Component Steady-State Availability의 Bayes 추정

박 춘 일*

Bayes Estimation of Component Steady－State Availablity

Choon-Il Park

〈目　　次〉

Abstract

This paper presents a class of Bayes estimation of component steady-state availability. Throughout this paper, we will denote the mean time between failure and the mean time between repair by MTBF and MTBR respectively.

In section 2, we investigate Bayes estimation of the steady-state availability for noninformative prior density function and in section 3, we compute Bayes estimation for conjugate prior density function.

1. Introduction

Let us denote the distribution of failure time X and repair Y by H(X) and G(Y) respectively. Then the repairable component with respect to these distribution can be determined by steady-state availablity $A=E(X)/(E(X)+E(Y))$, which is the probaility that the repairable component is in operation in the long fraction of time.

Gave and Mazumder(3) investigated the estimation of parameters in case that two probability distributions are specified on the two-state pro-

* 正會員 韓國海洋大學校 理工大學 敎授

cess in operation and under repair. By investigating the failure of previous try and the period of repairs, Nelson(4) determined the interval of prediction for the availability of repairable component. Brender(1) worked on theh Bayesian estimation of the steady-state availability by using failure rate and repair rate.

On the other hand, Thompson and Palicio(5) obtained a method to compute the Bayesian interval of the availability of a series or parallel system consisting of several statistically independent two-state subsystems having exponential of failure times and repair times.

The main purpose of this paper is to compute Bayesian estimation of component steady-state availability. In section 2, we will investigate Bayesian estimation of the steady-state availability ofr noninformative prior density function and in section3, we will compute Bayesian estmation for conjugate prior density function. Finally, some examples to compare several estimations numerically will be given in section 4.

2. Bayes Estimation for Noninformative Prior Density Function

Consider a repairable component for which theh failure time X is distributed as H(θ) with exponential probability density function(pdf)

$$h(x \mid \theta) = \frac{1}{\theta}, \exp\left(-\frac{x}{\theta}\right), \; x>0, \; \theta>0$$

and the repairable time Y is distributed as G(α, β) with Gamma-pdf

$$g(y \mid \alpha_0, \beta_0) = \frac{1}{\Gamma(\alpha_0)\,\beta_0^{a0}} \cdot y^{(\alpha 0-1)} \cdot \exp\left(-\frac{y}{\beta_0}\right), \; y>0, \; \alpha>0, \; \beta>0$$

where α (shape parameter)has a known value α_0 and β(scale parmeter)is unknown value β_0. Assume that X, Y independent the stady-state availability is given by

$$A = \frac{E(X)}{E(X)+E(Y)}, \; E(X) = 0, \; E(Y) = \alpha_0{}^0\beta_0$$

Then, the likelihood function of q, T_z, $T_y$given

$$(2-1)\quad L(q, T_x, T_y \mid \theta, \beta_0, \alpha_0) = \frac{1}{\theta^q} \exp\left(-\frac{T_x}{\theta}\right)\left[\frac{1}{\Gamma(\alpha_0)}\right]^q \left(\frac{1}{\beta_0}\right)^{q\alpha 0} \left(\prod_{i=1}^{\alpha} y_1\right)^{(\alpha 0-1)} \exp\left(\frac{T_y}{\beta_0}\right)$$

, where q : observed failure repair cycles

$T_x = \sum_{i=1}^{q} x_i$ (x_i is the i-th failure time) : total observed operating time

$T_y = \sum_{i=1}^{q} y_i$ (y_i is the i-th repair time) : total observed repair time

Let's introduce some integration for calculation of the expectation,

$$(2-2)\quad \int_0^\infty z^{(p-1)} \cdot \exp(-az)\, dz = \Gamma(p)a^{-p},$$

$$\int_0 z^{(b-1)})1-z^{(c-b-1)}(1-tz)^{-a}\, dz = \frac{\Gamma(b)\,\Gamma(c-b)}{\Gamma(c)}\,{}_2F_1(a, b ; c, t),$$

for $|t|<1$, $c<b<0$ and

$${}_2F_1(a, b ; c, t) = \sum_{i=0}^{\infty} \frac{(a_i)\,(b_i)}{(c_i)\cdot i\,!}\, t^i$$

is confluent hypergeometric functions of Gauss form for $|t| < 1$ with

$$(a_i) = \frac{\Gamma(a+i)}{\Gamma(a)}$$

From Erdelyi[2], we have

(2−3) ${}_2F_1(a, b ; d ; z) = (1-z)^{(c\text{-}a\text{-}b)}\, {}_2F_1(c-a, c-b ; c-z)$, for $|z| < 1$;

, where (a, b, c−a, c−b) is positive integer.

