS. Threshold($J=0.05$, $C_1=0.02$, $C_3=0.6$, $K_3=-0.5$)

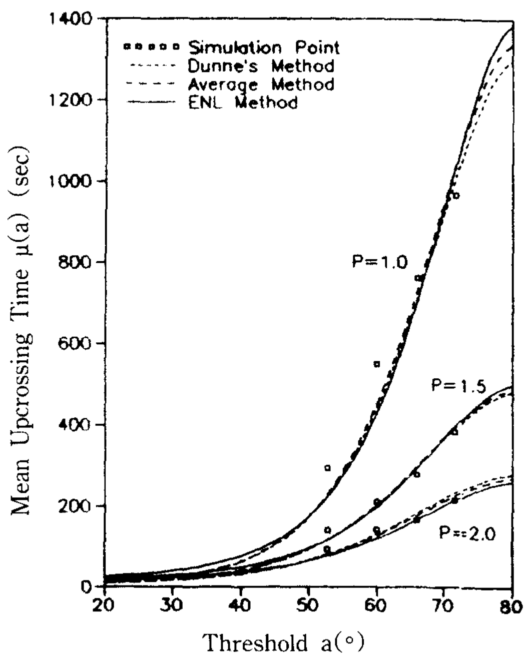

Fig. 5 Mean Upcrossing Time VS. Threshold($J=0.07$, $C_1=0.2$, $C_3=0.6$, $K_3=-0.5$)

6. Conclusions

The following conclusions are drawn from this study :

1) The crossing properties of the response can be approximately replaced by the excitation with a white noise process of suitable intensity.

2) The proposed schemes which reinstate the full nonlinear damping and nonlinear restoring with the equivalent white−noise intensity gives a relatively good agreement with the simulation results over the Dunne's results.[5]

3) The equivalent nonlinear method shows better results than those calculated by the average method.

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