Journal of the Korean
Statistical Society
Vol. 22, No. 1, 1993

Confidence Intervals in Three-Factor Nested Variance Component Model

Kwan-Joong Kang [1]

ABSTRACT

In the three-factor nested variance component model with equal numbers in the cells given by $y_{ijkm} = \mu + A_i + B_{ij} + C_{ijk} + \varepsilon_{ijkm}$, the exact confidence intervals of the variance component of σ_A^2, σ_B^2, σ_C^2, σ_ε^2, $\sigma_A^2/\sigma_\varepsilon^2$, $\sigma_B^2/\sigma_\varepsilon^2$, $\sigma_C^2/\sigma_\varepsilon^2$, σ_A^2/σ_C^2, σ_B^2/σ_C^2 and σ_A^2/σ_B^2 are not found out yet.

In this paper approximate lower and upper confidence intervals are presented.

1. INTRODUCTION

Consider the three-factor nested variance component model given by

$$y_{ijkm} = \mu + A_i + B_{ij} + C_{ijk} + \epsilon_{ijkm}, \tag{1.1}$$

$i = 1,2,...,I, j = 1,2,...,J, k = 1,2,...,K, m = 1,2,...,M.$
The A_i, B_{ij}, C_{ijk} and ϵ_{ijkm} are independent unobservable random variables and $A_i \sim N(0,\sigma_A^2)$, $B_{ij} \sim N(0,\sigma_B^2)$, $C_{ijk} \sim N(0,\sigma_C^2)$, $\epsilon_{ijkm} \sim N(0,\sigma_\epsilon^2)$, μ is

[1] Department of Mathematics, Dong-A University, Pusan, 604-714, Korea.

an unknown parameter, and the y_{ijkm} are observable random variables. An analysis of variance for this model is displayed in Graybill(1976).

The problem of confidence intervals on linear combinations of more than two variances is suggested by Smith(1936). Satterthwaite(1946) studied and expanded the method and the result of his studies has been known as "Satterthwaite procedure."

Lately, the procedure has been widely used and developed by many authors. Especially, Howe(1974), Leiva and Graybill(1986) got a good approximation. The precisions of confidence interval are satisfactory, but it is the result mostly found out in one or two factor model.

The purpose of this paper is to use and expand the approximation in three factor and determine $1-\alpha$ lower and upper confidence intervals on $\sigma_A^2, \sigma_B^2, \sigma_C^2$, σ_ϵ^2 of (1.1) in three-factor. There is no method availiable for setting exact $1-\alpha$ confidence intervals on the ratio of these, so by using this approximate $1-\alpha$ confidence intervals and the precisions will be treated in chapter 2,3 and 4.

2. CONFIDENCE INTERVALS

2.1. The confidence intervals on σ_ϵ^2.

The ANOVA table of (1.1) is as follows;

Source	*D.F.*	*S.S.*	*M.S.*	*E.M.S.*
Total	$IJKM$	$\Sigma\Sigma\Sigma\Sigma y_{ijkm}^2$		
Mean	1	$y^2/IJKM$		
Factor A	n_1	$\Sigma\Sigma\Sigma\Sigma(\bar{y}_{i...}-\bar{y}_{....})^2$	S_1^2	θ_1
B within A	n_2	$\Sigma\Sigma\Sigma\Sigma(\bar{y}_{ij..}-\bar{y}_{i...})^2$	S_2^2	θ_2
C within B, A	n_3	$\Sigma\Sigma\Sigma\Sigma(\bar{y}_{ijk.}-\bar{y}_{ij..})^2$	S_3^2	θ_3
Error	n_4	$\Sigma\Sigma\Sigma\Sigma(y_{ijkm}-\bar{y}_{ijk.})^2$	S_4^2	θ_4

where $n_1 = I-1,\ n_2 = I(J-1),\ n_3 = IJ(K-1),\ n_4 = IJK(M-1)$, $\theta_1 = \sigma_\epsilon^2 + M\sigma_C^2 + KM\sigma_B^2 + JKM\sigma_A^2$, $\theta_2 = \sigma_\epsilon^2 + M\sigma_C^2 + KM\sigma_B^2$, $\theta_3 = \sigma_\epsilon^2 + M\sigma_C^2$, $\theta_4 = \sigma_\epsilon^2$. The upper α probability point of Snedecor's F distribution and chi-square distribution are denoted by F_α and χ_α^2, respectively.

Since $\dfrac{n_4 S_4^2}{\sigma_\epsilon^2} = \chi_{n_4}^2$, $P[\dfrac{S_4^2}{\theta_4} \le \dfrac{\chi_{\alpha;n_4}^2}{n_4} = F_{\alpha:n_4,\infty}] = 1-\alpha$, we get an exact $1-\alpha$

lower(L) and upper(U) confidence limit on σ_ϵ^2 as follows;

$$L = \frac{S_4^2}{F_{\alpha_1;n_4,\infty}}, \quad U = \frac{S_4^2}{F_{1-\alpha_2;n_4,\infty}}, \quad \text{where} \quad \alpha_1 + \alpha_2 = \alpha. \qquad (2.1)$$

2.2. The confidence intervals on σ_C^2.

Since, $\frac{n_3 S_3^2}{\theta_3} = \frac{n_3 S_3^2}{\sigma_\epsilon^2 + M\sigma_C^2} = \chi_{n_3}^2$, $P[\frac{n_3 S_3^2}{\chi_{\alpha_1;n_3}^2} \le \sigma_\epsilon^2 + M\sigma_C^2 \le \frac{n_3 S_3^2}{\chi_{1-\alpha_2;n_3}^2}] = 1 - \alpha$, when $\sigma_\epsilon^2 = 0$, an exact $1-\alpha$ lower(L) and upper(U) confidence limit on σ_C^2 are as follows;

$$L = \frac{S_3^2}{MF_{\alpha_1,n_3,\infty}}, U = \frac{S_3^2}{MF_{1-\alpha_2;n_3,\infty}}, \quad \text{where} \quad \alpha_1 + \alpha_2 = \alpha. \qquad (2.2.1)$$

