교량 부재의 사하중효과 및 저항의 확률적 특성

Probabilistic Characteristics of Dead Load Effect and Resistance Variables for Bridge Members

정철헌* 오병환**
Chung, Chul Hun Oh, Byung Hwan

……

요약

교량 구조물의 안전성(신뢰성) 평가는 중요한 작업으로서, 교량 구조물의 안전성은 부재 단면의 저항과 부재에 작용하는 외부하중에 의해서 결정된다. 부재의 강도는 부재를 구성하는 재료의 강도, 부재의 치수 및 단면의 저항을 계산하는 산정식등에 내재하는 오차등으로 인해서 공칭저항과 실제값과는 많은 차이가 발생하며, 교량 구조물에서 발생하는 사하중 모멘트는 해석변수와 단면 자중에 의한 하중변수에 의해서 영향을 받는다. 본 연구에서는 사하중효과와 부재 저항의 확률특성을 결정하기 위한 신뢰성 연구를 수행하기 위해서 이들 기본변수들에 대한 확률특성을 실측 및 실험자료를 통해서 우선적으로 평가하였다. 이들 기본변수의 이용하여 사하중 효과 및 저항의 확률적특성을 Monte Carlo Simulation 기법을 이용하여 결정하였다. 이들을 구성하는 각 기본변수들의 확률특성은 기존 연구결과 및 본 연구의 현상 실측 자료를 통해서 결정하였다. 본 연구는 교량의 안전도 평가 및 교량의 신뢰성 해석을 합리적으로 수행할 수 있는 유용한 토대를 제공하는 것으로 사료된다.

Abstract

The safety of a bridge structure depends on the resistance, R, of the member and the action, S, (load or load effect) on the member. The strength of a bridge member may vary from the calculated or "nominal resistance" due to variations in the material strengths and in the dimensions of the members, as well as variabilities inherent in the equations used to calculate the resistances of members. Two basic variables used in predicting the dead load effect, D, are the analysis variables to account for the uncertainties and bias of the analytical idealization which transforms loads to load effects and the load intensity and load placement on the bridge. The fundamental requirement in the probability study is the collection of data on the strength and other physical properties of the materials and the geometric parameters of the structures. In this study, probabilistic characteristics (bias coefficient, C.O.V) of dead load effects and

---

* 정회원, 서울대학교 공학연구소 연구원, 공학박사
** 정회원, 서울대학교 공과대학 토목공학과 교수

• 본 논문에 대한 토의를 1994년 3월 31일까지 학회로 보내주시면 1994년 6월호에 토의회답을 게재하겠습니다.

resistances have been calculated using Monte Carlo Simulation technique. Information on statistics of basic variables, namely, physical properties of concrete, reinforcing steel bars and prestressing steels is furnished, based on actual field data.

Keywords : probabilistic characteristics, dead load effect, resistances, random variables, bias coefficient, coefficient of variation.

1. INTRODUCTION

The safety of a structure depends on the resistance, R, of the structure and the action, S, (load or load effect) on the structure. The first step in the safety analysis and design of structure is to study the variability of the strength of the structural(R.C, steel, prestressed concrete, etc.) member in flexure. The strength of a structural member may vary from the calculated or "nominal resistance" due to variations in the material strengths and in the dimensions of the members, as well as variabilities inherent in the equations used to calculate the resistances of members.

In this study, comprehensive Monte Carlo analyses were conducted to generate an ensemble of bias(R/R_n, D/D_n) for each load effect. R_n is the nominal resistance value calculated by the current code equation and R is the resistance obtained from the Monte Carlo Simulation considering variation of each variable. D_n is the nominal dead load effects calculated by the self-weight of the structural section and D is the dead load effect obtained from the Monte Carlo Simulation.

2. DETERMINATION OF PROBABILISTIC CHARACTERISTICS BY MONTE CARLO SIMULATION TECHNIQUE

Monte Carlo Simulation technique is a powerful engineering tool which enables one to perform a statistical analysis of the uncertainties in structural engineering problems, being particularly useful for complex problems where numerous random variables are related through nonlinear equation.

In this study, probabilistic characteristics (bias coefficient and C.O.V) of load effect and resistances have been calculated using Monte Carlo Simulation technique. The procedure has been programmed and calculations have been carried out by the computer.