Assume that MTBF in the exponential failure time distributed and scale parameter in the Gamma repair time distribution has independent noninformative prior distribution given by

$$(2-4) \qquad f(\theta) \propto \frac{1}{\theta^{m1}}, \qquad m_1 > 0$$

$$(2-5) \qquad f(\beta) \propto \frac{1}{\beta_0^{m2}}, \qquad \beta_l > 0,\ m_2 >$$

Then we have the following lemma.

LEMMA 2.1 The joint posterior density function of θ and βis given by

$$f(\theta, \beta_0 \mid q, T_x ; , T_y ; \alpha_0) = \frac{(T_x)^{(q+m1-1)}\,(T_y)^{qc0+m2-1)}\exp[-(T_x/\theta)+(T_y/\beta_0)]}{\theta^{q+m1)}\,\beta_0^{(q\alpha0+m2)}\,\Gamma(q+m_1-1)\,\Gamma(q\alpha_0+m_2-1)}$$

, where $\theta > 0$, $\beta >$ andΓ ()is a Gammn function.

PROOF. From(2-1), (2-2) and (2−3), the joint posterior distribution of θ, β_0 reduces to (2−6) $f(\theta, \beta_0 \mid q, T_x, T_y ; a_0)$

$$= \frac{L(q, T_x, T_y \mid \theta, \beta_0 p\alpha_0)\cdot f_1(\theta)\ f_2(\beta_0)}{\int_0^x\int_0^x L(q, T_x, T_y \mid \theta, \beta_0 ; \alpha_0)\cdot f_1(\theta)\ f_2(\beta_0)\,d\beta_0\,d\theta}$$

$$= \frac{e^{-q}\exp\left(-\frac{T_x}{\theta}\right)\left[\frac{1}{\Gamma(\alpha_0)}\right]\cdot \beta_0^{-q\alpha0}\left(\prod_{i=1}^{q} y_i\right)\exp\left(-\frac{T_y}{\beta_0}\right)\theta^{-m1}\theta^{-m2}}{\int_0^\infty\int_0^\infty e^{-q}\exp\left(-\frac{T_x}{\theta}\right)\left[\frac{1}{\Gamma(\alpha_0)}\right]^q \beta_0^{q\alpha0}\left(\prod_{i=1}^{q} y_i\right)^{(\alpha0-1)}\exp\left(-\frac{T_y}{\beta_0}\right)\theta^{-m1}\theta^{-m2}\,d\beta_0\,d\theta}$$

By the cancellation, the denominator of (2−6) becomes

$f(\theta, \beta \mid q, T_x, T_y ; a)$

$$= \int_0^\infty\int_0^\infty e^{-(q-m1)}\ \beta^{-(q\alpha0+m2)}\ \exp\left[-\left(\frac{T_x}{\theta}+\frac{T_y}{\beta_0}\right)\right]d\beta_0\,d\theta$$

$$= \int_0^\infty e^{-(q+m1)}\ \exp\left(-\frac{T_x}{\theta}\right)\left[\int_0^\infty \beta^{-(q\alpha0+m2)}\ \exp\left(-\frac{T_y}{\beta_0}\right)d\beta_0\right]d\theta$$

Suppose $\beta_0 = 1/\alpha$, then the inner integral is evaluated as

$$\int_0^\infty \beta_0^{-(q\alpha0+m2)}\ \exp\left(-\frac{T_y}{\beta_0}\right)d\beta_0$$

$$= \int_0^\infty \alpha^{(q\alpha0+m2-2)}\cdot\exp(-T_y\alpha)\,d\alpha$$

$$= \frac{\Gamma(q\alpha_0+m_2-1)}{(T_y)^{(q\alpha0+m2-1)}}$$

Suppose $\theta = 1/\beta$, then the outer integral of (2−6) is evaluated as

$$\int_0^\infty \varepsilon^{-(q+m)}\exp\left(-\frac{T_x}{\theta}\right)d\alpha$$

$$= \int_0^\infty \beta^{(q+m1-2)}\ \exp(-T_x\cdot\beta)\,d\beta$$

$$= \frac{\Gamma(q+m_1-1)}{(T_x)^{(q+m1-1)}}$$

(Q. E. D.)