When $\sigma_\epsilon^2 \ne 0$, we can use the method of Graybill, and thus we get $1-\alpha$ lower(L) and upper(U) confidence limit on σ_C^2 as follows;

$$L = \frac{n_3 S_3^2 - n_3 S_4^2 F_{\alpha_1;n_3,n_4}}{M\chi_{\alpha_1;n_3}^2}, \quad U = \frac{n_3 S_3^2 - n_3 S_4^2 F_{1-\alpha_2;n_3,n_4}}{M\chi_{1-\alpha_2;n_3}^2}, \quad \alpha_1 + \alpha_2 = \alpha. \qquad (2.2.2)$$

On another hand, as in Howe(1974), we can use Cornish-Fisher modification expansion. Now, $1-\alpha$ lower(L) and upper(U) confidence limits on σ_C^2 are as follows;

$$\begin{aligned} L &= \tfrac{1}{M}(S_3^2 - S_4^2 - [(1 - \tfrac{1}{F_{\alpha_1;n_3,\infty}})^2 S_3^4 \\ &\quad +\{(F_{\alpha_1;n_3,n_4} - 1)^2 \\ &\quad -F_{\alpha_1;n_3,n_4}^2(1 - \tfrac{1}{F_{\alpha_1;n_3,n_4}})^2\} S_4^4]^{\frac{1}{2}}), && \text{if } \tfrac{S_3^2}{S_4^2} > F_{\alpha_1;n_3,n_4}, \\ &= 0, && \text{if } \tfrac{S_3^2}{S_4^2} \le F_{\alpha_1;n_3,n_4}, \end{aligned}$$

$$\begin{aligned} U &= \tfrac{1}{M}(S_3^2 - S_4^2 + [(\tfrac{1}{F_{1-\alpha_2;n_3,\infty}} - 1)^2 S_3^4 \\ &\quad +\{(1 - F_{1-\alpha_2;n_3,n_4})^2 \\ &\quad -F_{1-\alpha_2;n_3,n_4}^2(\tfrac{1}{F_{1-\alpha_2;n_3,\infty}} - 1)^2\} S_4^4]^{\frac{1}{2}}, && \text{if } \tfrac{S_3^2}{S_4^2} > F_{1-\alpha_2;n_3,n_4}, \\ &= 0, && \text{if } \tfrac{S_3^2}{S_4^2} \le F_{1-\alpha_2;n_3,n_4}, \end{aligned} \qquad (2.2.3)$$

where $\alpha_1 + \alpha_2 = \alpha$.

2.3. The confidence intervals on σ_B^2.

Since $\frac{n_2 S_2^2}{\theta_2} = \chi^2_{n_2}$, and thus $P[\frac{n_2 S_2^2}{\chi^2_{\alpha_1;n_2}} \le \sigma_\epsilon^2 + M\sigma_C^2 + KM\sigma_B^2 \le \frac{n_2 S_2^2}{\chi^2_{1-\alpha_2;n_2}}] = 1-\alpha$, we get $1-\alpha$ lower(L) and upper(U) confidence limit on σ_B^2 as follows;

$$L = \frac{n_2 S_2^2 - n_2 S_3^2 F_{\alpha_1;n_2,n_3}}{KM\chi^2_{\alpha_1;n_2}},\quad U = \frac{n_2 S_2^2 - n_2 S_3^2 F_{1-\alpha_2;n_2,n_3}}{KM\chi^2_{1-\alpha_2;n_2}},\quad \alpha_1+\alpha_2 = \alpha. \quad (2.3.1)$$

Using Howe's method, $1-\alpha$ lower(L) and upper(U) confidence limits on σ_B^2 are as follows;

$$\begin{aligned} L &= \tfrac{1}{KM}(S_2^2 - S_3^2 - [(1-\tfrac{1}{F_{\alpha_1;n_2,\infty}})^2 S_2^4 \\ &\quad +\{(F_{\alpha_1;n_2,n_3}-1)^2 \\ &\quad -F^2_{\alpha_1;n_2,n_3}(1-\tfrac{1}{F_{\alpha_1;n_2,\infty}})^2\}S_3^4]^{\frac{1}{2}}), && \text{if } \tfrac{S_2^2}{S_3^2} > F_{\alpha_1;n_2,n_3}, \\ &= 0, && \text{if } \tfrac{S_2^2}{S_3^2} \le F_{\alpha_1;n_2,n_3}, \end{aligned}$$

$$\begin{aligned} U &= \tfrac{1}{KM}(S_2^2 - S_3^2 + [(\tfrac{1}{F_{1-\alpha_2;n_2,\infty}}-1)^2 S_2^4 \\ &\quad +\{(1-F_{1-\alpha_2;n_2,n_3})^2 \\ &\quad -F^2_{1-\alpha_2;n_2,n_3}(\tfrac{1}{F_{1-\alpha_2;n_2,\infty}}-1)^2\}S_3^4]^{\frac{1}{2}}), && \text{if } \tfrac{S_2^2}{S_3^2} > F_{1-\alpha_2;n_2,n_3}, \\ &= 0, && \text{if } \tfrac{S_2^2}{S_3^2} \le F_{1-\alpha_2;n_2,n_3}, \end{aligned} \quad (2.3.2)$$

where $\alpha_1 + \alpha_2 = \alpha$.

2.4. The confidence intervals on σ_A^2.

Since $\frac{n_1 S_1^2}{\theta_1} = \chi^2_{n_1}$, and thus

$P[\chi^2_{1-\alpha_2;n_1} \le \frac{n_1 S_1^2}{(\sigma_\epsilon^2 + M\sigma_C^2 + KM\sigma_B^2 + JKM\sigma_A^2)} \le \chi^2_{\alpha_1;n_1}] = 1-\alpha$, we get $1-\alpha$ lower(L) and upper(U) confidence limit on σ_A^2 as follows;

$$L = \frac{n_1 S_1^2 - n_1 S_2^2 F_{\alpha_1;n_1,n_2}}{JKM\chi^2_{\alpha_1;n_1}}, \quad U = \frac{n_1 S_1^2 - n_1 S_2^2 F_{1-\alpha_2;n_1,n_2}}{JKM\chi^2_{1-\alpha_2;n_1}}, \quad \alpha_1 + \alpha_2 = \alpha. \tag{2.4.1}$$