3. STATISTICAL PROPERTIES OF BASIC RANDOM VARIABLES

The statistical properties of member resistance and dead load moment depend on the properties which describe the member, such as the cross sectional dimensions and material strength properties.

3.1 STATISTICS OF PROPERTIES OF CONCRETE

Under current design, production, testing, and quality-control procedures, the strength of concrete in an actual structure may differ from its specified design strength and may not be uniform throughout the structure.

The total variation in concrete strength measured by control cylinders includes the variation in concrete strength within a single batch. This in-batch variation may be considered as a variation in testing procedures, mixer inefficiencies, and actual concrete strength. The in-batch coefficients of variation of laboratory tests(16) varied from 0.5% – 8.1% with an overall average of 3.6%. American Concrete Institute(ACI) Committe 214 recommends that the level of control for within-batch tests could be divided into three classes with corresponding coefficients of variation as follows : 4% – 5% for good control, 5% – 6% for average control, and above 6% for poor control.

Recent test results(13) indicate that the average ratios of the in-situ compressive strength to the specified design strength of concrete range from 0.65 to 0.80. The average coefficient of variation is also found to be about 0.19. The lognormal distribution gives safer and better representation for the strengths of in-situ normal weight concrete.

3.2 STATISTICS OF PROPERTIES OF STEEL

If the bars are supplied to the site by different manufactures, the variation in strengths may be high due to different rolling practices and quality control procedures adopted by different manufactures.

Table 1 and Table 2 show the results of the statistical analysis of the data on the strength of steel and reinforcing bars collected by the authors at various places in KOREA(7,12,13).

Table 1 Results of statistical analysis of yield strength of reinforcing bars(KOREA)[12, 13]

Type of bars	Nominal yield strength(kg /cm²)	Bias coefficient	C.O.V (%)	Number of samples
SD30	3,000	1.20	9.5%	822
SD35	3,500	1.13	7.5%	80
SD40	4,000	1.09	8.5%	773

Table 2 Results of statistical analysis of yield strength of steel(KOREA)[7]

[strength unit : kg /cm²]

Type of steel	Plate thickness (mm)	Nominal yield strength	Estimated mean static σ_y / specified σ_y	C.O.V (%)	Number of samples
SWS50	30	3300	1.14	5.7	30
	50	3000	1.22	8.0	30
	80	3000	1.19	3.8	42

The probability distribution of the strengths of steels and bars is found to follow the lognormal curve.

3.3 STATISTICS OF PROPERTIES OF PRESTRESSING STEEL

In this study, tension tests were carried out on prestressing steels. The yield strength, σ_y, was 150 kg /mm², and the tensile strength was 177 kg /mm², based on an area of 0.987 cm² and a length of 562 cm.

Besed on test results(8), it seems reasonable to assume a lognormal distribution for the tensile strength of prestressing steel. The bias coefficient(mean value /nominal value) for the tensile strength of prestressing steel is found to be 1.01 with a C.O.V taken as 3.0 percent.

3.4 DIMENSIONAL VARIATIONS

The work reported is based primarily on data obtained from a number of published sources and involves additional field work. While a number of researchers including Connolly(21) and Tichy(22) have recommended the use of a log-normal distribution for the probability models of dimensions, others have preferred the use of a normal distribution for certain dimensions(19,23). It will be assumed in the present study that a log-normal distribution may safely represent the distribution of the geometric imperfections of reinforced concrete members.

3.4.1 SLAB DIMENSIONS

Slab Thickness

Variations in slab thickness affect the effective depth and weight of slabs, and thus influence their strengths and deflections.

Results from the field investigation of the distribution properties of in-situ slab is given in Table 3.

Table 3 Distribution properties of slab thickness [1-5,9,10,11,13]

Number of samples	Nominal specified, (cm)	Mean deviation from nominal, (cm)	Standard deviation (cm)
21	18.0	+0.33	0.651
32	20.0	+0.50	0.501
24	25.0	−0.50	1.343
37	40.0	+0.52	1.379
19	50.0	+1.25	1.031
22	55.0	−1.00	1.399
155[a]	18.0-55.0[a]	+0.201[a]	1.011[a]

[a] Weighted values of all data shown.