LEMMA 2.2 The posterior distribution of component steady-state availability $A = 1/(1+\alpha_0\delta)$ is given by

$R(A \mid q, T_x, T_y ; \alpha_0)$

$$= \left(\frac{\alpha_0 T_y}{T_x}\right)^{(q\alpha 0+m2-1)} A^{(q\alpha 0+m2-2)} (1-A)^{(q+m1-2)}$$

$$= B(q+m_1-1,\ q\alpha_0-m_2-1) \left[1-A\left(-\frac{\alpha_0 T_y}{T_x}\right)\right]^k,$$

$$(0 < A < 1)$$

, where B(. , .)is Beta function, $k=(q+m_1+q\alpha_0+m_2-2)$ and $\delta=\beta_0/\theta$ is the service factor.

PROOF. First, we find the posterior distribution of $= \delta_0/\theta$. According to LEMMA2. 1, we express that

$$R_\delta(\delta \mid q,\ T_x\ T_y\ ;\ \alpha_0)\ \beta_0\delta^{-2} d\,\beta_0$$

$$= \int_0^\infty S\left(\frac{\beta_0}{\delta},\ \beta_0 \mid q,\ T_x,\ T_y\ ;\ \alpha_0\right) \beta_0\delta^{-2}\, d\beta_0$$

$$= \frac{(T_x)^{(q+m1-1)}\,(T_y)^{(q\alpha 0+m2-1)}}{\Gamma(q-m_1-1)\ \Gamma(q\alpha+m_2-1)} \int_0^\infty \left(\frac{\beta_0}{\delta}\right)^{-(1+m1)} \beta^{-(q\alpha 0+m2)} \exp\left[-\left(\frac{\delta}{\beta_0}T_x+\frac{1}{\beta_0}T_y\right)\right] \beta_0\delta^{-2} d\beta_0$$

$$= \frac{(T_x)^{(q+m1-1)}\,(T_y)^{(q\alpha 0+m2-1)}}{\Gamma(q-m_1-1)\ \Gamma(q\alpha_0+m_2-1)} \int_0^\infty \beta_0^{-(q+m1+q\alpha 0+m2-1)} \exp\left[-\frac{1}{\beta_0}(\delta T_x+T_y)\right] d\theta_0$$

Let $\beta_0 = 1/\alpha$, then we have integration with respect to β_0 that

$$R_\delta(\delta \mid q,\ T_x,\ T_y\ ;\ \alpha)$$

$$= \frac{(T_x)^{(q+m1-1)}\,(T_y)^{(a\alpha 0+m2-1)}\,\delta^{(q+m1-2}\Gamma\ (q+m_1+q\alpha_0+m_2-2)}{\Gamma(q+m_1-1)\cdot\Gamma(q\alpha_0+m_2-1)\cdot(\delta T_x+T_y)^{(q+m1+q\alpha 0+m2-2)}}$$

$$= \frac{(T_x)^{(q+m1-1)}\,(T_y)^{(q\alpha 0+m2-2)}}{B(q-m_1-1,\ q\alpha_0+m_2-1)\ (\delta T_x+T_y)^{(q-m1+q\alpha 0+m2-2)}},\ 0<\delta<\infty$$

Accordingly, the posterior distribution of component steady-state availability $A = 1/(1+\alpha_0\delta)$is given by

$$R(A \mid q,\ T_x,\ T_y\ ;\ \alpha_0)$$

$$= R_\delta\left[\frac{(A^{-1}-1)}{\alpha_0} \mid q,\ T_x,\ T_y\ ;\ \alpha_0\right]\left(\frac{A^{-2}}{\alpha_0}\right)$$

$$= \frac{(T_x)^{(q+m1-1)}\,(T_y)^{(q\alpha 0+m1-1)}\,(1-A)^{(q+m1-2)}}{B(q+m_1-1,\ q\alpha_0+m_2-1)\left[T_y+T_x\left(1-\frac{A^{-1}-1}{\alpha_0}\right)\right]^{(q+m1+q\alpha 0+m2-2)}}$$

$$= \frac{\left(\frac{\alpha T_y}{T_x}\right)^{(q\alpha 0+m2-2)}\ (1-A)q+m1-2)}{B(q+m_1-1,\ q\alpha_0+m_2-1)\left[1-A\left(1-\frac{\alpha_0 T_y}{T_x}\right)\right]^{(q+m1+q\alpha 0+m2-2)}}$$

(Q. E. D.)