Using Howe's method, $1-\alpha$ lower(L) and upper(U) limit on σ_A^2 are as follows;

$$\begin{aligned} L &= \tfrac{1}{JKM}(S_1^2 - S_2^2 - [(1 - \tfrac{1}{F_{\alpha_1;n_1,\infty}})^2 S_1^4 \\ &\quad +\{(F_{\alpha_1;n_1,n_2} - 1)^2 \\ &\quad -F^2_{\alpha_1;n_1,n_2}(1 - \tfrac{1}{F_{\alpha_1;n_1,\infty}})^2\} S_2^4]^{\frac{1}{2}}), \qquad \text{if } \tfrac{S_1^2}{S_2^2} > F_{\alpha_1;n_1,n_2}, \\ &= 0, \qquad \text{if } \tfrac{S_1^2}{S_2^2} \le F_{\alpha_1;n_1,n_2}, \end{aligned}$$

$$\begin{aligned} U &= \tfrac{1}{JKM}(S_1^2 - S_2^2 + [(\tfrac{1}{F_{1-\alpha_2;n_1,\infty}} - 1)^2 S_1^4 \\ &\quad +\{(1 - F_{1-\alpha_2;n_1,n_2})^2 \\ &\quad -F_{1-\alpha_2;n_1,n_2}(\tfrac{1}{F_{1-\alpha_2;n_1,\infty}} - 1)^2\} S_2^4]^{\frac{1}{2}}), \qquad \text{if } \tfrac{S_1^2}{S_2^2} > F_{1-\alpha_2;n_1,n_2}, \\ &= 0, \qquad \text{if } \tfrac{S_1^2}{S_2^2} \le F_{1-\alpha_2;n_1,n_2}, \end{aligned} \tag{2.4.2}$$

where $\alpha_1 + \alpha_2 = \alpha$.

3. The CONFIDENCE INTERVALS ON THE RATIO OF VARIANCES

3.1 Confidence Intervals on $\sigma_A^2/\sigma_\epsilon^2$.

Since $\overline{y}....$, S_1^2, S_2^2, S_3^2 and S_4^2 are complete sufficient statistics for this problem, the upper confidence interval (see Wang,Graybill(1981)) should be a function of them. Using the ANOVA table, $(\theta_1 - \theta_2)/\theta_4 = JKM\sigma_A^2/\sigma_\epsilon^2$, and thus an upper confidence interval on $(\theta_1 - \theta_2)/\theta_4$ is equivalent to an upper confidence interval on $\sigma_A^2/\sigma_\epsilon^2$. Let $\widehat{\theta} = (S_1^2 - S_2^2)/S_4^2$, then the mean and variance of $\widehat{\theta}$ are as follows;

$$
\begin{aligned}
E(\hat{\theta}) &= E\left(\frac{S_1^2 - S_2^2}{S_4^2}\right) = \frac{n_4(\theta_1 - \theta_2)}{(n_4-2)\theta_4}, \\
Var(\hat{\theta}) &= Var\left(\frac{S_1^2 - S_2^2}{S_4^2}\right) = \frac{2n_4^2\theta_1^2}{n_1(n_4-2)^2\theta_4^2} + \frac{2n_4^2\theta_2^2}{n_2(n_4-2)^2\theta_4^2} + \frac{2n_4^2}{(n_4-4)(n_4-2)^2}\left(\frac{\theta_1}{\theta_4} - \frac{\theta_2}{\theta_4}\right)^2 \\
&\quad + \frac{4n_4^2\theta_1^2}{n_1(n_4-4)(n_4-2)^2\theta_4^2} + \frac{4n_4^2\theta_2^2}{n_2(n_4-4)(n_4-2)^2\theta_4^2}.
\end{aligned}
$$

If we replace θ_i by its UMVU estimators and denote the resulting $Var(\hat{\theta})$ by $\widehat{Var}(\hat{\theta})$,

$$
\widehat{Var}(\hat{\theta}) = c_1\frac{S_1^4}{S_4^4} + c_2\frac{S_2^4}{S_4^4} + \left(c_3\frac{S_1^2}{S_4^2} - c_4\frac{S_2^2}{S_4^2}\right)^2,
$$

where c_1, c_2, c_3 and c_4 are appropriate constants which are function of n_1, n_2 and n_4. So a large sample lower confidence point of $(\theta_1 - \theta_2)/\theta_4$ is

$$
\begin{aligned}
\hat{\theta} - N_\alpha\{\widehat{Var}(\hat{\theta})\}^{\frac{1}{2}} &= \frac{S_1^2 - S_2^2}{S_4^2} - N_\alpha\left\{c_1\frac{S_1^4}{S_4^4} + c_2\frac{S_2^4}{S_4^4} + \left(c_3\frac{S_1^2}{S_4^2} - c_4\frac{S_2^2}{S_4^2}\right)^2\right\}^{\frac{1}{2}} \\
&= \frac{S_2^2}{S_4^2}\left[\frac{S_1^2}{S_2^2} - 1 - N_\alpha\left\{c_1\left(\frac{S_1^2}{S_2^2}\right)^2 + c_2 + \left(c_3\frac{S_1^2}{S_2^2} - c_4\right)^2\right\}^{\frac{1}{2}}\right] \\
&= \frac{S_2^2}{S_4^2}q\left(\frac{S_1^2}{S_2^2}\right) = h\left(\frac{S_1^2}{S_2^2}, \frac{S_4^2}{S_2^2}\right).
\end{aligned}
$$

Therefore in general we require the lower confidence point $h(\frac{S_1^2}{S_2^2}, \frac{S_4^2}{S_2^2})$ of $\frac{\theta_1 - \theta_2}{\theta_4}$ to be of the form $\frac{S_2^2}{S_4^2}q(\frac{S_1^2}{S_2^2})$, where $q(.)$ is the function to be determined.

(a) When the hypothesis $H_0 : \sigma_A^2 = 0$ vs $H_1 : \sigma_A^2 > 0$ is accepted for a size α test the confidence interval should include zero, and when H_0 is rejected, $h(\frac{S_1^2}{S_2^2}, \frac{S_4^2}{S_2^2})$ should be an increasing function of $\frac{S_1^2}{S_2^2}$. To test $H_0 : \sigma_A^2 = 0$ vs $H_1 : \sigma_A^2 > 0$ the hypothesis H_0 is accepted if and only if $\frac{S_1^2}{S_2^2} < F_{\alpha;n_1,n_2}$. This test is uniformly most powerful unbiased. Thus $h(\frac{S_1^2}{S_2^2}, \frac{S_4^2}{S_2^2}) = 0$ when $\frac{S_1^2}{S_2^2} \le F_{\alpha;n_1,n_2}$. $h(\frac{S_1^2}{S_2^2}, \frac{S_4^2}{S_2^2}) > 0$ and increasing in $\frac{S_1^2}{S_2^2}$ when $\frac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}$. And using the results of $\hat{\theta}, q(\frac{S_1^2}{S_2^2}) = 0$ when $\frac{S_1^2}{S_2^2} \le F_{\alpha;n_1,n_2}$. $q(\frac{S_1^2}{S_2^2}) > 0$ and increasing in $\frac{S_1^2}{S_2^2}$ when $\frac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}$.

