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A Goodness-of-Fit Test for Exponentiality with Censored Samples[1)]

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Abstract

A goodness-of-fit test for the two-parameter exponential distribution, for use with the singly Type I and Type II right censored samples, is proposed. The test statistic is based on the L_1-norm of discrepancy between the cumulative distribution function and the empirical distribution function. To deal with the unknown parameters problem, the K- transformation is considered and modified to be applied to the censored samples. Rosenblatt's transformation is extended to the cases of Type I and Type II censored samples, in order to transform the censored samples into the complete ones. The critical values of the test statistic are obtained by Monte Carlo simulations for some finite sample sizes. The power studies are conducted to compare the proposed test with the Pettitt(1977) test for exponentiality with censored samples. It appears that the proposed test has relatively good power properties for moderate and large sample sizes.

1. Introduction

In reliability theory and survival analysis, statistical inferences are often based on the censored samples. A crucial aspect of data analysis with a censored sample is the problem of testing the goodness-of-fit of the sample to a specified distribution model. The importance of goodness-of-fit test for censored samples is noted in many literatures such as Lawless(1982) and D'Agostino and Stephens(1986).

This article is concerned with the goodness-of-fit test for the two-parameter exponential distribution with unknown parameters when a censored sample is obtained. Among several types of censoring, we consider here only the single Type I and Type II right censoring which are very commonly used in practical situations. A sample is said to be singly Type I right censored at L if the exact values of some observations are not known but only that those are greater than a fixed time L. As illustration, consider a life testing in which n items are placed on test. If a decision is made to terminate the testing after a predetermined time L has elapsed, then only r ($r \leq n$) observations are taken until L, in which case r is random. On the other hand, a Type II right censored sample is one for which only the r smallest observations in a random sample of n items are obtained, with r a fixed number predetermined. A sample without any censoring is referred to as a complete sample.

Barr and Davidson (1973) introduce a goodness-of-fit test for censored samples which is the modification of the Kolmogorov - Smirnov test, Koziol and Byar (1975) present the asymptotic distribution of Barr and Davidson's test statistic, and Dufour and Maag (1978)

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provide tables of critical values of this statistic. Pettitt (1977) shows how to apply the Cramer - von Mises test to the censored sample problem. He simply adjusts the upper limit in integral part of the definition of the Cramer-von Mises statistic, and uses the maximum likelihood estimates of the parameters in the probability integral transformation.

These test statistics above are based only on the restricted information from a subset of the observations. However in the cases of censored samples the censoring time L and the size of r, as well as the observations, play a crucial role in the assessment of fit. So a goodness-of-fit test procedure which can reflect more information available is required. In this context, O'Reilly and Stephens(1988) suggest an approach which employs the transformation technique to convert an ordered censored uniform(0,1) sample to an ordered complete uniform sample. This approach is the extension of Rosenblatt(1952) transformation, and it considers the completely specified cases only.

In this article we consider the K-transformation instead of the probability integral transformation to deal with the unknown parameters problem, and modify it to handle the censored samples. As the result of applying the modified K-transformation, an ordered censored uniform sample is obtained from a censored exponential sample. And then the Rosenblatt's transformation is applied to transform the ordered censored uniform sample into the ordered complete uniform one. To test the uniformity of the ordered complete sample, we employ the Kim(1991) test statistic which is based on the L_1-norm of discrepancy between the empirical distribution function and the uniform distribution function on the unit interval.

2. Proposed Test Procedure

Let $X_1, X_2, \cdots, X_n$ represent a random sample of size n from a continuous distribution and let $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ be the corresponding order statistics. Let $X_{(1)}, X_{(2)}, \cdots, X_{(r)}$ $(r \le n)$ denote an ordered right censored sample from a single Type I or Type II censoring. Now consider the K-transformation to transform the censored exponential samples into the ordered censored uniform samples.

2.1 K-transformation

The K-transformation is equivalent to the combination of the N-transformation and the J-transformation. We first consider the K-transformation of the complete samples. Let X_i $(i=1,2,\cdots,n)$ be a random sample from the two-parameter exponential distribution, Exp(α,β). And assume that the location parameter α is known. The normalized spacing of order statistics, which is also called the N-transformation,

$$S_i = (n-i+1)\{X_{(i)} - X_{(i-1)}\},\ X_{(0)} = \alpha,\ i = 1,2,\cdots,n$$

produces i.i.d. sample from the exponential distribution. It is proved by Seshadri *et al.* (1969) that the J-transformation

$$Z_{(j)}=\sum_{i=1}^{j}S_i/\sum_{i=1}^{n}S_i\ ,\quad j=1,2,\cdots,n-1$$

yields ordered sample of size $n-1$ from the uniform distribution on (0, 1).

However in this article both the location parameter and scale parameter are assumed to be unknown. Therefore $X_{(1)}$ can be used as an estimator of α. Accordingly the N-transformation

$$S_i'=(n-i)\{X_{(i+1)}-X_{(i)}\},\quad i=1,2,\cdots,n-1,$$

and then the J-transformation

$$Z_{(j)}'=\sum_{i=1}^{j}S_i'/\sum_{i=1}^{n-1}S_i',\quad j=1,2,\cdots,n-2 \tag{2.1}$$

produce ordered uniform sample of size $n-2$.