By using **LEMMA 2.1** and **LEMMA 2.2** we can prove the following theorem.

THEOREM 2.1 Under a squarred-error loss function, Bayesian estimation of component steady-state availability A is given by

$$A_1^* = \frac{q\alpha_0+m_2-1}{q+m_1+1\alpha_0+m_2-2}\cdot {}_2F_1(1,\ q+m_1-1\ ;\ q-m_1+q\alpha_0+m_2-1\ ;\ 1-\frac{T_y\alpha_0}{T_x})$$

, Where $0 < \frac{\alpha_0 T_y}{T_x} < 2$, ${}_2F_1(a, b ; c ; t)$ is a confluent hypergeometric function in Gauss form.

PROOF. Bayes estimation of component steady-state availability A is average of posterior distrbution. From (2−2) and (2−3), we reduce as follows ;

$$A_1^* = \int_0^1 A\ h(A \mid q, T_x, T_y ; \alpha_0)dA$$

$$= \frac{\left(\frac{\alpha_0 T_y}{T_x}\right)^{(q\alpha_0+m2-1)}}{B(q+m_1-1,\ q\alpha_0+m_2-1)} \int_0^1 \frac{A^{(q\alpha 0+m2-1)}(1-A)^{(q+m1-2)}}{\left[1-A\left(1-\frac{\alpha_0 T_y}{T_x}\right)\right]^{(q+m1+q\alpha 0+m2-2)}} \cdot dA$$

$$= \frac{\left(\frac{\alpha_0 T_y}{T_x}\right)^{(q\alpha 0+m2-1)} \Gamma(q+m_1+q\alpha_0+m_2-2)\ \Gamma(q\alpha_0+m_2)\ \Gamma(q+m_1-1)}{\Gamma(q-m_1-1)\ \Gamma(q\alpha_0+m_1+q\alpha_0+m_2-1)} \cdot Q1$$

$$= \frac{\left(\frac{\alpha_0 T_y}{T_x}\right)^{(q\alpha 0+m2-1)} \cdot (q\alpha_0+m_2-1)}{q-m_1+1\alpha_0+m_2-2} \left(\frac{\alpha_0 T_y}{T_x}\right)^{-(q\alpha 0+m2-1)} \cdot Q2$$

$$= \frac{q\alpha_0+m_2-1}{q+m_1+q\alpha_0+m_2-2} \cdot Q2$$

, where $Q1 = {}_2F_1(q+m_1+q\alpha_0+m_2-2 ; q\ \alpha_0+m_1+q\alpha_0+m_2-1 ; 1-(\alpha_0 T_y / T_x))$ and $Q2 = {}_\lambda F_1(1, q+m_1-1 ; q+m_1+q\alpha_0+m_2-1 ; 1-(\alpha_0 T_y / T_x))$.
(Q. E. D.)

3. Bayes Estimation for Conjugate Prior Distribution.

In this section, by employing the technique similar to that of the previous section, we shall obtain bayes estimation of steady-state availability for conjugate prior distribution. Suppose MTBF in the exponential failure time distribution and the scale parameter in Gamma repair time distribution have independent conjugate density function ;

$$\text{(3−1)} \qquad K_1(\theta) = \frac{b^a}{\Gamma(\theta)}\left(\frac{1}{\theta}\right)^{(a+1)} \exp\left(-\frac{b}{\theta}\right),\ \theta > 0,\ a,\ b \geq 0$$

$$\text{(3−2)} \qquad K_2(\theta) = \frac{d^c}{\Gamma(c)}\left(\frac{1}{\theta}\right)^{(c+1)} \exp\left(-\frac{d}{\theta_0}\right),\ \beta_0 > 0,\ c,\ d \geq 0$$

Then, we have the following lemma.