(b) When $J \to \infty$, then $n_2, n_4 \to \infty$, and if $n_2, n_4 \to \infty$, then $S_2^2 \to \theta_2$ and $S_4^2 \to \theta_4$ in probability. Therfore

$$P[\frac{S_1^2\theta_4}{S_4^2\theta_1} \leq F_{\alpha;n_1,n_4}] = 1-\alpha, \qquad P[\frac{S_1^2}{\theta_1} \leq F_{\alpha;n_1,\infty}] = P[\frac{S_1^2}{F_{\alpha;n_1,\infty}} \leq \theta_1] = 1-\alpha,$$

and we get $P[\frac{S_2^2}{S_4^2}(\frac{S_1^2}{S_2^2 F_{\alpha;n_1,\infty}} - 1) \leq \frac{\theta_1-\theta_2}{\theta_4}] = 1-\alpha$. In case of $J \to \infty$,

$$q(\frac{S_1^2}{S_2^2}) = \frac{S_1^2}{S_3^2 F_{\alpha;n_1,\infty}} \text{ if } \frac{S_1^2}{S_2^2} \leq F_{\alpha;n_1,\infty}, \quad q(\frac{S_1^2}{S_2^2}) = 0, \text{ if } \frac{S_1^2}{S_2^2} > F_{\alpha;n_1,\infty}.$$

(c) If $\sigma_A^2 \to \infty (S_1^2 \to \theta_2)$ then $P[\frac{S_2^2}{S_4^2}(\frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_4}}) \leq \frac{\theta_1}{\theta_4}] = 1-\alpha$, and exist l such that

$$q(\frac{S_1^2}{S_2^2}) = \frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_4}}\{1 + l(\frac{S_1^2}{S_2^2})\}, \quad \text{where} \quad l(\frac{S_1^2}{S_2^2}) \to \infty \quad \text{as} \quad \frac{S_1^2}{S_2^2} \to \infty.$$

Any function $q(\frac{S_1^2}{S_2^2})$ satisfying these conditions will give an exact coefficient in the three limiting cases $\frac{\theta_1}{\theta_2} = 1, \frac{\theta_1}{\theta_2} \to \infty$ and $J \to \infty$.

Thus the simplest function satisfying those conditions is the linear function; $q(\frac{S_1^2}{S_2^2}) = a_1 \frac{S_1^2}{S_2^2} + b_1$.

From the condition (c), $a_1 = 1/F_{\alpha;n_1,n_4}$. And from condition (a),

$$q(\frac{S_1^2}{S_2^2}) = q(F_{\alpha;n_1,n_2}) = \frac{F_{\alpha;n_1,n_2}}{F_{\alpha;n_1,n_4}} + b_1 = 0 \quad \text{and } b_1 = -\frac{F_{\alpha;n_1,n_2}}{F_{\alpha;n_1,n_4}}.$$

Thus, the $1-\alpha$ lower(L) confidence limit on $\sigma_A^2/\sigma_\epsilon^2$ is as follows:

$$\begin{aligned} L &= \frac{S_2^2}{JKMS_4^2}(\frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_2}} - \frac{F_{\alpha;n_1,n_2}}{F_{\alpha;n_1,n_4}}), \quad \text{if} \quad \frac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}, \\ &= 0, \qquad \text{if} \quad \frac{S_1^2}{S_2^2} \leq F_{\alpha;n_1,n_2}. \end{aligned} \tag{3.1.1}$$

Note that $L = 0$ if and only if the α test of $H_0 : \sigma_A^2 = 0$ is accepted. Also

$$P[L=0] = P[\frac{S_1^2}{S_2^2} \leq F_{\alpha;n_1,n_2}] \leq 1-\alpha, \quad P[L=0] = 1-\alpha \quad \text{if and only if } \sigma_A^2 = 0.$$

$$P[\frac{\theta_1-\theta_2}{\theta_4} \le f(S_1^2,S_2^2,S_4^2)] = 1 - P[f(S_1^2,S_2^2,S_4^2) \le \frac{\theta_1-\theta_2}{\theta_4}],$$

therefore the upper limit of the lower confidence interval can be obtained by the use of the confidence coefficient α in the lower limit of the upper confidence interval discussed in 3.1.1. First consider

$$q(\frac{S_1^2}{S_2^2}) = \frac{S_1^2}{S_2^2 F_{1-\alpha;n_1,n_4}} - \frac{F_{1-\alpha;n_1,n_2}}{F_{1-\alpha;n_1,n_4}}.$$

From this $q(.)$, the $1-\alpha$ upper(U) confidence limit on $\sigma_A^2/\sigma_\epsilon^2$ is as follows;

$$\begin{aligned} U &= \frac{S_2^2}{JKMS_4^2}(\frac{S_1^2}{S_2^2 F_{1-\alpha;n_1,n_4}} - \frac{F_{1-\alpha;n_1,n_2}}{F_{1-\alpha;n_1,n_4}}), && \text{if } \frac{S_1^2}{S_2^2} > F_{1-\alpha;n_1,n_2}, \\ &= 0, && \text{if } \frac{S_1^2}{S_2^2} \le F_{1-\alpha;n_1,n_2}. \end{aligned} \tag{3.1.2}$$

Note that $U = 0$ if and only if the $1-\alpha$ level test of $H_0 : \sigma_A^2 = 0$ is accepted. Also $P[U = 0] \le \alpha$ and $P[U = 0] = \alpha$.

On another hand, now let $q(\frac{S_1^2}{S_2^2}) = \frac{1}{F_{\alpha;n_1,n_4}}\{a_1(\frac{S_1^2}{S_2^2}) + b_1 + c_1\frac{S_2^2}{S_1^2}\}$.

