응용통계연구 제 6권 2호
1993년 9월 pp.237-245.

A Study on a Nonparametric Test for Ordered Alternatives in Regression Problem[1)]

Ki Hoon Lee[2)]

Abstract

A nonparametric test for the parallelisim of k regression lines against ordered alternatives is proposed. The test statistic is a weighted Jonckheere-type statistic applied to slope estimators obtained from each lines. The distribution of the proposed test statistic is asymptotically distribution-free. From the viewpoint of efficiencies, the proposed test has desirable properties and is more efficient than the other nonparametric tests.

1. Introduction

Consider the linear regression model

$$Y_{ij}=\alpha_i+\beta_j(x_{ij}-\bar{x}_i)+e_{ij},\ \ i=1,2,\cdots,k;\ j=1,2,\cdots,n_i,$$

where α_i (intercept) and β_j (slope) are unknown parameters and x_{ij}'s are known constants with $\bar{x}_i$ denoting the average of the i-th group of x's. The Y_{ij}'s are observable, while error terms are mutually independent and identically distributed with continuous symmetric distribution function F.

The problem is to test the null hypothesis

$$H_0:\beta_1=\beta_2=\cdots=\beta_k=\beta\ \ (\text{unknown})$$

against the ordered alternatives

$$H_1:\beta_1\le\beta_2\le\cdots\le\beta_k,$$

where at least one inequality is strict.

Hollander (1970) has suggested a distribution-free signed rank test for the parallelism of two regression lines, assuming that each line has the same number of design points. Potthoff (1974) has also proposed a nonparametric test when k=2. His idea is intuitively appealing, but the test is neither distribution-free nor asymptotically distribution-free.

Adichie (1976) has proposed parametric and nonparametric tests for the parallelism of several regression lines against ordered alternatives. Parametric tests are the likelihood ratio test and a test based on linear combinations of the least squares estimators. Nonparametric tests are rank versions of parametric tests and asymptotically distribution-free. Rao and Gore (1984) have suggested a distribution-free Jonckheere type test with a restrictions such that design points are equispaced.

In this paper, we propose a nonparametric test based on a weighted Jonckheere-type

1) This paper was supported by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1991.
2) Department of Statistics, Jeonju University, Chonju, Korea.

statistic. That is, the test statistic is a sum of weighted Mann-Whitney counts which are applied to the slope estimators.

Section 2 deals with the proposed test. We show the asymptotic normality of the proposed test statistic under some regularity conditions and also calculate the asymptotic relative efficiency. Section 3 contains the result of a small-sample Monte-Carlo study. In section 4, summary and conclusion are remarked.

2. The Proposed Test Statistic and Its Properties

Let $S_{ij}(\alpha_i)$ be the statistics defined by

$$S_{ij}(\alpha_i)=\begin{cases} \dfrac{Y_{ij}-\alpha_i}{x_{ij}-\overline{x}_j}, & \text{if } x_{ij}\neq \overline{x}_i \\ 0, & \text{if } x_{ij}=\overline{x}_i, \quad i=1,\cdots,k \ ; j=1,\cdots,n_i \end{cases}$$

Let $S_{ij}(\hat{\alpha}_i)$ be the statistic $S_{ij}(\alpha_i)$ in which α_i is replaced by $\hat{\alpha}_i$, a consistent estimator of α_i. Then, for each j, $S_{ij}(\hat{\alpha}_i)$ is an estimator of β_i.

We now construct a weighted Jonckheere-type statistic applied to the slope estimators $S_{ij}(\hat{\alpha}_i)$. Jonckheere (1954) statistic is the pairwise sum of Mann -Whitney counts for testing homogeneity of location parameters against ordered alternatives. The proposed test statistic is defined by

$$U(\hat{\underline{\alpha}})=\sum_{u<v}\sum_{i}\sum_{j}W_{ui}W_{vj}\phi(S_{vj}(\hat{\alpha}_v)-S_{ui}(\hat{\alpha}_u)),$$

where W_{ab}'s are weights defined by $W_{ab}=(x_{ab}-\overline{x_a})^2/\sum_j(x_{aj}-\overline{x_a})^2$ and $\phi(t)=1$ or 0 according as $t>0$ or $t\leq 0$. Under the ordered alternatives H_1, the value of $U(\hat{\underline{\alpha}})$ tends to be large. We thus reject H_0 for large value of $U(\hat{\underline{\alpha}})$ in favor of H_1.