LEMMA 3.1 The joint posterior density function of θ and θ is given by $K(\theta, \beta_0 \mid q, t_x, t_y ; \alpha_0)$

$$= \frac{(T_x+b)^{(q+a)}\ (T_y+d)^{(q\alpha 0+c)} \exp[-(1/\theta)(T_x+b)-(1/\beta_0)(T_y+d)]}{\theta^{q+a+1)}\ \beta_0^{(q\alpha 0+c+1)} \cdot \Gamma(q+a)\ \Gamma(q\alpha_0-c)}$$

PROOF From (3−1) and (3−2), the joint distribution of θ and β_0 is as follows,

$K(\theta, \beta_0 \mid q, T_x, T_y ; \alpha_0)$

$$= \frac{L(q,\ T_x,\ T_y \mid \theta,\ \beta_0\ ;\ \alpha_0)\ k_1(\theta)\ k_2(\beta_0)}{\int_0^\infty \int_0^\infty L(q,\ T_x,\ T_y \mid \theta,\ \beta_0\ ;\ \alpha_0)\ k_1(\theta)\cdot k_2(\beta_0)\ d\beta_0 d\theta}$$

$$= \frac{\theta^{-q}\exp(-T_x/\theta)\ [1/\Gamma(\alpha_0)]^q\cdot\theta_0^{-q\alpha 0}\ (\prod_{i=1}^{q} y_i)^{(\alpha 0-1)}\ \exp(-T_y/\theta_0)\ [b^c/\Gamma(a))\theta^{-(a+1)}}{\int_0^\infty\int_0^\infty \theta^{-q}\ \exp\left(-\frac{T_x}{\theta}\right)\left[\frac{1}{\Gamma(\alpha_0)}\right]^q\cdot\theta_0^{-q\alpha 0}\left(\prod_{i=1}^{q} y_i\right)^{()\sigma(:}\exp\left(-\frac{T_y}{\beta_0}\right)\frac{T_y}{\Gamma(a)}\theta^{-(a+1)}\exp\left(-\frac{b}{\theta}\right)}$$

$$* \frac{\exp(-b/\theta)\ \ (d^c/\Gamma(c))\ \theta_0^{-(c+1)}\ \exp(-d/\theta_0)}{(d^c/\Gamma(c))\ \theta_0^{-(c+1)}\exp(-d/\theta_0)d\theta_0 d\theta}$$

By the cancellation,

$$\int_0^\infty\int_0^\infty \theta^{-(q+a+1)}\ \beta^{-(q\alpha 0+c+1)}\ \exp\left[-\frac{1}{\theta}(T_x+b)-\frac{1}{\beta_0}(T_y+d)\right]d\theta_0 d\theta$$

$$(2-8) = \int_0^\infty \theta^{-(q+a+1)}\ \exp\left[-\frac{1}{\theta}(T_x+b)\right]\int_0^\infty \beta_0^{-(q\alpha 0+c+1)}\ \exp\left\{-\frac{1}{\beta_0}(T_y+d)\right\}d\theta_0\Big]d\theta$$

Suppose $\theta_0 = 1/\alpha$, then the inner integral of (2−8) as follows ;

$$\int_0^\infty \beta_0^{-(q\alpha 0+c+1)}\ \exp\left[-\frac{1}{\theta_0}(T_y+d)\right]d\theta_0 = \int_0^\infty \alpha^{(q\alpha 0+c+1)}\ \exp\left[-(T_y+d)\right]d\theta$$

$$= \frac{\Gamma(q\alpha_0+c)}{(T_y+d)^{(q\alpha 0+c)}}$$

and suppose $\theta = 1/\alpha$, then the outer integral of (2−8) as

$$\int_0^\infty \theta_0^{-(q+a+1)}\ \exp\left[\{-\frac{1}{\theta}(T_y+b)\}\right]d\theta_0 = \int_0^\infty \beta^{(q+a+1)}\ \exp\left[\{-\beta(T_y+d)\}\right]d\beta$$

$$= \frac{\beta^{q+a)}}{(T_y+d)^{(q+a)}}$$

(Q. E. D.)