From condition (c) of 3.1.1, $a_1 = 1$. And from condition (b) of 3.1.1,

$$b_1 = b_1(n_1,\infty,\infty) = -F_{\alpha;n_1,\infty}, \quad c_1 = c_1(n_1,\infty,\infty) = 0.$$

From condition (a) of 3.1.1, $c_1 = -F_{\alpha;n_1,n_2}(F_{\alpha;n_1,n_2} + b_1)$. Let $b_1(n_1,n_2,n_4) = -F_{\alpha;n_1,\infty}$ for all n_2 and n_4, then $c_1 = F_{\alpha;n_1,n_2}(F_{\alpha;n_1,\infty} - F_{\alpha;n_1,n_2})$,

i.e. $q(\frac{S_1^2}{S_2^2}) = \frac{1}{F_{\alpha;n_1,n_3}}[\frac{S_1^2}{S_2^2} - F_{\alpha;n_1,\infty} + F_{\alpha;n_1,n_2}(F_{\alpha;n_1,\infty} - F_{\alpha;n_1,n_2})\frac{S_2^2}{S_1^2}]$.

Thus a $1-\alpha$ lower(L) confidence limit on $\sigma_A^2/\sigma_\epsilon^2$ is as follows;

$$\begin{aligned} L &= \frac{S_2^2}{JKMS_4^2 F_{\alpha;n_1,n_4}}[\frac{S_1^2}{S_2^2} - F_{\alpha;n_1,\infty} \\ &\quad + F_{\alpha;n_1,n_2}(F_{\alpha;n_1,\infty} - F_{\alpha;n_1,n_2})\frac{S_2^2}{S_1^2}], && \text{if} \frac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}, \\ &= 0, && \text{if} \frac{S_1^2}{S_2^2} \le F_{\alpha;n_1,n_2}. \end{aligned} \tag{3.1.3}$$

In the lower confidence interval on $\sigma_A^2/\sigma_\epsilon^2$(3.1.3), replace α with $1-\alpha$, the $1-\alpha$

upper(U) confidence limit on $\sigma_A^2/\sigma_\epsilon^2$ is as follows;

$$\begin{aligned} U &= \frac{S_2^2}{JKMS_4^2 F_{1-\alpha;n_1,n_4}} \\ &\quad \cdot[\frac{S_1^2}{S_2^2} - F_{1-\alpha;n_1,\infty} \\ &\quad +F_{1-\alpha;n_1,n_2}(F_{1-\alpha;n_1,\infty} - F_{1-\alpha;n_1,n_2})\frac{S_2^2}{S_1^2}], \quad \text{if} \quad \frac{S_1^2}{S_2^2} > F_{1-\alpha;n_1,n_2}, \\ &= 0, \quad \text{if} \quad \frac{S_1^2}{S_2^2} \leq F_{1-\alpha;n_1,n_2}. \end{aligned} \tag{3.1.4}$$

3.2. The confidence intervals on $\sigma_B^2/\sigma_\epsilon^2$.

From $(\theta_2-\theta_3)/\theta_4 = KM\sigma_B^2/\sigma_\epsilon^2$, similar to that of (3.1.3), the $1-\alpha$ lower(L) and upper(U) confidence limit are as follows;

$$\begin{aligned} L &= \frac{S_3^2}{S_4^2 F_{\alpha_1;n_2,n_4}}[\frac{S_2^2}{S_3^2} - F_{\alpha_1;n_2,\infty} \\ &\quad +F_{\alpha_1;n_2,n_3}(F_{\alpha_1;n_2,\infty} - F_{\alpha_1;n_2,n_3})\frac{S_3^2}{S_2^2}], \quad \text{if} \quad \frac{S_2^2}{S_3^2} > F_{\alpha_1;n_2,n_3}, \\ &= 0, \quad \text{if} \quad \frac{S_2^2}{S_3^2} \leq F_{\alpha_1;n_2,n_3}, \end{aligned}$$

$$\begin{aligned} U &= \frac{S_3^2}{S_4^2 F_{1-\alpha_2;n_2,n_4}}[\frac{S_2^2}{S_3^2} - F_{1-\alpha_2;n_2,\infty} \\ &\quad +F_{1-\alpha_2;n_2,n_3}(F_{1-\alpha_2;n_2,\infty} - F_{1-\alpha_2;n_2,n_3})\frac{S_3^2}{S_2^2}], \quad \text{if} \quad \frac{S_2^2}{S_3^2} > F_{1-\alpha_2;n_2,n_3}, \\ &= 0, \quad \text{if} \quad \frac{S_2^2}{S_3^2} \leq F_{1-\alpha_2;n_2,n_3}, \end{aligned} \tag{3.2}$$

where $\alpha_1 + \alpha_2 = \alpha$.

3.3. The confidence intervals on $\sigma_C^2/\sigma_\epsilon^2$.

Since $\dfrac{\theta_3}{\theta_4} = \dfrac{\sigma_\epsilon + M\sigma_C^2}{\sigma_\epsilon^2} = 1 + \dfrac{M\sigma_C^2}{\sigma_\epsilon^2}$, and thus $P[\dfrac{S_3^2/\theta_3}{S_4^2/\theta_4} \leq F_{\alpha;n_3,n_4}] = 1-\alpha$, we get

$$P[\frac{S_3^2}{S_4^2 F_{\alpha;n_3,n_4}} \leq 1 + \frac{M\sigma_C^2}{\sigma_\epsilon^2}] = P[\frac{1}{M}(\frac{S_3^2}{S_4^2 F_{\alpha;n_3,n_4}} - 1) \leq \frac{\sigma_C^2}{\sigma_\epsilon^2}] = 1-\alpha.$$

$P[F_{1-\alpha;n_3,n_4} \leq \dfrac{S_3^2/\theta_3}{S_4^2/\theta_4}] = 1-\alpha$, and thus $P[\frac{\theta_3}{\theta_4} \leq \frac{S_3^2}{S_4^2 F_{1-\alpha;n_3,n_4}}] = 1-\alpha$, we get

$$P[\frac{\sigma_C^2}{\sigma_\epsilon^2} \le \frac{1}{M}(\frac{S_3^2}{S_4^2 F_{1-\alpha;n_3,n_4}} - 1)] = 1 - \alpha.$$

Therefore, the $1-\alpha$ lower(L) and upper(U) confidence limits on $\sigma_C^2/\sigma_\epsilon^2$ are as follows;

$$\begin{aligned} L &= \frac{S_3^2}{MS_4^2 F_{\alpha_1;n_3,n_4}} - 1, \quad \text{if } \frac{S_3^2}{S_4^2} > F_{\alpha_1;n_3,n_4}, \\ &= 0, \quad \text{if } \frac{S_3^2}{S_4^2} \le F_{\alpha_1;n_3,n_4}, \\ U &= \frac{S_3^2}{MS_4^2 F_{1-\alpha_2;n_3,n_4}} - 1, \quad \text{if } \frac{S_3^2}{S_4^2} > F_{1-\alpha_2;n_3,n_4}, \\ &= 0, \quad \text{if } \frac{S_3^2}{S_4^2} \le F_{1-\alpha_2;n_3,n_4}, \end{aligned} \tag{3.3}$$

where $\alpha_1 + \alpha_2 = \alpha$.