Based on the observations in Table 3, the probabilistic characteristics of distributions of slab thickness can be calculated from

$$\bar{h}=h_n+0.201(\sigma_h=1.011\text{ cm}) \qquad (1)$$

$$\text{Bias}=\bar{h}/h_n,\ \text{C.O.V}=\sigma_h/\bar{h} \qquad (2)$$

where, $\bar{h}$ = the mean value of slab thickness

h_n = the nominal value of slab thickness

Statistical analysis of the ratio of the actual mean thickness to the specified thickness gives a measure of the prediction uncertainty in slab thicknesses. By analyzing Table 3, a mean value of 1.01 for the ratio($h_{actual}/h_{nominal}$) is found. The standard deviation for the ratio is 0.047 and the C.O.V of the ratio is 0.046(= 0.047/1.01).

Assuming an average slab thickness of 25cm, Eq. (1) gives values of h = 25.201cm and σ_h = 1.011cm. Hence, the C.O.V is 0.04(1.011 / 25.201). Therefore, the total uncertainty in slab thickness in terms of C.O.V is $(0.046^2+0.04^2)^{1/2}=0.061$. This value is used in subsequent calculations.

Concrete Cover for Slab Steel

Available data(9,11-13) on concrete cover for top and bottom reinforcement of in-situ slabs are shown in Table 4. The errors in the location of the top reinforcement of in-situ slab exhibited higher mean and standard deviation than those for the bottom reinforcement.

Table 4 Distribution properties of concrete cover

Number of samples	Mean deviation from nominal, (cm)	Standard deviation (cm)
(a) Top concrete cover		
589	−3.20[a]	1.75[a]
(b) Bottom concrete cover		
554	−0.30[b]	0.81[b]

[a] Weighted values of data for top concrete cover

[b] Weighted values of data for bottom concrete cover

Effective Depth of Slab Reinforcement

The effective depth of top bar, d_t, and the concrete cover of bottoms bars, c_b, can be treated as independent random variables if the variability of the bar diameter is assumed to be zero. Since slab thickness h is also an independent random variables, the mean value of the effective depth of top bars, d_t, can be calculated from :

$$\bar{d}_t=\bar{h}-\bar{c}_t \qquad (3)$$

and its dispersion can be calculated from :

$$\sigma_{dt}^2 = \sigma_h^2 + \sigma_{ct}^2 \qquad (4)$$

Similarly, the mean value of the effective depth of bottom bars, $\bar{d}_b$, in-situ slab can be expressed by :

$$\bar{d}_b = \bar{h} - \bar{c}_b \qquad (5)$$

in which the concrete cover for the bottom bars, c_b, and the slab thickness, h, are independent random variables. Thus, the variability of the effective depth is

$$\sigma_{db}^2 = \sigma_h^2 + \sigma_{cb}^2 \qquad (6)$$

Based on Eqs. (3) – (6), the distribution properties of effective depth in the in-situ slabs are shown in Table 5.

Table 5 Distribution properties of effective depth of bars

Mean deviation from nominal (cm)	Standard deviation (cm)
(a) Effective depth of top bars	
−3.00	2.00
(b) Effective depth of bottom bars	
−0.10	1.29

3.4.2 BEAM DIMENSIONS

Beam Width

Results from various investigators for the variations in the widths of in- situ beam stem as well as beam flanges are shown in Table 6 (13,19,21,24,25). The weighted means and standard deviations of all data are also shown in the table.

Overall Depth of Beams

Table 7 summarizes the uncertainties for the beam depths obtained from various investigators(13). Weighted mean deviations from nominal dimensions and standard deviations are also shown in the table.

Table 6 Distribution properties of beam width

Number of samples	Nominal specified (cm)	Mean deviation from nominal (cm)	Standard deviation (cm)	Reference
(a) Width of Beam Rib				
60	29.21	+ 0.127	0.185	[24]
195	30.0	+ 0.356	0.404	[25]
101	30.48	+ 0.0	0.160	[21]
594	–	+ 0.094	0.60	[13]
909[a]	18.0-55.0[a]	+ 0.164[a]	0.50[a]	
(b) Flange Width				
119	50.0	+ 0.229	0.419	[19]
101	60.0	+ 0.610	0.681	[19]
220[b]	50.0-60.0[b]	+ 0.404[b]	0.539[b]	–

[a] Weighted values of data for beam width

[b] Weighted values for flange width

Table 7 Distribution properties of overall depth of beams

Number of samples	Nominal specified (cm)	Mean deviation from nominal (cm)	Standard deviation (cm)	Reference
60	67.31	+ 0.025	0.190	[24]
48	45.72	−0.635	0.410	[21]
230	–	−0.150	0.830	[13]
909[a]	45.72-67.31[a]	+ 0.188[a]	0.657[a]	–

[a] Weighted values of data for in-situ beam

Concrete Cover for Beam Reinforcement

Available data(9,13) on concrete cover of top and bottom reinforcement are shown in Table 8 along with weighted mean values and standard deviations of the available data.