LEMMA 3.2 The posterior density function of component steady-state availability for $A-1/1(1+\alpha\delta)$ is given by $h(A \mid q,\ T_x,\ T_y\ ;\ \theta_0)$

$$= \frac{(\alpha_0 T_y/T_x)^{(q\alpha 0+m1-1)}A^{(q\alpha 0+m2-2)}(1-A)^{(q+m1-2)}}{B(q+m_1-1,\ q\alpha_0+m_2-1)[1-A(1-\alpha_0 T_y/T_x)]^{(q+m1+q\alpha 0+m2-2)}}$$

,where B(.) is Beta-function.

PROOF First, according to LEMMA 3.1,

$S(\delta \mid q,\ T_x,\ T_y\ ;\ \alpha_0)$

$$= \int_0^\infty K(\frac{\beta_0}{\delta},\ \beta_0 \mid q,\ T_x,\ T_y\ ;\ \alpha_0)\beta_0\delta^{-2}d\beta_0$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}}{\Gamma(q+a)\Gamma(q\alpha_0+c)}\int_0^\infty (\frac{\beta_0}{\delta})^{-(q+a+1)}\ \beta_0^{-(q\alpha 0c+1)}exp[-\frac{\delta}{\beta_0}(T_x+b)+\frac{1}{\beta_0}(T_y+d)]\beta_0\delta^{-2}d\beta_0$$

$$=\frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}\delta^{(q+a-1)}}{\Gamma(q+a)\cdot\Gamma(q+c)}\int_0^{\infty}\beta_0^{-(q+a+q\alpha 0+c+1)}exp[\frac{1}{\beta_0}\{\delta(T_x+b)+(T_y+d)\}]d\beta_0$$

, whrer $1/\delta=\theta_0$, Suppose $\beta_0=1/\alpha$, then $S(\delta \mid q,\ T_x,\ T_y\ ;\ \alpha_0)$

$$=\frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}\delta^{(q+a-1)}\Gamma(q+a+q\alpha 0+c)}{\Gamma(q+a)\Gamma(q\alpha_0+c)[\delta(T_x+b)+(T_y+d)]^{(q+a+q\alpha 0+c)}}$$

$$=\frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}\delta^{(q+a-1)}}{B(q+a,\ q\alpha_0+c)\ [\delta(T_x+b)+(T_y+d)]^{(q+a+q\alpha 0+c)}}$$

Therefore, posterior distribution of steady-state availability is as follows

$P(A \mid q,\ T_x,\ T_y\ ;\ \alpha_0)$

$$=S[\frac{A^{-1}}{\alpha_0} \mid q,\ T_x,\ T_y\ ;\ \alpha_0]\ (\frac{A^{-2}}{\alpha_0})$$

$$=\frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}[(A^{-1}-1)/\alpha_0]^{(q+a-1)}(A^{-2}/\alpha_0)}{B(q+a,\ q\alpha_0+c)[\delta(T_y+d)(A^{-1}-1)(T_x+b)/\alpha_0]^{(q+a+q\alpha 0+c)}}$$

$$=\frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha 0+c)}[(A^{-1}-1)/\alpha_0]^{(q+a-1)}A^{-2}/\alpha_0}{B(q-a,\ q\alpha_0+c)\{1-A[\alpha_0(R_y+d)/(T_x+b))\}]\}^{(q+a+q\alpha 0+c)}}$$

$$=\frac{[\{\alpha_0(T_y+d)\}/(T_x+b)]^{(q\alpha 0+c)}A^{(q\alpha 0+c-1)}(1-A)^{(q+a-1)}}{B(q+a,\ q\alpha_0+c)[1-A[1-\{\alpha_0(T_yd)/(T_x+b)\}]]^{(q+a+q\alpha 0+c)}}$$

(Q. E. D)

THEOREM 3.1 Under the squarred-error loss function, Bayes estimation steady-state availability A is given by

$$A_2^*=\frac{q\alpha_0+c}{q+a+q\alpha_0+c}{}_2F_1[1,\ q+a\ ;\ q+a+q\alpha_0+1\ ;\ 1-\frac{\alpha_0(T_y+d)}{T_x+b)}]$$

where $0<\frac{T_y+d}{T_x+b}<2$, ${}_2F_1(a,b\ ;\ c\ ;\ t)$ is a confluent hypergeometric function in Gauss form.