Now the modification of the K-transformation is considered to deal with the censoring problem. We assume here that only the smallest r observations are obtained from Type II censoring, or from Type I censoring with censoring time L even though r is random in this case. (The effects of the censoring time L and the size of r on the transformation are fully taken into account in the Rosenblatt's transformation, Section 2.2.2.) Then the K-transformation above can be modified for the censored sample cases. As usual the N-transformation,

$$S_i^*=(n-i)\{X_{(i+1)}-X_{(i)}\},\quad i=1,2,\cdots,r-1$$

can be employed to produce a subset of the unordered Exp(0, β) samples. Here $S_r^*,\cdots,S_{n-1}^*$ may be considered to be masked. In the censored sample cases, however, the denominator in the J-transformation has to be adjusted properly since from S_r^* to S_{n-1}^* are not known. Since S_i^* yields a subset of the exponential random sample, the denominator in the J-transformation can be replaced by $(n-1)\sum_{i=1}^{r-1}S_i^*/(r-1)$, which is based on the maximum likelihood estimator of β. Thus we suggest a slight modification of (2.1) as,

$$Z_{(j)}^*=(r-1)\sum_{i=1}^{j}S_i^*/(n-1)\sum_{i=1}^{r-1}S_i^*,\quad j=1,2,\cdots,r-2\ . \tag{2.2}$$

Monte Carlo simulation experiments are performed to investigate whether the modification (2.2) works well. 10,000 exponential random samples are generated with the subroutine of IMSL for each size of n and r, and then

$$U=\sum_{i=1}^{n-1}S_i^*,\quad V=(n-1)\sum_{i=1}^{r-1}S_i^*/(r-1)$$

are calculated. The average values of $(U-V)/U$ and $|U-V|/U$ are obtained and presented in Table 1. The simulation results show that the deviations between U and V are small enough, and hence imply that the modified K-transformation yields the ordered censored approximate uniform samples.

Table 1. Average Values of $(U-V)/U$ and $|U-V|/U$ for each n and r

n	r	$(U-V)/U$	$\|U-V\|/U$	n	r	$(U-V)/U$	$\|U-V\|/U$
20	18	-0.00076	0.05768	30	27	-0.00012	0.04719
	16	-0.00104	0.08778		24	-0.00070	0.07170
	14	-0.00164	0.11578		21	-0.00089	0.09474
50	45	-0.00009	0.03688	100	90	-0.00037	0.02666
	40	-0.00096	0.05649		80	0.00037	0.03955
	35	0.00075	0.07293		70	0.00020	0.05237
150	135	-0.00009	0.02141	300	270	0.00007	0.01535
	120	0.00044	0.03233		240	0.00049	0.02270
	105	0.00097	0.04241		210	0.00076	0.03006
400	360	0.00005	0.01340				
	320	0.00024	0.01997				
	280	0.00026	0.02599				

2.2 Rosenblatt's transformation

The ordered censored uniform sample transformed from the ordered censored exponential sample should be converted to the ordered complete uniform sample. For this purpose we next consider the well-known transformation suggested by Rosenblatt(1952). For simplicity, let $Y_1<Y_2<\cdots<Y_n$ denote the ordered uniform sample instead of $Z^*_{(1)}<Z^*_{(2)}<\cdots<Z^*_{(n)}$ yielded by the modified K-transformation.

It is well known that the joint probability density function of $Y_1,Y_2,\cdots,Y_n$ is

$$g(y_1,y_2,\cdots,y_n)=\begin{cases} n!, & 0<y_1<y_2<\cdots<y_n<1, \\ 0 & \text{elsewhere,}\end{cases} \tag{2.3}$$

the marginal probability density function of Y_k is

$$g_k(y_k)=\begin{cases} [n!/(k-1)!(n-k)!](y_k)^{k-1}(1-y_k)^{n-k}, & 0<y_k<1, \\ 0 & \text{elsewhere,}\end{cases} \tag{2.4}$$

and the joint probability density function of Y_i and Y_j is

$$g_{ij}(y_i,y_j)=\begin{cases} [n!/(i-1)!(j-i-1)!(n-j)!](y_i)^{i-1}(y_j-y_i)^{j-i-1}(1-y_j)^{n-j}, 0<y_i<y_j<1, \\ 0 \quad \text{elsewhere.}\end{cases} \tag{2.5}$$