Considering the slope estimators $S_{ij}(\hat{\alpha}_i)$, we expect that the estimators are less sensitive to variations in the Y's when the x's are far apart from $\overline{x}$. Hence, it is reasonable to give more weights to the slopes determined from distant points than those determined from close points. We thus assign weights proportional to the distance of x's from $\overline{x}$'s to the Mann-Whitney counts of the slope estimators, $\phi(S_{vj}(\hat{\alpha}_v)-S_{ui}(\hat{\alpha}_u))$.

Since $S_{ij}(\hat{\alpha}_i)$, $j=1,2,\cdots,n_i$, are not independent, the exact distribution of $U(\hat{\underline{\alpha}})$ may be too complicated to be obtained. We derive the distribution of $U(\underline{\alpha})$, and prove that $U(\underline{\alpha})$ and $U(\hat{\underline{\alpha}})$ have the same limiting distribution.

Note that the proposed test statistic is the pairwise sum of

$$U_{uv}(\hat{\underline{\alpha}})=\sum_{i}\sum_{j}W_{ui}W_{vj}\phi(S_{vj}(\hat{\alpha}_v)-S_{ui}(\hat{\alpha}_u)),\ 1\leq u<v\leq k,$$

where $\hat{\alpha}_i$ is a $\sqrt{n_i}$-consistent estimator of α_i. We now want to prove the asymptotic normality of $U_{uv}(\alpha)$ following the line of Theorem 8.1 of Hoeffding (1948), which deals with U-statistics for independent but not identically distributed random variables.

Let F_{ij} be the distribution function of $e_{ij}/(x_{ij}-\overline{x}_i)$, which have the corresponding density functions f_{ij} to be continuous and symmetric about zero. For simplicity, we will express $S_{ij}(\alpha_i)$ as S_{ij}, hereafter.

Theorem 1. Assume that

$$\frac{\max_j(x_{ij}-\overline{x}_i)^2}{\sum_j(x_{ij}-\overline{x}_i)^2} \rightarrow 0, \text{ as } n\rightarrow\infty;\ i=1,\cdots,k. \tag{2.1}$$

Then under H_o,

$$\sqrt{12}(U_{uv}(\alpha)-1/2)/\sqrt{\sum W_{ui}^2+\sum W_{vj}^2}$$

converges in law to a standard normal distribution.

Proof First, we find a U-statistic which is related to $U_{uv}(\alpha)$. Let

$$U_{uv}^* = \binom{n}{2}^{-1}\sum{}'\Phi(X_{a_1},X_{a_2}),\ n=n_u+n_v,$$

where the summation $\sum{}'$ is extended overall $\binom{n}{2}$ sets satisfying $1\leq\alpha_1<\alpha_2\leq n$,

$$X_a=(X_a^{(1)},X_a^{(2)},X_a^{(3)})=\begin{cases}(S_{ua},W_{ua},1), & 1\leq a\leq n_u\\ (S_{v,a-n_u},W_{v,a-n_u},2), & n_u+1\leq a\leq n\end{cases}$$

and

$$\Phi(X_{a_1},X_{a_2})=\sum{}_2\xi(X_h,X_{h'})(\phi(X_{h'}^{(1)}-X_h^{(1)})-1/2),$$

where the sum $\sum_2$ is extended overall 2! permutations (h,h') of (a_1,a_2) and

$$\xi(X_h,X_{h'})=\begin{cases}X_h^{(2)}\cdot X_{h'}^{(2)}, & \text{if } X_h^{(3)}=1,X_{h'}^{(3)}=2\\ 0, & \text{otherwise.}\end{cases}$$

$X_a^{(2)}$ and $X_a^{(3)}$ are constants, but we regard them as random variables for which all the probality mass is concentrated at a single point. Thus Φ is symmetric in its arguments and X_a, $a=1,2,\cdots,n$, are mutually independent random variables.

