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Multi-Level Skip-Lot Sampling Plan[1)]

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Abstract

This paper is a generalization of single- and two-level skip-lot sampling plans to n-level, which can considerably reduce inspection cost when the level of submitted quality is high. In every skipping inspection of the generalized sampling plan, not only skipping parameters but also inspection fractions can be freely choosed. The general formula of the operating characteristic function for the n-level skip-lot sampling plan is derived. Also the operating characteristic curves of a reference plan, two-level and three-level skip-lot sampling plans are compared.

1. Introduction

Dodge(1955) proposed a skip-lot sampling plan applicable to bulk materials or products produced or furnished in successive batches or lots. Perry(1973a, 1973b) developed it to sigle- and two-level skip-lot sampling plans of which the latter has no restriction on the fraction of lots to be inspected. Parker(1981) presented a modified single-level skip-lot sampling plan. Hess and Kittleman(1989) applied Perry's (1973b) result to skip-period inspection plans to assure good performance of a plant. Under those plans, no matter how good the quality of submitted lots may be, no more than two-level skipping inspection is allowed. But it needs to allow for a minimum amount of inspection when quality is definitely good so that the cost of inspection is reduced and the manufacturer's will to produce is raised. It is, therefore, desirable to extend the sigle- and two-level skip-lot sampling plans to multi-level applicable to the skip-lot sampling plans of more than two-level.

In this paper, a general n-level or multi-level skip-lot sampling plan(MLSkSP) is presented, which has no restriction on the level n and inspection fractions f_k's and skipping parameters i_k's, where f_k is the fraction of lots to be inspected and i_k is the number of lots to be consecutively inspected and accepted on the $k^{th}, k=1,2,\cdots,n,$ skipping inspection. Note that $0<f_k\leq 1$ and i_k's are natural numbers for $k=1,2,\cdots,n.$ Also note that Dodge(1943), Lieberman and Solomon (1955) and Perry(1973b) have not kaken into account the different numbers between i_k's.

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2. Procedure of the Plan MLSkSP

The procedure of the plan MLSkSP is the following.

(1) Start with normal inspection which inspects every lot, using the *reference sampling plan* that is a given lot-inspection plan by the method of attributes (single sampling, double sampling , etc.).

(2) When i_1 consecutive lots are accepted on normal inspection, swich to the first skipping inspection at rate f_1.

(3) During the first skipping inspection:

- When i_2 consecutively inspected lots are accepted, swich to the second skipping inspection at rate f_2.
- When a lot is rejected, swich to the normal inspection.

(4) During the $k^{th}, k=2,3,\cdots,n-1$, skipping inspection:

- When i_{k+1} consecutively inspected lots are accepted, swich to the $(k+1)^{th}$ skipping inspection at rate f_{k+1}.
- When a lot is rejected, swich to the $(k-1)^{th}$ skipping inspection.

(5) During the n^{th} skipping inspection at rate f_k:

When a lot is rejected, swich to the $(n-1)^{th}$ skipping inspection.

3. Derivation of Operating Characteristic Function

In order to obtain the probability of acceptance for the plan MLSkSP, we can also apply the Markov chain approach taken in Perry (1973a, 1973b). The state space of the Markov chain for the plan MLSkSP is

$$\{N_R, N_1, N_2, \cdots, N_{i_1}, S_{1A1}, \cdots, S_{1Ai_2}, S_{1R}, S_{1N0}, S_{1N1}, \cdots, S_{1N(i_2-1)}, \cdots,$$
$$S_{(n-1)A1}, \cdots, S_{(n-1)Ai_n}, S_{(n-1)R}, S_{(n-1)N0}, S_{(n-1)N1}, \cdots, S_{(n-1)N(i_n-1)}, S_{nA}, S_{nR}, S_{nN}\}.$$

All the elements of the state space are defined in terms of Perry (1973b) as follows:

N_R=lot rejected on normal inspection.

N_j=number of consecutively accepted lots during normal inspection is j, $j=1, 2, \cdots, i_1$.

S_{kAj}=number of consecutively inspected and accepted lots during the k^{th}, $k=1,2,\cdots,n-1$, skipping inspection at rate f_k is j, $j=1,2,\cdots,i_{k+1}$.