2.2.1 Type II censored samples

If the exponential sample is censored at $X_{(r)}$ due to Type II censoring, then only the r smallest observations $X_{(1)},X_{(2)},\cdots,X_{(r)}$ are obtained, and transformed by the modified K-transformation into the ordered censored uniform samples Y_1, $Y_2,\cdots,Y_{r-2}$. In our problem, since the joint probability density function of $Y_1,Y_2,\cdots,Y_n$, is known and absolutely continuous, the marginal and conditional distributions can be defined as follows,

$$Y_1' = G_1(y_1),\ Y_2' = G_{2/1}(y_2/y_1),\ \cdots,\ Y_m' = G_{m/1,\dots,m-1}(y_m/y_1,\cdots,y_{m-1}),$$

where $m = r - 2$. It can be shown from (2.3) - (2.5) that

$$Y_i' = 1 - [(1-Y_i)/(1-Y_{i-1})]^{n-i+1},\ i = 1,\cdots,m, \tag{2.6}$$

where $Y_0 = 0$, by applying the Markov property of order statistics. Then Y_i' becomes a random sample from the uniform distribution, and hence the ordered censored uniform sample is transformed to the unordered censored uniform sample of size m out of n. Now to transform the unordered censored uniform sample to the ordered complete uniform sample, we develop, recursively, the following inverse relationship of (2.6) with $n = m$

$$Y_i^* = 1 - \prod_{j=1}^{i}(1-Y_j')^{-(m-j+1)},\ i = 1,\cdots,m. \tag{2.7}$$

As the result of this transformation, the ordered complete uniform sample of size m is obtained.

[Example 1] Michael and Schucany(1979) present a Type II right censored data set with $n = 9$ and $m = 5$. Application of Rosenblatt's transformation (2.6) and (2.7) yields the following results.

Y_i	Y_i'	Y_i^*
0.1794	0.8313	0.2995
0.3588	0.8610	0.5722
0.5382	0.8995	0.8011
0.7176	0.9477	0.9545
0.8970	0.9935	0.9997

In the case of highly censored sample, in other words, when m is much smaller than n, Y_m may appear to be far from 1 so that it is transformed into Y_m^* which is also far from 1. Accordingly these results inflate the values of any test statistics.

2.2.2 Type I censored samples

Assume that due to the predetermined censoring time L, only the r smallest observations $X_{(1)}, X_{(2)}, \cdots, X_{(r)}$ are obtained, and transformed by the modified K- transformation into the ordered censored uniform samples $Y_1, Y_2, \cdots, Y_m$. The transformation

$$Y_i' = 1 - [(t-Y_i)/(t-Y_{i-1})]^{m-i+1},\quad i = 1,\cdots,m, \tag{2.8}$$

where $Y_0 = 0$, drived by O'Reilly and Stephens (1988) provides the unordered censored uniform sample. Here t is the image of L in the interval (0,1), which is similar to the value $1-\exp\{-(L-\alpha)/\beta\}$ in the probability integral transformation. In particular, t can be obtained when L is treated as an additional observation $X_{(r+1)}$ in the modified K-transformation. The inverse relationship of (2.8)

$$Y_i^* = 1 - \prod_{j=1}^{i}(1 - Y_j')^{-(m-j+1)}, \quad i = 1,2,\cdots,m, \tag{2.9}$$

makes it possible to transform the unordered censored uniform sample into the ordered complete uniform sample.

[Example 2] Assume that the data set given by Michael and Schucany(1979) is obtained from a Type I right censoring with n = 9 and t = 0.9. Application of Rosenblatt's transformation (2.8) and (2.9) produces the following results, which are different from the results in case of Type II censoring.

Y_i	Y_i'	Y_i^*
0.1794	0.6710	0.1993
0.3588	0.6818	0.3987
0.5382	0.7012	0.5980
0.7176	0.7458	0.7973
0.8970	0.9835	0.9967

It is important to note that even though we have the same data set of r observations from two types of censoring, we obtain different results in the transformed complete uniform data set and hence in the value of test statistic since the predetermined censoring time L does affect the transformations.

2.3 Test Statistic

To test the uniformity of the ordered complete samples which are derived by applying the modified K-transformation and Rosenblatt's transformation, we employ the Kim (1991) test statistic which is based on the L_1-norm of discrepancy between the empirical distribution function and the uniform distribution function on the unit interval,

$$\mathrm{CL1} = \|i/m - F_m\{F^{-1}(i/m)\}\|_1, \quad i = 1,\cdots,m-1$$

where $\|\cdot\|_1$ denotes the L_1-norm, $F(\cdot)$ the distribution function of uniform, and $F_m(\cdot)$ the empirical distribution function. If the calculated value of CL1 exceeds the critical value in Table 2 or 3, we reject, at a specified significance level, the hypothesis that the censored sample follows the exponential distribution.

Kim(1991) investigates some properties and the null distribution of the test statistic L1 for the complete sample cases. In our problem we start with the censored samples and finally obtain the complete samples of uniform. So the proposed test statistic CL1 has the same properties and distribution as the statistic L1.

2.4 Critical Values

The critical values are obtained by Monte Carlo simulations. The exponential random samples are generated for sample sizes n = 10 (10) 30 (20) 90, 100 (50) 400. The smallest r random numbers are selected as the observations of Type II censored sample. We set the proportion of censored observations as p = 1.0, 0.9, 0.8, 0.7 for each sample size, and r is determined with the relationship, $p=r/n$. If p = 1.0, a complete sample is to be generated. On the other hand, Type I censoring time L is determined as a ratio to $X_{(r)}$, that is, $L=X_{(r)}(1+\delta)$. We set δ = 0.0, 0.01, 0.02, and p = 1.0, 0.9, 0.8, 0.7 in Type I censoring cases. If δ = 0.0, the rth observation happens to be taken exactly on the censoring time L. For each n, p and δ, 10000 Monte Carlo runs are conducted. The critical values are presented in Table 2 and 3 for significance levels α = 0.3, 0.25, 0.2, 0.15, 0.1, 0.05, 0.025, 0.01.