3.4. The confidence intervals on σ_A^2/σ_B^2.

From the ANOVA table $\frac{\theta_1-\theta_2}{\theta_2-\theta_3} = \frac{J\sigma_A^2}{\sigma_B^2}$, and thus a confidence interval on $\frac{\theta_1-\theta_2}{\theta_2-\theta_3}$ is equvalent to a confidence interval on $\frac{\sigma_A^2}{\sigma_B^2}$. The lower confidence is a function of S_1^2, S_2^2 and S_3^2. Thus, we say $f(S_1^2, S_2^2, S_3^2)$ such that $P[f(S_1^2, S_2^2, S_3^2) \le \frac{\theta_1-\theta_2}{\theta_2-\theta_3}]$ is approximately a specified confidence coefficient $1-\alpha$. This probability is a function of $\sigma_A^2/\sigma_C^2, \sigma_B^2/\sigma_C^2$ two unknown parameters which vary from zero to infinity. Let $f(S_1^2, S_2^2., S_3^2)$ be $q(S_1^2/S_2^2, S_3^2/S_2^2)$. To determine the function of $q(S_1^2/S_2^2, S_3^2/S_2^2)$ the following conditions are imposed.

Since the MLE of $\frac{\theta_1-\theta_2}{\theta_2-\theta_3} = \frac{\theta_1/\theta_2 - 1}{1-\theta_3/\theta_2}$ is of the form $\frac{S_1^2/S_2^2 - 1}{1 - S_3^2/S_2^2}$, we require $q(..)$ to be monotone increasing function of $\frac{S_1^2}{S_2^2}$ and $\frac{S_3^2}{S_2^2}$, respectively. From $\frac{\theta_1-\theta_2}{\theta_2-\theta_3} = \frac{J\sigma_A^2}{\sigma_B^2} = \frac{\theta_1/\theta_2 - 1}{1-\theta_3/\theta_2}$, if $\theta_3 = 0$, then $\frac{\theta_1-\theta_2}{\theta_2-\theta_3} = \frac{\theta_1}{\theta_2} - 1$ and $P[\frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_2}} - 1 \le \frac{\theta_1}{\theta_2} - 1] = P[\frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_2}} - 1 \le \frac{\theta_1-\theta_2}{\theta_2-\theta_3}] = 1-\alpha$. Thus the lower(L) limit of the upper confidence interval on σ_A^2/σ_B^2 is $L = \frac{1}{J}(\frac{S_1^2}{S_2^2 F_{\alpha;n_1,n_2}} - 1)$.

When $J \to \infty$, it follows that $S_2^2 \to \theta_2$ and $S_3^2 \to \theta_3$ in probability, and also $(S_2^2 - S_3^2) \to (\theta_2 - \theta_3)$ in probability. From $P[\frac{S_1^2}{F_{\alpha;n_1,\infty}} \leq \theta_1] = 1 - \alpha$, we get

$$P[\frac{S_1^2}{F_{\alpha;n_1,\infty}} - S_2^2 \leq \theta_1 - \theta_2] = P[\frac{S_1^2 - S_2^2 F_{\alpha;n_1,\infty}}{F_{\alpha;n_1,\infty}} \leq \theta_1 - \theta_2] = 1 - \alpha.$$

Next dividing the left and right hand sides respectively by the equivalent expressions $S_2^2 - S_3^2$ and $\theta_2 - \theta_3$ (we assume that $\theta_2 > \theta_3, S_2^2 > S_3^2$) and we obtain

$$P[\frac{S_1^2 - S_2^2 F_{\alpha;n_1,\infty}}{(S_2^2 - S_3^2) F_{\alpha;n_1,\infty}} \leq \frac{\theta_1 - \theta_2}{\theta_2 - \theta_3}] = 1 - \alpha.$$

If $\sigma_B^2 \neq 0$, and if the hypothesis $H_0 : \sigma_A^2 = 0$ is accepted for a size α test, the confidence interval on $\frac{\theta_1 - \theta_2}{\theta_2 - \theta_3}$ should include zero.

For testing $H_0 : \sigma_A^2 = 0$ vs $H_a : \sigma_A^2 > 0$ the hypothesis H_0 is accepted if and only if $\frac{S_1^2}{S_2^2} < F_{\alpha;n_1,n_2}$. Thus we require $q(\frac{S_1^2}{S_2^2}, \frac{S_3^2}{S_2^2}) = 0$ when $\frac{S_1^2}{S_2^2} \leq F_{\alpha;n_1,n_2}$. $q(\frac{S_1^2}{S_2^2}, \frac{S_3^2}{S_2^2}) > 0$ when $\frac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}$.

The simplest function satisfying those conditions is a function given by

$$\begin{aligned} q(\tfrac{S_1^2}{S_2^2}, \tfrac{S_3^2}{S_2^2}) &= \frac{\frac{S_1^2}{S_2^2} - F_{\alpha;n_1,n_2}}{(1 - \frac{S_3^2 F_{1-\alpha;n_2,n_3}}{S_2^2}) F_{\alpha;n_1,n_2}}, && \text{if } \tfrac{S_2^2}{S_3^2} > F_{1-\alpha;n_2,n_3}, \tfrac{S_1^2}{S_2^2} > F_{\alpha;n_1,n_2}, \\ &= 0, && \text{if } \tfrac{S_2^2}{S_3^2} > F_{1-\alpha;n_2,n_3}, \tfrac{S_1^2}{S_2^2} \leq F_{\alpha;n_1,n_2}. \end{aligned}$$