S_{kNj}=lot skipped during the $k^{th}, k=1,2,\cdots,n-1$, skipping inspection at rate f_k, and previous number of inspected and accepted lots on the k^{th} skipping inspection at rate f_k is j, $j=0,1,\cdots,i_{k+1}-1$.

S_{kR}=lot rejected during the $k^{th}, k=1,2,\cdots,n$, skipping inspection at rate f_k.

S_{nA}=lot inspected and accepted during n^{th} skipping inspection at rate f_n.

S_{nN}=lot skipped during n^{th} skipping inspection at rate f_n.

Let P denote the probability of accepting a lot according to the reference plan and let $Q = 1-P$ throughout this paper. Then the one-step transition probability matrix M for the plan MLSkSP is given by

$$M=\begin{pmatrix} P_{00} & & & & & \\ P_{10} & P_{11} & & & 0 & \\ & P_{21} & P_{22} & & & \\ & & P_{32} & \ddots & & \\ & 0 & & \ddots & P_{(n-1)(n-1)} & \\ & & & & P_{n(n-1)} & P_{nn} \end{pmatrix},$$

where the submatrices of M are the followings together with their corresponding states. In these submatrices, all the values of elements that have no entry equal to zeros.

$$P_{00}=\begin{array}{c} \\ N_R \\ N_1 \\ \vdots \\ N_{(i_1-1)} \end{array}\begin{array}{c} \begin{array}{ccccc} N_R & N_1 & N_2 & \cdots & N_{i_1} \end{array} \\ \begin{pmatrix} Q & P & & & \\ Q & & P & & \\ \vdots & & & \ddots & \\ Q & & & & P \end{pmatrix} \end{array},$$

$$P_{10}=\begin{array}{c} \\ N_{i_1} \\ S_{1A1} \\ \vdots \\ S_{1A(i_2-1)} \\ S_{1R} \\ S_{1N0} \\ \vdots \\ S_{1N(i_2-1)} \end{array}\begin{array}{c} \begin{array}{ccccc} N_R & N_1 & N_2 & \cdots & N_{i_1} \end{array} \\ \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ Q & P & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix} \end{array},$$

and for $k=1,2,\cdots,n-1,$

$$P_{kk}=\begin{array}{c} \\ S_{(k-1)Ai_k} \\ S_{kA1} \\ \vdots \\ S_{kA(i_{k+1}-1)} \\ S_{kR} \\ S_{kN0} \\ S_{kN1} \\ \vdots \\ S_{kN(i_{k+1}-1)} \end{array}\begin{array}{c} \begin{array}{cccccccc} S_{kA1} & S_{kA2} & \cdots & S_{kAi_{k+1}} & S_{kR} & S_{kN0} & \cdots & S_{kN(i_{k+1}-1)} \end{array} \\ \begin{pmatrix} f_kP & & & & f_kQ & 1-f_k & & \\ & f_kP & & & f_kQ & & & \\ & & \ddots & & \vdots & & \ddots & \\ & & & f_kP & f_kQ & & & 1-f_k \\ & & & & & & & \\ f_kP & & & & f_kQ & 1-f_k & & \\ & f_kP & & & f_kQ & & & \\ & & \ddots & & \vdots & & \ddots & \\ & & & f_kP & f_kQ & & & 1-f_k \end{pmatrix} \end{array},$$

where $S_{(k-1)Ai_k} = N_{i_1}$ when k=1. Also for $k = 2, 3, \cdots, n-1$,

$$P_{k(k-1)} = \begin{array}{c} \\ S_{(k-1)Ai_k} \\ S_{kA1} \\ \vdots \\ S_{kA(i_{k-1}-1)} \\ S_{kR} \\ S_{kN0} \\ \vdots \\ S_{kN(i_{k-1}-1)} \end{array} \begin{array}{c} \begin{array}{cccccccc} S_{kA1} & \cdots & S_{kAi_{k-1}} & S_{kR} & S_{kN0} & \cdots & S_{kN(i_{k-1}-1)} \end{array} \\ \left(\begin{array}{cccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ f_{k-1}P & & & f_{k-1}Q & 1-f_{k-1} & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right) \end{array},$$