It is to be noted that these critical values can be used for the test of exponentiality with complete samples since Type II censored samples with $r=n$ (that is, p = 1.0) or Type I censored samples with p = 1.0 and δ = 0.0 are considered as the complete samples. It is obvious that the critical values for Type II censored samples with p = 1.0 are the same as the ones for Type I censored samples with p = 1.0 and δ = 0.0.

3. Power Comparisons

It is well known that the Cramer-von Mises test is more powerful than the Kolmogorov-Smirnov test. Therefore we compare the proposed test with the Pettitt's test which is the modification of the Cramer-von Mises test, and not with the Barr and Davidson's test which is the modification of the Kolmogorov-Smirnov test. Monte Carlo power studies are conducted to compare the power of tests. For the purpose of more precise power comparisons, new critical values for Pettit's test statistic are obtained by the same way as for the statistic CL1. For the sake of completeness, we define the computational formulas of Pettit's test statistics as follows

$$\mathrm{PET2}=\sum_{i=1}^{r-1}\{Z_{(i)}-(2i-1)/2n\}^2-(r-1)\{4(r-1)^2-1\}/12n^2$$
$$+nZ_{(r)}\{(r-1)^2/n^2-(r-1)Z_{(r)}/n+Z_{(r)}^2/3\}$$

where $Z_{(i)}=1-\exp\{-(X_{(i)}-a)/b\}$, $a=X_{(1)}$, $b=\{\sum_{i=1}^{r}X_{(i)}+(n-r)X_{(r)}-nX_{(1)}\}/r$, and

$$\mathrm{PET1}=\sum_{i=1}^{r}\{W_{(i)}-(2i-1)/2n\}^2-r(4r^2-1)/12n^2+nW_{(c)}\{r^2/n^2-rW_{(c)}/n+W_{(c)}^2/3\}$$

where $W_{(i)}=1-\exp\{-(X_{(i)}-a)/d\}$, $a=X_{(1)}$, $d=\{\sum_{i=1}^{r}X_{(i)}+(n-r)L-nX_{(1)}\}/r$, $W_{(c)}=1-\exp\{-(L-a)/d\}$, for Type II and Type I censored samples, respectively.

A variety of widely used alternatives to the exponential distribution are considered. Ten thousand random samples are generated for sample sizes n = 30, 50, 100, 150 from the distributions, including the exponential, Weibull (2.0, 3.0), Weibull (1.0, 1.5), gamma(0.5, 1.0), gamma(2.0, 1.0), normal(2.0, 5.0), log-normal(1.0, 0.5), beta(2.0, 3.0), and beta(2.0, 2.0) distribution. We select p = 1.0, 0.9, 0.8, 0.7 for Type II censoring, and δ = 0.02, p = 1.0, 0.9, 0.8, 0.7 for Type I censoring. For each distribution model and sample size combination, the proposed test and Pettit's test are performed. The estimates of power shown in Table 4 and 5 are the percentage of 10,000 Monte Carlo samples declared significant by the test at a given significance level α = 0.1, 0.05, 0.01, respectively.

Our power studies reveal that the proposed test has higher power than Pettitt's test, for moderate and large sample sizes, against a broad class of alternatives except for gamma(0.5, 1.0) distribution. Also it is found that the test statistic CL1 is superior in power properties to the statistic L1 proposed by Kim(1991) except for the case of log-normal(1.0, 0.5) distribution. There is, however, one potential drawback with the proposed test. The proposed test yields conservative test results, particularly for small samples since the test statistic is a discrete random variable. Thus the estimates of power for small sample sizes ($r < 21$) are not presented here. To cope with this problem, other test statistics such as Cramer-von Mises and Anderson-Darling which are continuous but less powerful than L_1-norm test statistic might be employed to test the uniformity after both the modified K-transformation and Rosenblatt's transformation are conducted.

Simulation results which are not presented in this article have been tabled by the author and are available upon request.