PROOF. From LEMMA 3.2, (2−2) (2−3) we have following Bayes estimation of component steady-state availability A ;

$$A_2=\int_0^1 A\ P(A \mid q,\ T_x,\ T_y\ ;\ \alpha_0)DA$$

$$=\frac{[\alpha_0(T_y+d)/(T_x+b)]^{(q\alpha 0+c)}}{B(q+a,\ q\alpha_0+c)}\int_0^1\frac{A^{(q\alpha 0}c)(1-A)^{(q+a+1)}}{[1-A\{1-\alpha_0(T_y+d)/(T_x+b)\}]^{(q+a+q\alpha 0+c)}}dA$$

$$=\frac{[\alpha_0(T_y+d)/(T_x+b)]^{(q\alpha 0+c)}\Gamma(q+a+q\alpha_0+c)\Gamma(q\alpha_0+c+1)\Gamma(q+a)}{\Gamma(q+a)\ \Gamma(q\alpha_0+c)\ \Gamma(q+a+q\alpha_0+c+1)}*K1$$

$$= \frac{[\alpha_0(T_y+d)/(T_x+b)]^{(q\alpha_0+c)}(q\alpha_0+c)}{(q+a+q\alpha_0+c)} \left[\frac{\alpha_0(T_y+d)}{T_x b} \right]^{(-q\alpha_0+c)} \cdot K2$$

$$= \frac{q\alpha_0+c}{q+a+q\alpha_0+c} \cdot K2$$

where $K1 = {}_2F_1[q+a+q\alpha_0+c,\ q\alpha_0+c+1;\ q+a+q\alpha_0+c+1;\ 1-\alpha_0(T_y+d)/(T_x+b)]$,

and $K2 = F[1,\ q+a;\ q+a+q\alpha_0+c+1 - \frac{\alpha_0(T_y+d)}{T_x b}]$

(Q. E. D.)

4. Numerical results

In this section, several numerical estimations were compared with each other. As is seen from Table 4.3, the extimations nearest to true value are considered as those A_1^* and A_2^*, which we reduced from the above sections.

Example

Cycle	Failure times	Repair times
1	725.67	18.34
2	280.04	16.84
3	850.58	13.69
4	845.76	17.83
5	195.10	16.76
6	732.36	15.78
7	528.40	20.96
8	610.12	18.83
9	327.84	19.73
10	310.12	17.84

Table. 4.1

$q=5,\ \theta=500,\ \alpha_0=3,\ \beta_0=5$

(m_1, m_2)	(1, 2)	(3, 5)	(5, 9)	(4, 8)
True A	A_1^*	A_1^*	A_1^*	A_1^*
0.971	0.966	0.863	0.957	0.943

Table. 4.2

$q=5,\ \theta=500,\ \alpha_0=3,\ \beta_0=5$

(a, b), (a, d)	(1, 5), (1, 5)	(2, 10), 3, 7)
True A	A_2^*	A_2^*
0.971	0.985	0.962
(2, 5), (3, 4)	(3, 3), (4, 8)	
A_2^*	A_2^*	

Table. 4.3

True A	A_1^*	A_2^*	MLE
0.971	0.863	0.962	0.957

where, A_1^*(3, 5), A_2^*(2, 10), (3, 7)

References

(1) Brender, D.M. The Prediction and Mearsurement of System availability ; Advanced Application and Technique, IEEE Transaction of Reliability, vol. 17(1968), 138－147.

(2) Erdelyi, A ; Higher Transcendental Function, vol. 1, Mc-Grow Hill, N.Y. 1953.

(3) Gaver, D.P. and Mazumder, M ; Statistical Estimation in a problem of System Reliability, Naval Research Logistic Quarterly, vol. 14, (1967) 473－488.

(4) Nelson, W ; A Statistical Prediction Interval of Availability, IEEE Transaction on Reliability, vol. 19, (1970) 179－182.

(5) Thompson, W.E and Palicio, P.A ; Baysian Confidence Limites for the Availability of System, IEEE transaction of Reliability, vol. 24 (1975) 118－120.

(6) Rupert, G. and Miller Jr. ; Survival Analysis, John Willy and Sons, 1981.