Under H_0, U_{uv}^* is related to $U_{uv}(\alpha)$ by the equation

$$U_{uv}^* = \binom{n}{2}^{-1}(U_{uv}(\alpha)-1/2). \tag{2.2}$$

Let $\overline{\Psi}_n=\binom{n}{2}^{-1}\sum_i\sum_{j\neq i}\Psi_{1(i)j}(X_i)$, where $\Psi_{1(i)j}(x_i)=E[\Phi(x_i,X_j)]-E[\Phi(X_i,X_j)]$, then $\overline{\Psi}_n$ has the limiting normal distribution from the boundness of $\Phi(X_{a_1},X_{a_2})$ by Liapounov's form of the central Limit Theorem. It is easily shown that U_{uv}^* and $\overline{\Psi}_n$ have the same limiting distribution.

Since $\overline{\Psi}_n$ can be written as

$$\overline{\Psi}_n = \frac{2}{n(n-1)}\left[\sum_i^{n_u} W_{ui}\left(\frac{1}{2} - \overline{F}_v(S_{ui})\right) + \sum_j^{n_v} W_{vj}\left(\overline{F}_u(S_{vj}) - \frac{1}{2}\right)\right],$$

where $\overline{F}_a(S_{bi}) = \sum_j W_{aj}F_{aj}(S_{bi})$, we have $E(\overline{\Psi}_n) = 0$ and $Var(\overline{\Psi}_n) = \binom{n}{2}^{-1}(\sum_i^{n_u} W_{ui}^2 + \sum_j^{n_v} W_{vj}^2)/12$. From (2.2), $U_{uv}(\underline{\alpha})$ converges in law to a normal distribution with mean 1/2 and variance $(\sum W_{ui}^2 + \sum W_{vj}^2)/12$, which completes the proof.

The next theorem is an extension of Theorem 1, which shows the limiting distribution of $U(\underline{\alpha}) = \sum_{u<v} U_{uv}(\underline{\alpha})$.

Theorem 2 Under the conditions in Theorem 1,

$$\sqrt{12}\left(U(\underline{\alpha}) - \frac{k(k-1)}{4}\right)\Big/\sqrt{\sum_h^k \sum_i^{n_k} W_{hi}^2 (2h-1-k)^2}$$

converges in law to a standard normal distribution.

Randles (1982, 1984) has investigated the limiting distributions of U-statistics with estimated parameters where the terms are not identically distributed. We now wish to show that under some regularity conditions, $U(\underline{\alpha})$ and $U(\hat{\underline{\alpha}})$ have the same limiting distribution by applying the results of Randles (1984).

Theorem 3 Assume that the design points x_{ij}, $i = 1, 2, \cdots, n_i$ are symmetric about their mean $\overline{x_i}$ and

$$\lim_{N\to\infty} \frac{n_i}{N} = \lambda_i, \quad 0<\lambda_i<1, \quad i=1,2,\cdots,k \quad \text{with } N = \sum n_i$$

and

$$\lim_{n_i\to\infty} \frac{\sum_j (x_{ij}-\overline{x_i})^2}{\sum_i \sum_j (x_{ij}-\overline{x_i})^2} = \rho_i, \quad 0<\rho_i<1 \ ; \ i=1,2,\cdots,k.$$

Then, under H_0

$$\sqrt{N}(U(\hat{\underline{\alpha}}) - U(\underline{\alpha})) \xrightarrow{p} 0,$$

where $\hat{\underline{\alpha}} = (\hat{\alpha}_1, \cdots, \hat{\alpha}_k)'$ and $\hat{\alpha}_i$ is a $\sqrt{n_i}$-consistent estimator of α_i, $i = 1, 2, \cdots, k$.

We find the asymptotic efficiency of the proposed test under translation alternatives. For simplicity, we assume that all the regression lines have the same design points, that is, x_{ij}, $i = 1, 2, \cdots, k$ are all equal for $j = 1, 2, \cdots, n$. A sequence of translation alternatives $\{H_N\}$ will be specified by

$$H_N : \beta_i = \beta + \theta_i / C_n^{1/2}, \quad \theta_1 \le \theta_2 \le \cdots \le \theta_k \ (\theta_1 < \theta_k),$$

where $C_n = \sum (x_{ij} - \overline{x_i})^2$.

The asymptotic normality of $U(\hat{\alpha})$ under $\{H_N\}$ can be shown by following the previous line.