Table 8 Distribution properties of concrete cover of beams

Number of samples	Nominal specified (cm)	Mean deviation from nominal (cm)	Standard deviation (cm)	Reference
48	3.81	+ 0.28	1.57	[9]
230	–	+ 0.24	0.98	[13]
278[a]	3.81[a]	+ 0.188[a]	1.08[a]	–

[a] Weighted values of data for in-situ beam

Effective Depth of Beam Reinforcement

No data are available for effective depth of beam reinforcement.

Therefore, the distribution properties of the effective depth of beam reinforcement were calculated from the recommended distributions of beam thickness and concrete cover using Eq. (3)-Eq. (6). Recommended distributions are shown in Table 9.

Table 9 Recommended distribution properties of effective depth of beams

Mean deviation from nominal (cm)	Standard deviation (cm)
+ 0.062	1.264

3.4.3 ASPHALT THICKNESS AND SPAN LENGTH DIMENSION

In this study, the probabilistic characteristics of asphalt thickness and span length is determined using the results of a recent survey. Available data on the asphalt thickness and span length of existing bridges were collected at several sites throughout the KOREA(1-6, 9, 10, 11). The means and standard deviations of all data are shown in Table 10-11.

Table 10 Distribution properties of asphalt thickness

Number of samples	Nominal specified (mm)	Estimated mean measured / specified	Standard deviation (mm)	c.o.v (%)
58	50.0	1.42	3.58	7.7

Table 11 Distribution properties of span length

Number of samples	Nominal specified (mm)	Estimated mean measured / specified	Standard deviation (mm)	c.o.v (%)
67	8.0-50.0	1.007	0.28	1.6

4. PROBABILISTIC CHARACTERISTICS OF DEAD LOAD EFFECT

The dead load moment on a simple-supported bridge member is given by

$$D = W_d \cdot \ell^2 / 8 \tag{7}$$

$$W_d = \gamma_a \cdot A_a + \gamma_c \cdot A_c + \gamma_s \cdot A_s \tag{8}$$

where W_d = dead load ; γ_a = unit weight of asphalt(ton/m^3) ; γ_c = unit weight of reinforcement concrete(ton/m^3) ; γ_s = unit weight of steel(ton/m^3) ; A_a = area of asphalt ; A_c = area of concrete ; A_s = area of steel ; and ℓ = span length. The statistical parameter of "A" and "ℓ" are established in the Chap. 3. It has been found that basic random variable is log-normal distributed with certain bias factor (mean-to-nominal ratio) and coefficient of variation.

Using the Monte Carlo simulation technuque(14), random deviates of various variables are generated and the values of D are then generated. D_n is obtained by substituting the nominal value of the basic random variables in the prediction equation. Fig. 1-3 show examples of bridge model selected for Monte Carlo Simulation.

It is found that the log-normal distribution fits the generated data well. The ratio D/D_n is obtained for various bridge types using Monte Carlo Simulation and summarized in Table 12.

Table 12 Dead load moment statistics

Bridge Type	R.C Slab	P/C Girder	Steel I-beam
Bias (D/D_n)	1.036	1.065	1.046
C.O.V (%)	8.7	10.0	4.4