$$P_{n(n-1)} = \begin{array}{c} \\ S_{(n-1)Ai_n} \\ S_{nA} \\ S_{nR} \\ S_{nN} \end{array} \begin{array}{c} \begin{array}{ccccccc} S_{(n-1)A1} & \cdots & S_{(n-1)Ai_n} & S_{(n-1)R} & S_{(n-1)N0} & \cdots & S_{(n-1)N(i_n-1)} \end{array} \\ \left(\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ f_{n-1}P & & & f_{n-1}Q & 1-f_{n-1} & & \\ & & & & & & \end{array} \right) \end{array},$$

and

$$P_{nn} = \begin{array}{c} \\ S_{(n-1)Ai_n} \\ S_{nA} \\ S_{nR} \\ S_{nN} \end{array} \begin{array}{c} \begin{array}{ccc} S_{nA} & S_{nR} & S_{nN} \end{array} \\ \left(\begin{array}{ccc} f_kP & 1-f_n & f_kQ \\ f_kP & 1-f_n & f_kQ \\ & & \\ f_kP & 1-f_n & f_kQ \end{array} \right) \end{array}.$$

Since the Markov chain of the plan MLSkSP has the same properties as Perry's (1973a, 1973b), we can uniquely obtain the long-run or stationary probabilities, π_i's, of all the given states by solving the system of equations

$$\begin{cases} \pi_i = \sum_j \pi_j p_{ji}, & \text{for all states } i, \\ \sum_i \pi_i = 1, \end{cases}$$

where p_{ji} is the one-step transition probability of going from state j to state i (Parzen; 1964).

The probability of acceptance for the plan MLSkSP can be obtained from

$$P_a(f_1, \cdots, f_n; i_1, \cdots, i_n) = 1 - (\pi_{N_R} + \pi_{S_{1R}} + \pi_{S_{2R}} + \cdots + \pi_{S_{nR}}),$$

where $\pi_{N_R}, \pi_{S_{1R}}, \pi_{S_{2R}}, \cdots, \pi_{S_{nR}}$ are long-run probabilities of lot rejection in the normal, the first, the second, $\cdots$, and the n^{th} skipping inspections, respectively. Those probabilities are derived from the above system of equations after some tedious calculations and the solutions are given by

$$\pi_{N_R} = \frac{Q}{B}\frac{(1-P^{i_1})(1-P^{i_2})\cdots(1-P^{i_n})}{P^{i_1}P^{i_2}\cdots P^{i_n}},$$
$$\pi_{S_{1R}} = \frac{Q}{B}\frac{(1-P^{i_2})(1-P^{i_3})\cdots(1-P^{i_n})}{P^{i_2}P^{i_3}\cdots P^{i_n}},$$
$$\cdots,$$
$$\pi_{S_{(n-1)R}} = \frac{Q}{B}\frac{(1-P^{i_n})}{P^{i_n}},$$
$$\pi_{S_{nR}} = \frac{Q}{B},$$

where

$$B = \frac{1}{f_n}+\frac{1}{f_{n-1}}\frac{(1-P^{i_n})}{P^{i_n}}+\cdots+\frac{1}{f_1}\frac{(1-P^{i_2})(1-P^{i_3})\cdots(1-P^{i_n})}{P^{i_2}P^{i_3}\cdots P^{i_n}} + \frac{(1-P^{i_1})(1-P^{i_2})\cdots(1-P^{i_n})}{P^{i_1}P^{i_2}\cdots P^{i_n}}.$$

Thus the general formula of the operating characteristic(OC) function for the plan MLSkSP is explicitly given by

$$P_a(f_1,\cdots f_n;i_1,\cdots,i_n)=1-Q\frac{A}{B},$$

where

$$A=1+\frac{(1-P^{i_n})}{P^{i_n}}+\frac{(1-P^{i_{n-1}})(1-P^{i_n})}{P^{i_{n-1}}P^{i_n}}+\cdots+\frac{(1-P^{i_1})(1-P^{i_2})\cdots(1-P^{i_n})}{P^{i_1}P^{i_2}\cdots P^{i_n}}.$$

4. Comparisons

All the acceptance probabilities of the n-level skip-lot sampling plan for $n=1,2,\cdots$, can be derived from the general formula of the plan MLSkSP by suitably adjusting f_k's and i_k's for $k=1,2,\cdots,n$.