If $\frac{S_2^2}{S_3^2} \leq F_{1-\alpha;n_2,n_3}$, we assume that no confidence interval exists for $\frac{\sigma_A^2}{\sigma_B^2}$, i.e. the $1 - \alpha$ level test of $H_0; \sigma_B^2 = 0$ is accepted. From this a $1 - \alpha$ upper confidence interval on $\frac{\sigma_A^2}{\sigma_B^2}$ is $P[L \leq \frac{\sigma_A^2}{\sigma_B^2}]$, where

$$\begin{aligned} L &= \frac{\frac{S_1^2}{S_2^2} - F_{\alpha_1;n_1,n_2}}{JF_{\alpha_1;n_1,n_2}(1 - \frac{S_3^2 F_{1-\alpha_2;n_2,n_3}}{S_2^2})}, && \text{if } \frac{S_2^2}{S_3^2} > F_{1-\alpha_2;n_2,n_3}, \frac{S_1^2}{S_2^2} > F_{\alpha_1;n_1,n_2}, \\ &= 0, && \text{if } \frac{S_2^2}{S_3^2} > F_{1-\alpha_2;n_2,n_3}, \frac{S_1^2}{S_2^2} \le F_{\alpha_1;n_1,n_2}, \end{aligned} \tag{3.4.1}$$

where $\alpha_1 + \alpha_2 = \alpha$.

If $\frac{S_2^2}{S_3^2} \le F_{1-\alpha_2;n_2,n_3}$, then L does not exist.

Further the upper(U) limit of the lower confidence interval on σ_A^2/σ_B^2 is obtained from the expression L in (3.4.1) by replacing α with $1-\alpha$ and $1-\alpha$ with α in the tabulated $F's$. That is, $P[\sigma_A^2/\sigma_B^2 \le U]$, where

$$\begin{aligned} U &= \frac{\frac{S_1^2}{S_2^2} - F_{1-\alpha_2;n_1,n_2}}{JF_{1-\alpha_2;n_1,n_2}(1 - \frac{S_3^2 F_{\alpha_1;n_2,n_3}}{S_2^2})}, && \text{if } \frac{S_2^2}{S_3^2} > F_{\alpha_1;n_2,n_3}, \frac{S_1^2}{S_2^2} > F_{1-\alpha_2;n_1,n_2}, \\ &= 0, && \text{if } \frac{S_2^2}{S_3^2} > F_{\alpha_1;n_2,n_3}, \frac{S_1^2}{S_2^2} \le F_{1-\alpha_2;n_1,n_2}, \end{aligned} \tag{3.4.2}$$

where $\alpha_1 + \alpha_2 = \alpha$.

When $J \to \infty, \sigma_C^2 = 0, U$ is exactly equal to $1-\alpha$. If $\frac{S_2^2}{S_3^2} \le F_{\alpha_1;n_2,n_3}$, then U do not exist.

3.5. The confidence intervals on σ_B^2/σ_C^2.

Since $(\theta_2 - \theta_3)/(\theta_3 - \theta_4) = K\sigma_B^2/\sigma_C^2$, similar to that of chapter 3.4, the $1-\alpha$ lower(L) and upper(U) confidence limits on σ_B^2/σ_C^2 is as follows;

$$\begin{aligned} L &= \frac{\frac{S_2^2}{S_3^2} - F_{\alpha_1;n_2,n_3}}{KF_{\alpha_1;n_2,n_3}(1 - \frac{S_4^2}{S_3^2} F_{1-\alpha_2;n_3,n_4})}, && \text{if } \frac{S_3^2}{S_4^2} > F_{1-\alpha_2;n_3,n_4}, \frac{S_2^2}{S_3^2} > F_{\alpha_1;n_2,n_3}, \\ &= 0, && \text{if } \frac{S_3^2}{S_4^2} > F_{1-\alpha_2;n_3,n_4}, \frac{S_2^2}{S_3^2} \le F_{\alpha_1;n_2,n_3}, \end{aligned} \tag{3.5.1}$$

$$
\begin{aligned}
U &= \frac{\frac{S_2^2}{S_3^2} - F_{1-\alpha_2;n_2,n_3}}{KF_{1-\alpha_2;n_2,n_3}(1-\frac{S_4^2}{S_3^2}F_{\alpha_1;n_3,n_4})}, && \text{if} \quad \frac{S_3^2}{S_4^2} > F_{\alpha_1;n_3,n_4}, \frac{S_2^2}{S_3^2} > F_{1-\alpha_2;n_2,n_3}, \\
&= 0, && \text{if} \quad \frac{S_3^2}{S_4^2} > F_{\alpha_1;n_3,n_4}, \frac{S_2^2}{S_3^2} \le F_{1-\alpha_2;n_2,n_3},
\end{aligned}
\tag{3.5.2}
$$

where $\alpha_1 + \alpha_2 = \alpha$.

If $\frac{S_3^2}{S_4^2} \le F_{1-\alpha_2;n_3,n_4}$ and $\frac{S_3^2}{S_4^2} \le F_{\alpha_1;n_3,n_4}$, then both L and U do not exists.

3.6. The confidence interval on σ_A^2/σ_C^2.

Since $\frac{\theta_1-\theta_2}{\theta_4} = \frac{JKM\sigma_A^2}{\sigma_\epsilon^2}$ and $\frac{\theta_3-\theta_4}{\theta_4} = \frac{M\sigma_C^2}{\sigma_\epsilon^2}$, we get

$$P[\frac{S_1^2}{S_4^2F_{\alpha;n_1,n_4}} \le \frac{\theta_1}{\theta_4}] = P[\frac{-S_2^2}{S_4^2F_{1-\alpha;n_2,n_4}} \le -\frac{\theta_2}{\theta_4}] = 1-\alpha,$$

$$P[\frac{\theta_3}{\theta_4} - 1 \le \frac{S_3^2}{S_4^2F_{1-\alpha;n_3,n_4}} - 1] = P[\frac{1}{\frac{S_3^2}{S_4^2F_{1-\alpha;n_3,n_4}}-1} \le \frac{1}{\frac{\theta_3-\theta_4}{\theta_4}}] = 1-\alpha.$$

The lower limit of the upper confidence interval on $\frac{\theta_1-\theta_2}{\theta_3-\theta_4} = \frac{JK\sigma_A^2}{\sigma_C^2}$ is

$$P[\frac{\frac{S_1^2}{S_4^2F_{\alpha;n_1,n_4}} - \frac{S_2^2}{S_4^2F_{1-\alpha;n_2,n_4}}}{\frac{S_3^2}{S_4^2F_{1-\alpha;n_3,n_4}} - 1} \le \frac{\theta_1-\theta_2}{\theta_3-\theta_4}] = 1-\alpha, \frac{S_3^2}{S_4^2} > F_{1-\alpha;n_3,n_4}.$$