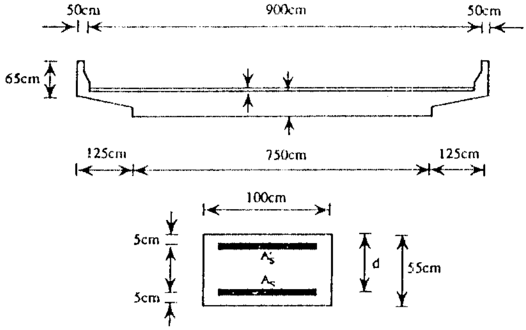


Fig. 1 Reinforced concrete slab example bridge

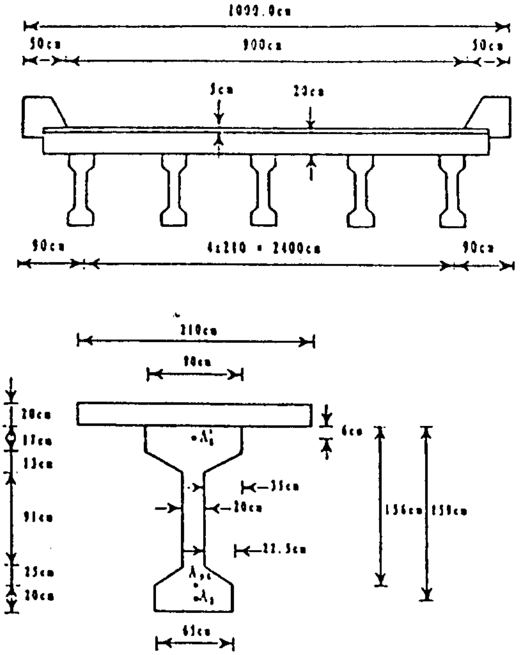


Fig. 2 Prestressed concrete girder example bridge

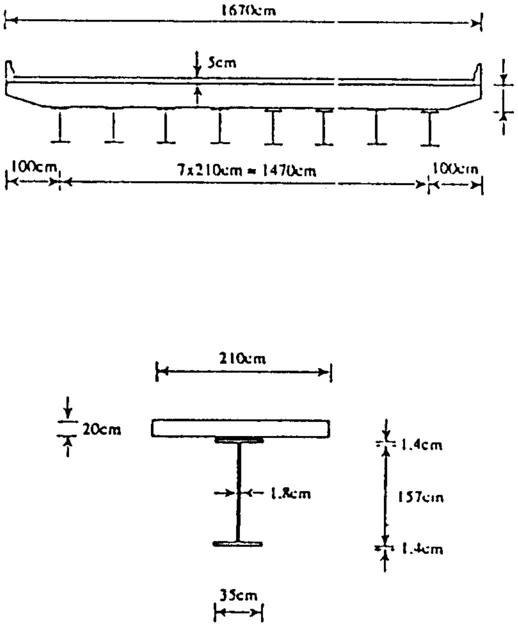


Fig. 3 Steel I-beam example bridge

5. PROBABILISTIC CHARACTERISTICS OF RESISTANCES

Comprehensive Monte Carlo analyses were conducted to generate an ensemble of R/R_n for each selected cross section. R_n is the nominal resistance value calculated by the current code procedure and R is the resistance obtained from the Monte Carlo analyses. The dimensions and member properties of the concrete slab are given in Fig. 1.

The nominal resistance, M_n, may be computed as follows :

$$\text{If} \quad \left(\frac{A_s - A_s'}{b \cdot d}\right) \ge 0.85 \cdot k_1 \cdot \left(\frac{\sigma_{ck} \cdot d'}{\sigma_y d}\right)\left(\frac{6120}{6120+\sigma_y}\right) \tag{9}$$

$$\text{then} \quad M_n = [(A_s - A_s')\sigma_y(d - a/2) + A_s'\sigma_y(d - d')] \tag{10}$$

where b = width of compression face of member ; d = distance from extreme compression fiber to the centroid of tension reinforcement (cm) : d′ = distance from extreme compression fiber to the centroid of compression reinforcement(cm) ; σ_y = specified yield strength of reinforcement(kg/cm^2) ; σ_{ck} = specified compressive strength of concrete (kg/cm^2) ; a = depth of equivalent rectangular stress block ; k_1 = ratio of depth of equivalent compression zone to depth from fiber of maximum compressive strain to the neutral axis ; A_s = area of tension reinforcement (cm^2) ; and A_s' = area of compression reinforcement(cm^2). When the value of $(A_s - A_s')/bd$ is less than the value specified in Eq. (9), so that the stress in the compression reinforcement is less than the yield strength, σ_y, or when effects of compression reinforcement are neglected, the nominal resistance may be computed by rectangular sections with tension reinforcement only.