If we let $f_1=f_2=\cdots=f_{n-1}=1, f_n=f$ and $i_1=i_2=\cdots=i_{n-1}=0$, $i_n=i$, $P_a(f_1,\cdots f_n;i_1,\cdots,i_n)$ is reduced to

$$P_a(f,i)=\frac{fP+(1-f)P^i}{f+(1-f)P^i},$$

which is exactly Perry's (1973a) formula for the single-level skip-lot sampling plans SkSP-2. Also by letting $f_1=f_2=\cdots=f_{n-2}=1$, $f_{n-1}=f_1$, $f_n=f_2$ and $i_1=i_2=\cdots=i_{n-2}=0$, $i_{n-1}=i_1$, $i_n=i_2$, we can obtain the probabilities of acceptance of the two-level skip-lot sampling plan as follows:

$$P_a(f_1,f_2;i_1,i_2)=\frac{f_2P^{i_1}+f_1f_2\{P(1-P^{i_2})-P^{i_1}(1-P^{i_{2-1}})\}+(f_1-f_2)P^{i_1+i_2}}{f_2\{P^{i_1}+f_1(1-P^{i_1})(1-P^{i_2})\}+(f_1-f_2)P^{i_1+i_2}},$$

which goes to Perry's (1973b) formula $P_a^{2L1}(f_1,f_2;i)$ for the plans *Plan 2L1* when $i_1=i_2$.

Note that in the numerator of Perry's formula, f_1f_2 is misprinted by f_1.

In order to obtain the OC function of more higher-level MLSkSP than two -level, it is sufficient to adjust the inspection fractions f_k's and skipping parameters i_k's similarly to the cases of the single- and two-level plans.

Figure 1 and Figure 2 show the OC curves of reference plans, P_a(1/2, 1/5 ; 4, 4), P_a(1/2, 1/5 ; 4, 8) and P_a(1/ 2, 1/5, 1/10 ; 4,8,12) for the reference plans of n=20, c=1 and n=50, c=2, respectively. Figure 3 compares the OC curves of P_a(1/2, 1/5, 1/10 ; 4, 4, 4), P_a(1/2, 1/5, 1/10 ;8, 8, 8) P_a(1/2, 1/5, 1/10 ;12, 12, 12) and P_a(1/2, 1/5, 1/10; 4, 8, 12) for the reference plans of n=20, c=1.

From Figure 1 and Figure 2 , we can easily see the following facts.
(1) Skip-lot sampling plans are more desirable than the reference plans.
(2) The OC property of the two-level skip-lot sampling plans for the case $i_1 \neq i_2$ seems similar to that of the three-level skip-lot sampling plans, and the latter plans seem more reasonable than the other two plans considered. The reason is that the acceptance probabilities of the three-level skip-lot sampling plans are largest among all the plans when the defective rate is less than about 3% and are smallest among the skip-lot sampling plans when the defective rate is greater than about 3%. Note that the three-level skip-lot sampling plans get the lower cost of inspection than the two-level when the level of submitted quality is high.

From Figure 3, we can see the following OC property of the three-level skip-lot sampling plans. When the defective rate is between about 3% and 10%, roughly speaking, the greater the sum of the skipping parameters, i_1, i_2 and i_3, is, the smaller the acceptance probability is.