Table 2. Critical Values for Test Statistic CL1 : Type II Censored Samples

		Sample size				Significance level			
n	*p*	0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
20	1.0	1.44445	1.55556	1.72222	1.88889	2.11111	2.38889	2.72223	3.16666
	0.9	1.35294	1.41177	1.52941	1.70588	1.88235	2.17647	2.47059	2.82353
	0.8	1.20000	1.26667	1.33333	1.46667	1.60000	1.86667	2.13333	2.40000
	0.7	1.00000	1.07692	1.23077	1.30769	1.46154	1.69231	1.84615	2.07692
30	1.0	1.85714	2.00000	2.14286	2.35714	2.60714	3.03571	3.42857	3.92857
	0.9	1.65385	1.76923	1.92308	2.07692	2.34615	2.69231	3.07692	3.50000
	0.8	1.47826	1.56522	1.69565	1.86957	2.08696	2.39130	2.69565	3.04348
	0.7	1.30000	1.40000	1.50000	1.65000	1.80000	2.10000	2.40000	2.65000
50	1.0	2.47917	2.64583	2.85417	3.12500	3.45833	4.02083	4.54167	5.22917
	0.9	2.18182	2.34091	2.50000	2.72727	3.04545	3.52273	4.00000	4.59091
	0.8	1.92308	2.05128	2.20513	2.41026	2.66667	3.10256	3.51282	3.92308
	0.7	1.73529	1.82353	1.97059	2.11765	2.35294	2.73529	3.05882	3.47059
100	1.0	3.51020	3.74490	4.03061	4.43877	4.93877	5.77551	6.50000	7.39796
	0.9	3.07865	3.28090	3.51685	3.85393	4.26966	5.00000	5.59551	6.46068
	0.8	2.77215	2.94937	3.15190	3.41772	3.78481	4.39240	4.93671	5.60760
	0.7	2.47826	2.63768	2.82609	3.05797	3.34783	3.85507	4.33333	4.91304
150	1.0	4.31757	4.62838	5.01351	5.50000	6.12163	7.06757	7.90541	9.03378
	0.9	3.80597	4.06716	4.35821	4.73134	5.26866	6.13433	6.91045	7.80597
	0.8	3.39496	3.62185	3.89916	4.23529	4.68067	5.43698	6.10084	6.98319
	0.7	3.07692	3.25000	3.49039	3.77885	4.17308	4.80770	5.35577	6.07692
300	1.0	6.12416	6.54362	7.11074	7.72819	8.59396	9.99329	11.35235	12.94295
	0.9	5.40892	5.75836	6.18216	6.72120	7.50929	8.64683	9.78437	11.17844
	0.8	4.82427	5.16736	5.51883	6.00000	6.62761	7.73222	8.65271	9.76568
	0.7	4.31100	4.57416	4.90909	5.32057	5.92344	6.76555	7.60765	8.60287
400	1.0	7.04774	7.56784	8.14322	8.89448	10.02262	11.69346	13.19599	14.90955
	0.9	6.23677	6.63510	7.12814	7.76881	8.67688	10.06129	11.33983	13.07800
	0.8	5.50783	5.87774	6.33228	6.83386	7.53918	8.78369	9.97492	11.43573
	0.7	4.96774	5.25806	5.62007	6.08244	6.71684	7.73476	8.72042	9.93906