Theorem 4 Assume the condition (2.1) and it is also assumed that $\int f(x)^2 dx < \infty$. Then under $\{H_N\}$,

$$\sqrt{12}\left(U(\alpha) - \frac{k(k-1)}{4} - \delta_N\right) \Big/ \sqrt{\sum_j W_{Nj}^2 \sum_h^k (2h-1-k)^2}$$

converges in law to a standard normal distribution, where $\delta_N = \sum_{u<v} C_n^{-1/2}(\theta_v - \theta_u) \int \bar{f}_n^2(x)dx$ and $\bar{f}_n(x) = \sum_j W_{ij} f_{ij}(x)$, $i = 1, 2, \cdots, k$.

Theorem 5 Under the assumption of theorem 3 and $\{H_N\}$

$$\sqrt{N}(U(\hat{\alpha}) - U(\alpha)) \xrightarrow{P} 0,$$

where $\hat{\alpha} = (\hat{\alpha}_1, \cdots, \hat{\alpha}_k)'$ and $\hat{\alpha}_i$ is a $\sqrt{n_i}$-consistent estimator of α_i.

For the comparison of efficiencies, we consider the parametric test statistic S which has a limiting normal distribution. Since the design points of each line are assumed to be equal, it is reasonable to suggest, as a parametric competitor, the statistic of the form

$$S = \sum_{u<v} (\bar{\beta}_v - \bar{\beta}_u),$$

where $\bar{\beta}_i$ is the least squares estimator of β_i, $i = 1, 2, \cdots, k$. Adichie (1976) has shown that S is asymptotically normally distributed under $\{H_N\}$. The mean and variance under $\{H_N\}$ are given by $E(S) = \sum_{u<v} C_n^{-1/2}(\theta_v - \theta_u)$ and $Var(S) = \sum_h^k (2h-1-k)^2 \sigma^2 / C_n$. From the result of Skillings and Wolfe (1978), we have

$$ARE(U(\hat{\alpha}), S) = 12\ \bar{\sigma}_n^2 \{\int \bar{f}_n^2(x)dx\}^2,$$

where $\bar{f}_n(x)$ is given in Theorem 4 with the variance $\bar{\sigma}_n^2$. Hodges and Lehmann (1956) have found the lower bound of this expression to be 0.864.

3. Monte Carlo Empirical Power Study

In this section, we examine the empirical powers of our proposed test by comparing with the parametric and nonparametric tests which are briefly restated in the following.

1) Adichie's parametric LR test:

$$\overline{E}_k^2 = \sum C_i(\bar{\beta} - \tilde{\beta}_i)^2/(N-2k)\ \hat{\sigma}^2,$$

where $\hat{\sigma}^2 = \sum_i \sum_j (Y_{ij} - \overline{Y}_i - \bar{\beta}_i(x_{ij} - \overline{x}_i))^2/(N-2k)$, $\tilde{\beta}_i$=isotonic regression of MLE $\overline{\beta}_i$ of β_i and $\bar{\beta} = \sum_i \rho_i \bar{\beta}_i$, $\rho_i = C_i / \sum_j C_j$. The statistic $\overline{E}_k$ has the E-bar- squared distribution under H_0. The critical values were obtained by simulation in our Monte Carlo study.

2) Adichie's rank version of LR test :

$$\overline{\chi}_k^2(\varphi) = \sum_i C_i(\tilde{T}_{n_i} - T_N^*)^2/12^{-1},$$

where $\tilde{T}_{n_i}$ = isotonic regression of $T^*_{n_i}$, $T^*_{n_i} = \sum_j (x_{ij} - \overline{x}_i)R^*_{ij}/(n_i+1)C_i$, R^*_{ij} = rank of jth residual among the ith sample and $T^*_N = \sum_i \rho_i T_{n_i}^*$. The critical values can be obtained using the asymptotic equivalence of $\overline{\chi}_k^2(\varphi)$ and $\overline{\chi}_k^2$.

3) Rao-Gore test statistic :

$$G = \sum_{u<v} \sum_i^r \sum_j^r \phi(S^*_{vj} - S^*_{ui}),$$

where $S^*_{ij} = (Y_{i,j+r} - Y_{ij})/d$; $d = x_{i,j+r} - x_{ij}$: $n_i = 2r$. Since the null distribution of G is the same as that of Jonckheere statistic, the exact critical values are available.