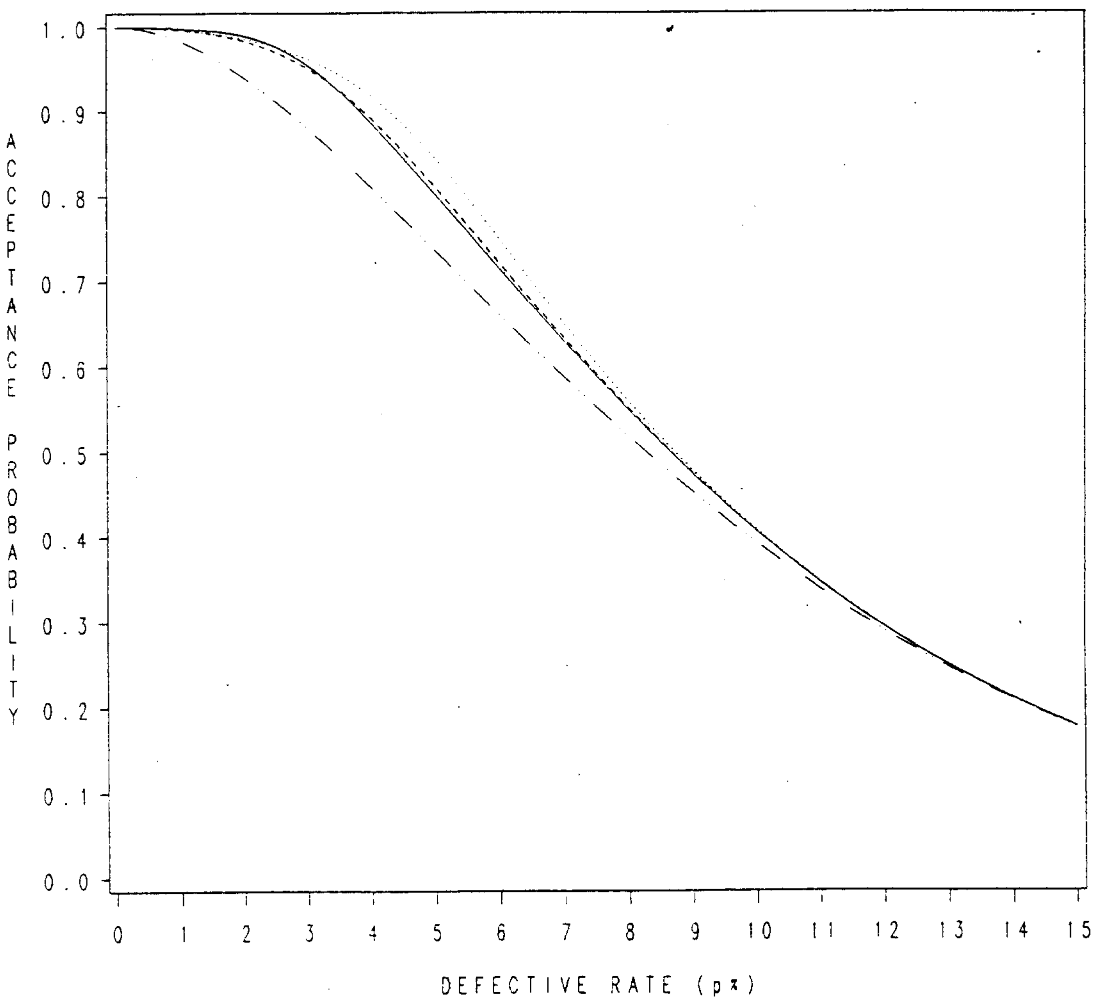

Figure 1. Operating Characteristic Curves with Reference Plan n=20, c=1.
1: Reference Plan 2: P_a(1/2, 1/5 ; 4, 4) 3: P_a(1/2, 1/5 ; 4, 8)
4: P_a(1/2, 1/5, 1/10 ; 4, 8, 12)