Table 3. Critical Values for Test Statistic CL1 : Type I Censored Samples

Sample size			Significance level							
n	δ	p	0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
20	0.00	1.0	1.44445	1.55556	1.72222	1.88889	2.11111	2.38889	2.72223	3.16667
		0.9	1.37500	1.50000	1.62500	1.75000	1.93750	2.31250	2.56250	2.93750
		0.8	1.28571	1.35714	1.50000	1.64286	1.78571	2.14286	2.42857	2.71429
		0.7	1.16667	1.25000	1.41667	1.50000	1.66667	2.00000	2.25000	2.50000
	0.01	1.0	1.57895	1.68421	1.84211	2.00000	2.26316	2.63158	3.00000	3.47368
		0.9	1.41176	1.52941	1.64706	1.76471	2.00000	2.35294	2.64706	3.00000
		0.8	1.33333	1.40000	1.53333	1.66667	1.86667	2.20000	2.53333	2.80000
		0.7	1.23077	1.30769	1.46154	1.53846	1.76923	2.07692	2.30769	2.61539
	0.02	1.0	1.57895	1.68421	1.84211	2.00000	2.26316	2.63158	3.00000	3.47368
		0.9	1.41176	1.52941	1.64706	1.76471	2.00000	2.29412	2.64706	3.00000
		0.8	1.33333	1.40000	1.53333	1.66667	1.86667	2.20000	2.46667	2.80000
		0.7	1.23077	1.30769	1.38462	1.53846	1.69231	2.00000	2.30769	2.53846
30	0.00	1.0	1.85714	2.00000	2.14286	2.35714	2.60714	3.03571	3.42857	3.92857
		0.9	1.76000	1.88000	2.04000	2.24000	2.48000	2.88000	3.24000	3.80000
		0.8	1.63636	1.77273	1.90909	2.09091	2.31818	2.72727	3.09091	3.50000
		0.7	1.52632	1.63158	1.78947	1.94737	2.15790	2.52632	2.89474	3.26316
	0.01	1.0	2.00000	2.17241	2.34483	2.58621	2.86207	3.37931	3.89655	4.34483
		0.9	1.80769	1.92308	2.07692	2.26923	2.50000	2.92308	3.30769	3.80769
		0.8	1.65217	1.78261	1.95652	2.13043	2.39130	2.78261	3.17391	3.56522
		0.7	1.55000	1.65000	1.80000	1.95000	2.20000	2.60000	2.95000	3.30000
	0.02	1.0	2.03448	2.17241	2.34483	2.58621	2.86207	3.37931	3.89655	4.37931
		0.9	1.76923	1.92308	2.07692	2.26923	2.50000	2.92308	3.30769	3.76923
		0.8	1.65217	1.78261	1.91304	2.08696	2.34783	2.73913	3.13043	3.52174
		0.7	1.55000	1.65000	1.75000	1.95000	2.20000	2.55000	2.90000	3.25000
50	0.00	1.0	2.47917	2.64583	2.85417	3.12500	3.45833	4.02083	4.54167	5.22917
		0.9	2.32558	2.48837	2.68768	2.95349	3.27907	3.83721	4.32558	4.88372
		0.8	2.15790	2.34211	2.52632	2.76316	3.07895	3.57895	4.05263	4.65790
		0.7	2.03030	2.15152	2.33333	2.54545	2.84848	3.30303	3.75758	4.21212
	0.01	1.0	2.65306	2.87755	3.10204	3.46939	3.85714	4.53061	5.12245	5.83673
		0.9	2.34091	2.52273	2.72727	2.97727	3.31818	3.86364	4.38636	4.97727
		0.8	2.17949	2.33333	2.51282	2.79487	3.07692	3.61539	4.10256	4.66667
		0.7	2.02941	2.17647	2.35294	2.55882	2.88235	3.35294	3.73529	4.26471
	0.02	1.0	2.67347	2.87755	3.12245	3.46939	3.87755	4.53061	5.14286	5.87755
		0.9	2.34091	2.52273	2.70455	2.97727	3.29545	3.86364	4.38636	4.93182
		0.8	2.17949	2.33333	2.51282	2.76923	3.07692	3.58974	4.07692	4.64102
		0.7	2.00000	2.14706	2.32353	2.52941	2.82353	3.29412	3.70588	4.20588
100	0.00	1.0	3.51020	3.74490	4.03061	4.43877	4.93877	5.77551	6.50000	7.39796
		0.9	3.32955	3.55682	3.84091	4.18182	4.67046	5.43182	6.13636	7.09091
		0.8	3.14103	3.35897	3.61538	3.94872	4.39744	5.14103	5.78205	6.61539
		0.7	2.94118	3.14706	3.39706	3.69118	4.07353	4.75000	5.32353	6.08823
	0.01	1.0	3.79798	4.10101	4.44444	4.89899	5.46465	6.33333	7.25252	8.24243
		0.9	3.34831	3.58427	3.86517	4.20225	4.69663	5.46067	6.17977	7.14607
		0.8	3.13924	3.35443	3.59494	3.96202	4.40506	5.15190	5.74684	6.60759
		0.7	2.92754	3.13043	3.36232	3.66667	4.02899	4.73913	5.33333	6.04348
	0.02	1.0	3.80808	4.11111	4.45455	4.90909	5.48485	6.35353	7.26263	8.27273
		0.9	3.34831	3.58427	3.87640	4.20225	4.68539	5.44944	6.22472	7.16854
		0.8	3.12658	3.34177	3.60759	3.94937	4.40506	5.12658	5.75949	6.60759
		0.7	2.94203	3.14493	3.36232	3.65217	4.07246	4.71014	5.26087	6.01449

Table 3. (Continued)

n	δ	p	Sample size				Significance level			
			0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
150	0.00	1.0	4.31757	4.62838	5.01351	5.50001	6.12163	7.06757	7.90541	9.03378
		0.9	4.11278	4.38346	4.73684	5.14285	5.75939	6.72180	7.58646	8.58647
		0.8	3.86441	4.13559	4.48305	4.87288	5.42373	6.31356	7.22034	8.35593
		0.7	3.63107	3.87379	4.17476	4.57282	5.10680	5.93204	6.67961	7.56311
	0.01	1.0	4.69128	5.00671	5.40939	5.93288	6.61745	7.74497	8.79195	10.01342
		0.9	4.11194	4.39552	4.76119	5.14925	5.75373	6.76866	7.62686	8.67164
		0.8	3.86554	4.12605	4.47899	4.86555	5.42016	6.29412	7.19328	8.33613
		0.7	3.60577	3.84615	4.16346	4.54808	5.07692	5.89423	6.64423	7.52884
	0.02	1.0	4.69128	5.01342	5.42282	5.93960	6.63758	7.75839	8.81879	10.04698
		0.9	4.11940	4.41045	4.73881	5.17164	5.77612	6.76866	7.71641	8.72388
		0.8	3.89076	4.15126	4.47059	4.88235	5.44537	6.37815	7.23530	8.39496
		0.7	3.64423	3.90385	4.21154	4.60577	5.10577	5.93269	6.72115	7.62500
300	0.00	1.0	6.12416	6.54362	7.11074	7.72819	8.59396	9.99329	11.35235	12.94295
		0.9	5.85448	6.27239	6.73881	7.35448	8.19404	9.47389	10.76493	12.34329
		0.8	5.50841	5.91597	6.38235	6.94538	7.72269	9.02521	10.26050	11.64286
		0.7	5.11058	5.45192	5.91346	6.45673	7.22596	8.32692	9.52403	10.83653
	0.01	1.0	6.52843	6.96990	7.52843	8.21406	9.17392	10.77594	12.24080	13.91639
		0.9	5.86245	6.28252	6.75092	7.39405	8.20818	9.51673	10.79554	12.55761
		0.8	5.50209	5.89958	6.38075	6.96652	7.74058	9.01674	10.28451	11.67782
		0.7	5.08134	5.47847	5.90909	6.50718	7.19617	8.33493	9.51196	10.81340
	0.02	1.0	6.53846	6.98663	7.54850	8.23077	9.19064	10.79935	12.25752	13.95986
		0.9	5.89963	6.30483	6.85130	7.49443	8.29368	9.62082	10.87361	12.57621
		0.8	5.61924	6.01674	6.51046	7.11716	7.89958	9.16737	10.42677	12.02929
		0.7	5.28708	5.65072	6.15789	6.69856	7.46411	8.61722	9.77990	11.19617
400	0.00	1.0	7.04774	7.56784	8.14322	8.89448	10.02262	11.69346	13.19599	14.90955
		0.9	6.74581	7.23464	7.76538	8.51955	9.50559	11.11733	12.60617	14.42460
		0.8	6.27358	6.73900	7.28617	7.95912	8.82389	10.29874	11.85220	13.57860
		0.7	5.88129	6.27698	6.78417	7.38849	8.21583	9.65468	10.90648	12.47122
	0.01	1.0	7.40850	7.96490	8.63908	9.39597	10.58143	12.31828	14.10774	15.86211
		0.9	6.76881	7.24512	7.83845	8.52646	9.49861	11.20614	12.70473	14.59331
		0.8	6.32602	6.76176	7.30721	7.98119	8.88715	10.34482	11.95923	13.60188
		0.7	5.90681	6.33692	6.81362	7.44803	8.28315	9.69535	10.91756	12.55913
	0.02	1.0	5.31250	5.58750	5.92501	6.34250	6.92249	7.88750	8.73499	9.75751
		0.9	6.16389	6.53611	6.98611	7.57222	8.33889	9.68333	10.94166	12.36112
		0.8	6.54545	7.00000	7.52978	8.21316	9.17869	10.76489	12.21943	13.86520
		0.7	6.21863	6.67383	7.20430	7.84229	8.73117	10.13977	11.43369	13.12903