On constructing our proposed test statistic, we use the Hodges-Lehmann type estimator of α_i considered by Adichie (1967), which is of the form

$$\hat{\alpha}_i = med_{s\le t}\{(Y_{it} + Y_{is} - \hat{\beta}_i(x_{it} + x_{is} - 2\overline{x}_i))/2\}.$$

And the critical values are obtained using the asymptotic normality of $U = U(\hat{\underline{\alpha}})$.

The number of regression lines considered are k=3 and k=5. Each regression line has equal sample sizes, $n_i = 10$ with the fixed design points $(1,2\cdots,10)$. The random numbers for uniform and normal distributions were obtained by using the IMSL subroutines.

Because of invariance of all test statistics with respect to α_i, we set $\alpha_i = 0$, $i = 1,2\cdots,k$, for convenience. Values for the slope β_i were specified by

$$\beta_i = 1 + (i-1)m\Delta, \quad i = 1,2,\cdots,k,$$

where Δ is the standard deviation of the least squares estimator of β for the combined sample. The values of m is given by (0, 1, 2, 3) for k=3 and (0, 0.6, 1.2, 1.8) for k=5. The increment of the values of m indicates the change of slopes from the null parameter space to the divergent alternatives.

For each sample generated according to the specific model, the values of the test statistics are calculated and compared with their respective critical values at significance levels of α=0.05 and α=0.10. In this simulation, 1000 replications are performed for each value of design constants. The empirical powers of G are lower than the other nonparametric competitors. It is because of the fact that G loses much information to pay for the exact distribution-free property. For short or medium tailed distributions such as uniform or normal , the empirical powers of our proposed test are obviously lower than parametric tests, but slightly or moderately higher than nonparametric competitors. For heavy tailed distribution, the proposed test performed better than the other tests in the most cases. From the small-sample Monte Carlo study, our proposed test is considered to be recommendable.

Table 1. Empirical Powers of Tests (×1000)

$k=3; n_1=n_2=n_3=10; \alpha=0.05 (\alpha=0.10)$								
	$m=0$	$m=1$	$m=2$	$m=3$	$m=0$	$m=1$	$m=2$	$m=3$
	(a) Uniform Distribution				(b) Normal Distribution			
$\overline{E}^2_k$	37(86)	187(313)	409(576)	737(863)	44(91)	157(299)	432(615)	735(850)
$\overline{\chi}^2_k(\varphi)$	45(88)	165(227)	364(517)	624(788)	44(97)	124(250)	377(539)	657(797)
G	44(106)	157(281)	336(517)	609(766)	37(98)	121(261)	358(520)	608(764)
U	50(98)	185(305)	391(554)	706(837)	51(108)	196(297)	473(617)	716(849)
	(c) Double Exponential Distribution				(d) Cauchy Distribution			
$\overline{E}^2_k$	44(94)	185(307)	453(619)	738(853)	49(101)	88(149)	132(229)	207(325)
$\overline{\chi}^2_k(\varphi)$	46(85)	202(318)	477(631)	727(856)	36(91)	136(248)	331(480)	541(667)
G	46(95)	162(286)	381(593)	661(806)	49(112)	114(202)	235(394)	363(548)
U	62(115)	238(376)	566(714)	823(910)	56(115)	182(290)	404(545)	599(716)
$k=5; n_1=n_2=n_3=n_4=n_5=10 ; \alpha=0.05 (\alpha=0.10)$								
	$m=0$	$m=0.6$	$m=1.2$	$m=1.8$	$m=0$	$m=0.6$	$m=1.2$	$m=1.8$
	(a) Uniform Distribution				(b) Normal Distribution			
$\overline{E}^2_k$	46(105)	194(321)	465(596)	765(858)	40(86)	180(302)	473(602)	759(856)
$\overline{\chi}^2_k(\varphi)$	31(97)	146(264)	346(518)	624(769)	38(80)	135(257)	364(518)	648(808)
G	48(105)	175(290)	379(542)	657(790)	48(106)	170(297)	403(533)	657(798)
U	59(106)	188(293)	437(600)	728(834)	53(111)	216(338)	498(632)	777(866)
	(c) Double Exponential Distribution				(d) Cauchy Distribution			
$\overline{E}^2_k$	43(90)	189(293)	490(615)	757(852)	57(84)	88(139)	127(209)	168(260)
$\overline{\chi}^2_k(\varphi)$	34(82)	190(327)	519(669)	770(871)	48(105)	144(246)	387(534)	552(702)
G	46(95)	195(321)	458(605)	739(848)	52(110)	114(220)	266(421)	417(585)
U	58(111)	254(384)	624(727)	858(931)	72(125)	210(311)	459(592)	640(756)

4. Concluding Remarks

In this paper, we proposed a nonparametric test for the parallelism of several regression lines against ordered alternatives. The proposed test statistic is a weighted Jonckheere-type statistic applied to the slope estimators. Though our proposed test statistic contains estimated parameters and the terms are not identically distributed, the proposed test is asymptotically distribution-free.