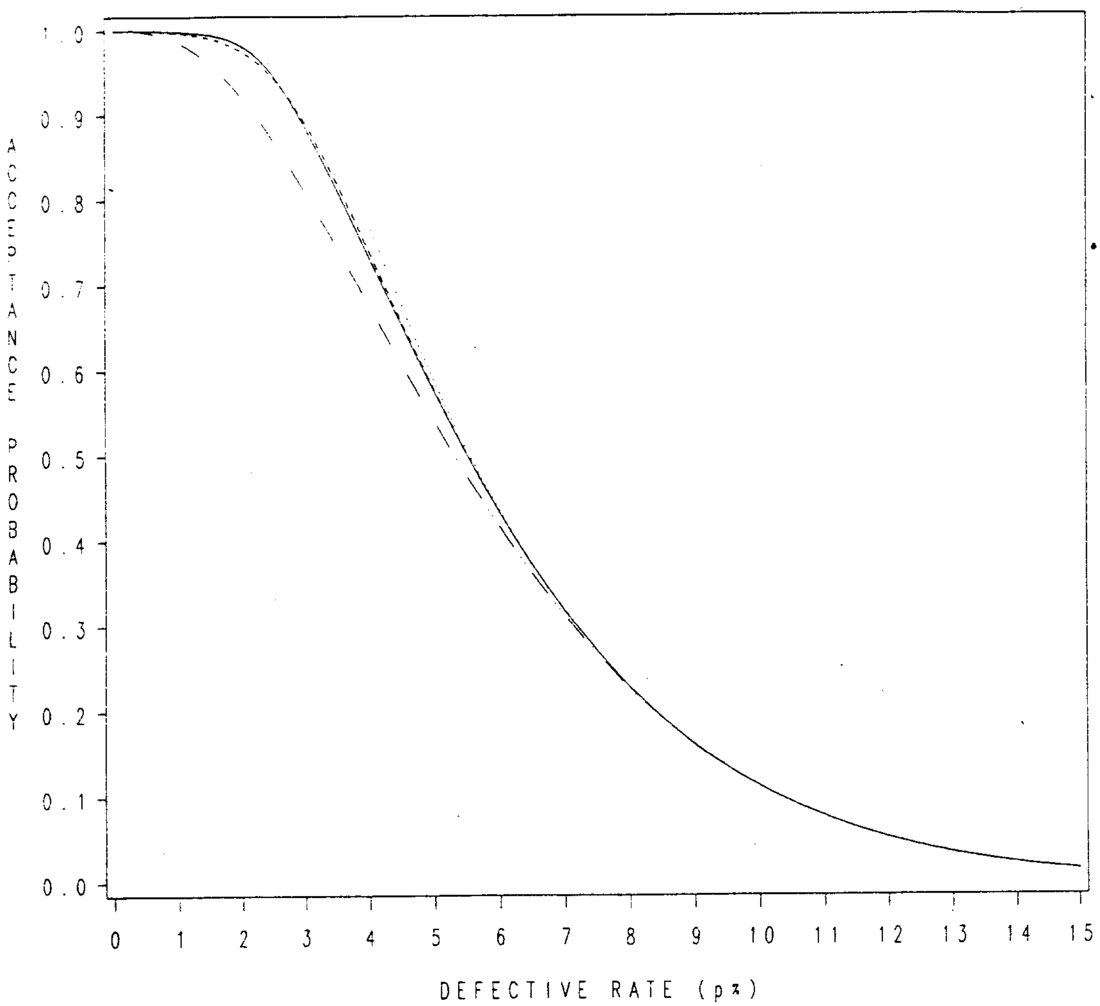

Figure 2. Operating Characteristic Curves with Reference Plan n=50, c=2.
1: Reference Plan 2: P_a(1/2, 1/5 ; 4, 4) 3: P_a(1/2, 1/5 ; 4, 8)
4: P_a(1/2, 1/5, 1/10 ; 4, 8, 12)