Table 4. Monte Carlo Power Estimates Under Some Alternatives : Type II Censored Samples

Alternative distributions	PET2	CL1	PET2	CL1	PET2	CL1	PET2	CL1
(1% Significance Level)								
	(n=30, p=0.7)		(n=50, p=0.8)		(n=100, p=0.9)		(n=150, p=1.0)	
Exponential	.0101	.0100	.0101	.0100	.0101	.0099	.0100	.0101
Weib.(2.0, 3.0)	.3861	.6319	.9228	.9750	1.0000	1.0000	1.0000	1.0000
Weib.(1.0, 1.5)	.0330	.1174	.1801	.3697	.7808	.8850	.9900	.9979
gamma(0.5, 1.0)	.3406	.2036	.6538	.5233	.9720	.9353	.9990	.9974
gamma(2.0, 1.0)	.0277	.1076	.1629	.3342	.7187	.8252	.9664	.9816
norm.(2.0, 5.0)	.5654	.7647	.9661	.9904	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.0955	.2556	.4357	.6214	.9496	.9717	.9984	.9983
beta(2.0, 3.0)	.1242	.3141	.6048	.8091	.9981	.9995	1.0000	1.0000
beta(2.0, 2.0)	.1990	.4371	.7851	.9166	1.0000	1.0000	1.0000	1.0000
(5% Significance Level)								
	(n=30, p=0.7)		(n=50, p=0.9)		(n=100, p=1.0)		(n=150, p=0.8)	
Exponential	.0501	.0486	.0500	.0495	.0500	.0501	.0500	.0499
Weib.(2.0, 3.0)	.6765	.8280	.9941	.9987	1.0000	1.0000	1.0000	1.0000
Weib.(1.0, 1.5)	.1459	.2963	.5316	.7262	.9737	.9898	.9728	.9883
gamma(0.5, 1.0)	.5589	.3754	.8624	.7655	.9958	.9927	.9994	.9978
gamma(2.0, 1.0)	.1327	.2839	.4599	.6444	.9310	.9539	.9681	.9841
norm.(2.0, 5.0)	.7911	.8912	.9985	.9994	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.2915	.4771	.7264	.8433	.9890	.9904	.9987	.9995
beta(2.0, 3.0)	.3584	.5626	.9274	.9824	1.0000	1.0000	1.0000	1.0000
beta(2.0, 2.0)	.4839	.6832	.9828	.9983	1.0000	1.0000	1.0000	1.0000
(10% Significance Level)								
	(n=30, p=0.8)		(n=50, p=0.8)		(n=100, p=0.8)		(n=150, p=1.0)	
Exponential	.1000	.0923	.1000	.0997	.1000	.0999	.1001	.1001
Weib.(2.0, 3.0)	.8656	.9444	.9907	.9973	1.0000	1.0000	1.0000	1.0000
Weib.(1.0, 1.5)	.2956	.4868	.5701	.7319	.9150	.9631	.9993	.9999
gamma(0.5, 1.0)	.7116	.5332	.8926	.8130	.9938	.9868	1.0000	1.0000
gamma(2.0, 1.0)	.2634	.4505	.5258	.6930	.8956	.9450	.9977	.9989
norm.(2.0, 5.0)	.9223	.9680	.9970	.9994	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.4517	.6390	.7792	.8802	.9876	.9952	.9998	.9999
beta(2.0, 3.0)	.6081	.7975	.9052	.9636	.9988	.9999	1.0000	1.0000
beta(2.0, 2.0)	.7572	.8890	.9666	.9913	1.0000	1.0000	1.0000	1.0000