We compared the efficiency of our proposed test with the parametric counterparts under some assumptions. The expression ARE is similar to that of well known efficiency of rank test with respect to t test in the two sample. Small-sample empirical power studies show that for the most cases, the proposed test perfomed better than other nonparametric tests. Hence our proposed test is the preferred test for the parallelism against ordered alternatives.

But the proposed test has a disadvantage that it is not robust, it is common in regression problem, to the outliers at influential design points which are far apart from $\overline{x}$ and a restriction that the design points are symmetric about their mean $\overline{x}$.

References

[1] Adichie, J. N. (1967). "Estimates of Regression Parameters Based on Rank Tests," *The Annals of Mathematical Statistics*, **38**, 894-904.

[2] Adichie, J. N. (1976). " Testing Parallelism of Regression Lines against Ordered Alternatives," *Communications in Statistics - Theory and Methods*, **A5(11)**, 985-997.

[3] Hodges, J. L., Jr. and Lehmann, E. L. (1956). "The Efficiency of Some Nonparametric Competitors of the t-test," *The Annals of Mathematical Statistics*, **27**, 324-335.

[4] Hoeffding, W. (1948). "A Class of Statistics with Asymptotically Normal Distribution," *The Annals of Mathematical Statistics*, **19**, 293-325.

[5] Hollander, M. (1970). "A Distribution-Free Test for Parallelism," *Journal of the American Statistical Associations*. **65**, 387-394.

[6] Jonckheere, A. R. (1954). "A Distribution-Free K-Sample Test against Ordered Alternatives," *Biometrika*, **41**, 133-145.

[7] Lee, K. H. (1990). "On a Nonparametric Tests for the Parallelism of Several Regression Lines against Ordered Alternatives," *Ph. D. Dissertation, Department of Computer Science and Statistics, Seoul National University, Seoul, Korea.*

[8] Potthoff, R. F. (1974). "A Nonparametric Test of Whether Two Simple Regression Lines are Parallel," *The Annals of Statistics*, **2**, 295-305.

[9] Randles, R. H. (1982). " On the Asymptotic Normality of Statistics with Estimated Parameters," *The Annals of Statistics*, **10**, 462-474

[10] Randles, R. H. (1984). "On Tests Applied to Residuals," *Journal of the American Statistical Associations*, **79**, 349-354.

[11] Rao, K. S. M. and Core, A. P. (1984). "Testing Concurrence and Parallelism of Several Sample Regressions against Ordered Alternatives," *Mathematische Operationsforschung Und Statistik Series Statistics*, **15**, 43-50.

[12] Skillings, J. H. and Wolfe, D. A. (1978). "Distribution-Free Tests for Ordered Alternatives in a Randomized Block Design," *Journal of the American Statistical Associations*, **73**, 427-431.

회귀직선에서 순서대립가설에 대한 비모수적 검정법 연구[1)]

이 기 훈[2)]

요 약

본 논문에서는 순서대립가설에 대하여 k 개의 회귀직선의 평행성을 검정하는 비모수적 검정법을 제안하였다. 검정통계량은 각 직선에서 얻은 기울기 추정량들에 가중치를 준 존키어(Jonckheere)형태의 통계량이다. 제안된 통계량의 분포는 점근적으로 정규분포를 따르게 되어, 검정법은 점근분포무관검정이 된다. 모수적 검정법과 비교한 점근상대효율은 바람직한 형태를 가지며, 기존의 비모수적 검정법과 비교하여도 더 효율적임을 보였다.

---

1) 이 논문은 1991년도 교육부지원 한국학술진흥재단의 지방대학 육성과제 학술연구 조성비의 지원에 의하여 연구되었음.

2) (560-759) 전라북도 전주시 완산구 효자동 전주대학교 통계학과.