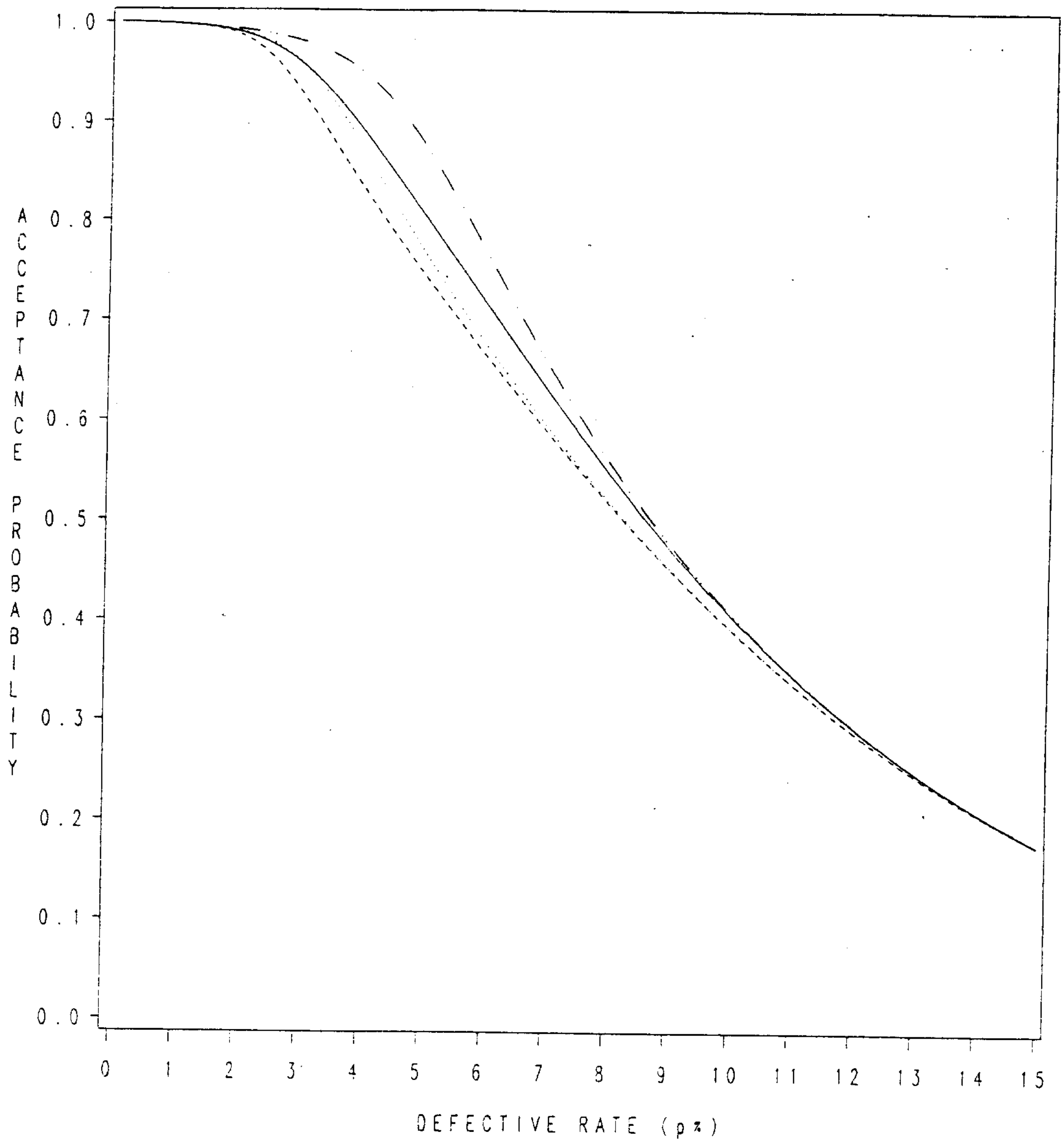

Figure 3. Operating Characteristic Curves with Reference Plan n=20, c=1.
1: P_a(1/2, 1/5, 1/10 ; 4, 4, 4) 2: P_a(1/2, 1/5, 1/10 ; 8, 8, 8)
3: P_a(1/2, 1/5, 1/10 ; 12, 12, 12) 4: P_a(1/2, 1/5, 1/10 ; 4, 8, 12)

5. Concluding Remarks

A general multi-level skip-lot sampling plan applicable to more than two-level skip-lot sampling plans is developed. The developed multi-level skip-lot sampling plans have merits that we can freely choose not only the number of consecutive lots to be accepted but also the fraction of lots to be inspected.

It has seen that the higher-level skip-lot sampling plans seem more reasonable than the lower-level plans in the aspect that the higher-level plans can reduce the cost of inspection when the level of submitted quality is high.

The proposed sampling plans, however, have a shortcoming that it may take much time for them to return to normal inspection from the higher-level skipping inspections when the quality of submitted lots suddenly grow worse. To overcome that demerit, another type, of multi-level skip-lot sampling plan, which immediately swich to normal inspection when a lot is rejected on any skipping inspection level, will be developed before long.

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요 약

본 논문은, 품질이 좋은 제품일수록 검사비용을 줄여서 생산성을 높일 수 있도록, 1단계와 2단계 스킵로트 샘플링검사 계획을 n단계 스킵로트 샘플링검사 계획으로 일반화한 것이다. 이 다단계 스킵로트 샘플링검사 계획에서는 매 검사단계마다 연속적으로 합격되어야 할 로트의 수 뿐만아니라 구분의 크기까지를 자유롭게 택할 수 있게 되어있다. 또한 검사특성곡선의 일반형태를 도출하였고, 이를 이용하여 2단계 및 3단계 스킵로트 샘플링검사 계획을 비교하였다.

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