Table 5. Monte Carlo Power Estimates Under Some Alternatives : Type I Censored Samples (δ = 0.02)

Alternative	Test Statistics							
distributions	PET1	CL1	PET1	CL1	PET1	CL1	PET1	CL1
				(1% Significance Level)				
	(n=30, p=1.0)		(n=50, p=0.8)		(n=100, p=0.7)		(n=150, p=0.9)	
Exponential	.0101	.0100	.0100	.0100	.0100	.0100	.0101	.0100
Weib.(2.0, 3.0)	.8205	.8730	.9056	.9702	.9982	.9997	1.0000	1.0000
Weib.(1.0, 1.5)	.1157	.1235	.1575	.3338	.4593	.6445	.9525	.9756
gamma(0.5, 1.0)	.5048	.4219	.6510	.5346	.9286	.8933	.9969	.9939
gamma(2.0, 1.0)	.0760	.0743	.1406	.2966	.4806	.6488	.9218	.9480
norm.(2.0, 5.0)	.8934	.9244	.9619	.9900	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.1927	.1633	.3999	.5705	.8661	.9253	.9958	.9976
beta(2.0, 3.0)	.5221	.6487	.5677	.7893	.9182	.9708	1.0000	1.0000
beta(2.0, 2.0)	.7400	.8584	.7551	.9095	.9760	.9947	1.0000	1.0000
				(5% Significance Level)				
	(n=30, p=0.9)		(n=50, p=1.0)		(n=100, p=0.7)		(n=150, p=0.8)	
Exponential	.0500	.0484	.0501	.0500	.0501	.0500	.0501	.0500
Weib.(2.0, 3.0)	.8761	.9491	.9990	.9989	1.0000	1.0000	1.0000	1.0000
Weib.(1.0, 1.5)	.2377	.4064	.6510	.6060	.7249	.8370	.9660	.9837
gamma(0.5, 1.0)	.6741	.5480	.8955	.8614	.9768	.9601	.9993	.9980
gamma(2.0, 1.0)	.1936	.3364	.5283	.4408	.7344	.8343	.9604	.9757
norm.(2.0, 5.0)	.9293	.9731	.9998	.9998	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.3717	.5086	.7560	.6513	.9550	.9748	.9983	.9991
beta(2.0, 3.0)	.6188	.8161	.9848	.9923	.9787	.9929	1.0000	1.0000
beta(2.0, 2.0)	.7890	.9192	.9988	.9996	.9962	.9989	1.0000	1.0000
				(10% Significance Level)				
	(n=30, p=0.9)		(n=50, p=0,7)		(n=100, p=0.8)		(n=150, p=1.0)	
Exponential	.1000	.0984	.1000	.1000	.1000	.1000	.1001	.1000
Weib.(2.0, 3.0)	.9258	.9739	.9691	.9881	1.0000	1.0000	1.0000	1.0000
Weib.(1.0, 1.5)	.3592	.5391	.4496	.6292	.8997	.9476	.9993	.9997
gamma(0.5, 1.0)	.7530	.6598	.8661	.7825	.9944	.9882	1.0000	1.0000
gamma(2.0, 1.0)	.2999	.4609	.4381	.6105	.8798	.9244	.9977	.9941
norm.(2.0, 5.0)	.9587	.9847	.9878	.9971	1.0000	1.0000	1.0000	1.0000
log-n.(1.0, 0.5)	.4835	.6205	.7089	.8278	.9850	.9915	.9998	.9995
beta(2.0, 3.0)	.7415	.8894	.7933	.9033	.9988	.9999	1.0000	1.0000
beta(2.0, 2.0)	.8744	.9583	.8901	.9569	1.0000	1.0000	1.0000	1.0000

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중도절단 표본의 지수분포성 적합도 검정을 위한 새로운 통계량[1)]

김 부 용[2)]

요 약

본 연구에서는 지수분포성 적합도검정 문제를 다루는데, 모수가 미지인 상황에서 제1종단일우측중도절단 표본과 제2종우측중도절단 표본인 경우에 각각 적용될 수 있는 새로운 검정통계량을 제안하였다. 미지의 모수 문제를 해결하기 위해서 K-변환을고려하였으며 중도절단 표본에 적용될 수 있도록 K-변환을 수정하였다. 한편, 수정된K-변환에 의해 얻어진 중도절단 균일표본을 완결 균일표본으로 전환시키기 위해서 Rosenblatt 변환을 적용하였다. 전환된 완결 균일표본의 적합도검정을 위한 통계량은분포함수와 경험분포함수의 편차의 L_1-norm 으로 정의되었다. 검정 통계량의 임계치는 Monte Carlo simulation에 의해 구했으며, 잘 알려진 Pettit 검정법과 검정력을 비교하였는데, 새로 제안된 검정통계량의 검정력이 대체로 우수한 것으로 평가되었다.

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