Similarly, the upper limit of the lower confidence interval on $\frac{JK\sigma_A^2}{\sigma_C^2}$ is

$$P[\frac{\theta_1-\theta_2}{\theta_3-\theta_4} \le \frac{\frac{S_1^2}{S_4^2F_{1-\alpha;n_1,n_4}} - \frac{S_2^2}{S_4^2F_{\alpha;n_2,n_4}}}{\frac{S_3^2}{S_4^2F_{\alpha;n_3,n_4}} - 1}] = 1-\alpha.$$

Therefore, $1-\alpha$ lower(L) and upper(U) confidence limit on σ_A^2/σ_C^2 are as follows;

$$L = \frac{\frac{S_1^2}{S_4^2F_{\alpha_1;n_1,n_4}} - \frac{S_2^2}{S_4^2F_{1-\alpha_2;n_2,n_4}}}{JK(\frac{S_3^2}{S_4^2F_{1-\alpha_2;n_3,n_4}}-1)}, \qquad \text{if} \quad \frac{S_3^2}{S_4^2} > F_{1-\alpha_2;n_3,n_4}, \tag{3.6.1}$$

$$U = \frac{\frac{S_1^2}{S_4^2 F_{1-\alpha_2;n_1,n_4}} - \frac{S_2^2}{S_4^2 F_{\alpha_1;n_2,n_4}}}{JK(\frac{S_3^2}{S_4^2 F_{\alpha_1;n_3,n_4}} - 1)}, \quad \text{if } \frac{S_3^2}{S_4^2} > F_{\alpha_1;n_3,n_4}. \tag{3.6.2}$$

If $\frac{S_3^2}{S_4^2} \le F_{1-\alpha_2;n_3,n_4}, \frac{S_3^2}{S_4^2} \le F_{\alpha_1;n_3,n_4}, \alpha_1 + \alpha_2 = \alpha$, then L and U do not exist.

4. DISCUSSION

In this chapter, I would like to discuss these precisions.

i) The confidence interval (2.1) on σ_ϵ^2 and the confidence interval (2.2.1) on σ_C^2 are quite accurate.
ii) The method of Graybill approximation was used in finding out the confidence interval (2.2.2) on σ_C^2 , the confidence interval (2.3.1) on σ_B^2 and the confidence interval (2.4.1) on σ_A^2. The precisions of his confidence intervals are not so good. Therefore the precisions of the above-mentioned intervals will be similar to those of Graybill.
iii) The method of Howe approximation was used in finding out the confidence interval (2.2.3) on σ_C^2, the confidence interval (2.3.2) on σ_B^2 and the confidence interval (2.4.2) on σ_A^2. When $1-\alpha = 0.95$, the precisions of the confidence intervals L and U lie between 0.948-0.950 and 0.939-0.950 respectively. Accordingly the precisions of the above-mentioned intervals will be similer to those of his. The confidence intervals of iii) are better than those of ii).
iv) Tukey-Williams approximation was used in finding out the confidence interval (3.1.2) on $\sigma_A^2/\sigma_\epsilon^2$. When $1-\alpha = 0.90$ and 0.95, the precisions of the confidence intervals lie between 0.9000-0.9241 and 0.9500-0.9681 respectively. So, the precisions of the above mentioned intervals will be similar to those of his.
v) Wang and Graybill approximation was used in finding out the confidence interval (3.1.3) and (3.1.4) on $\sigma_A^2/\sigma_\epsilon^2$. When $1-\alpha = 0.90$ and 0.95, the precisions of the confidence intervals lie between 0.9000-0.9115 and 0.950-0.959 respectively. So, the precisions of the above mentioned intervals will be similar to those of his. The confidence intervals of v) is better than those of iv).
vi) Wang and Graybill approximation was used in finding out the confidence interval (3.2) on $\sigma_B^2/\sigma_\epsilon^2$, the confidence interval (3.3) on $\sigma_B^2/\sigma_\epsilon^2$, the confidence interval (3.4.1),(3.4.2) on σ_A^2/σ_B^2, the confidence interval (3.5.1),(3.5.2)

on σ_B^2/σ_C^2 and the confidence interval (3.6.1), (3.6.2) on σ_A^2/σ_C^2. So, the precisions of the above mentioned intervals are similar to those of v).

REFERENCES

(1) Arteage,C.,Jeyaratnam,S. and Graybill,F.A.(1982). Confidence intervals for proportions of total variance in the two-way cross component of variance model. *Communication in Statistics Theory and Methods*, 11, 1643-1658.

(2) Burdik,R.K.and Graybill,F.A.(1984). Confidence intervals on linear combinations of variance components in the unbalanced one-way classification. *Technometrics*, 26, 131-136.

(3) Burdik,R.K. and Graybill,F.A.(1985). Confidence intervals on the total variance in an unbalanced two-fold nested classification with equal subsampling. *Communication in Statistics Theory and Methods*, 14(4), 761-774.

(4) Graybill,F.A.(1976). *Theory and Application of the Linear Model*, North Scituate, Mass, Duxbury Press.

(5) Howe,W.G.(1974). Approximate confidence limits on the mean of X+Y where X and Y are two tabled independent random variables. *Journal of the American Statistical Association*, 69, 789-794.

(6) Leiva,Ricardo A. and Graybill, F.A.(1986). Confidence intervals for variance components in the balanced two way model with interaction. *Communication in Statistics, Part B - Simulation and Computation*, 15(2), 301-322.

(7) Satterthwaite,F.E.(1946). An approximate distribution of estimates of variance components. *Biometrics Bulletin*, 2, 110-114.

(8) Smith,H.Fairfield(1936). The problem of comparing the result of two experiments with unequal errors. *Journal of Council of Scientific and Industrial Research*, 9, 211-212.

(9) Wang,C. and Graybill, F.A.(1981). Confidence intervals on a ratio of variances in the two-factor nested components of variance model. *Communication in Statistics Theory and Methods*, A, 10(14), 1357-1368.

(10) Williams,J.S.(1962). A confidence interval for variance components. *Biometrika* , 49, 278-281.