With known statistical properties for each of the variables in Eq. (10), conventional Monte Carlo simulation may be used to generate a distribution of resistance values. The dimensions and member properties of P / C girder bridge are given in Fig. 2. Prestressed concrete members may be assumed to act as uncracked members subjected to bending stresses within specified service loads. The nominal resistance, M_n, may be computed as follows :

$$\begin{aligned} M_n = {} & A_{pw}\sigma_{ps}(d_p - a/2) + A_s\sigma_y(d - d_p) \\ & + 0.85\sigma_{ck}(b - b_o)t_f\,(d_p - t_f/2) \\ & + A_s'\,\sigma_y(d_p - d') \end{aligned} \tag{11}$$

$$\begin{aligned} A_{pw} = [& A_{ps} \cdot \sigma_{ps} + A_s \cdot \sigma_y - 0.85 \cdot \\ & \sigma_{ck}(b - b_o)t_f - A_s' \cdot \sigma_y] / \sigma_{ps} \end{aligned} \tag{12}$$

where $\gamma_p = 0.85$ for stress-relieved strands, and 0.90 for low-relaxation strands ; σ_{py} = yield point stress of prestressing steel ; σ_{pu} = ultimate strength of prestressing steel ; σ_{ps} = average stress in prestressing steel at ultimate load ; q = the reinforcement index of the tension non-prestressed reinforcement ; q' = the reinforcement index of the compression non-prestressed reinforcement ; $\rho = A_s / bd$, ratio of non-prestressed tension reinforcement ; $\rho' = A_s' / bd$, ratio of compression reinforcement ; ρ_p = ratio of prestressing steel ; A_s = area of non-prestressed tension reinforcement ; A_s' = area of compression reinforcement ; A_{ps} = area of prestressing steel ; b = width of flange of flanged member or width of rectangular member ; b_o = width of a web of a flanged member ; t_f = average thickness of the flange of a flanged member ; d = distance from extreme compressive fiber to centroid of the tension steel ; d' = distance from extreme compressive fiber to centroid of the compression steel ; d_p = distance from extreme compressive fiber to centroid of the prestressing steel ; σ_y = yield strength of steel ; σ_{ck} = compressive strength of concrete at 28 days ; and A_{pw} = the steel area required to develop the ultimate compressive strength of the web of a flanged section. The dimensions and member properties of the steel I-beam bridge are given in Fig. 3. The nominal resistance, M_n, may be computed as follows :

$$M_n = \sigma_y \cdot z \tag{13}$$

where σ_y = specified yield stress of the steel ; and z = section modulus. Using the Monte Carlo technique, random deviates of various variables are generated and then the values of R are generated.

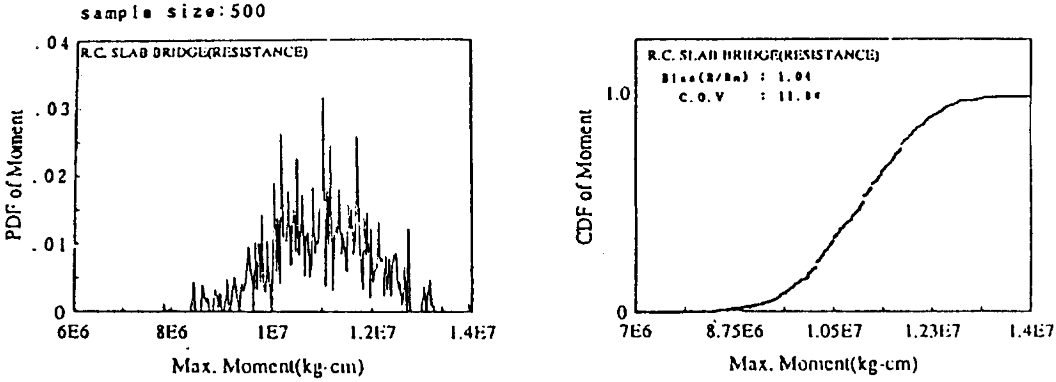
The probability density function, PDF, and cumulative distribution function, CDF, of generated data of resisting moment are shown in Fig. 4 − Fig. 6 according to each bridge type. The ratio R / R_n is found for each bridge type, and after a sufficient number of Monte Carlo trials, the mean and coefficient of variation can be determined. Some typical values of R / R_n and V_R are given in Table 13 for each bridge type.

Table 13 Resisting moment statistics

Bridge Type	R.C Slab	P / C Girder	Steel I-beam
Bias (R / R_n)	1.04	1.01	1.03
C.O.V (%)	11.8	10.2	10.3

6. CONCLUSION

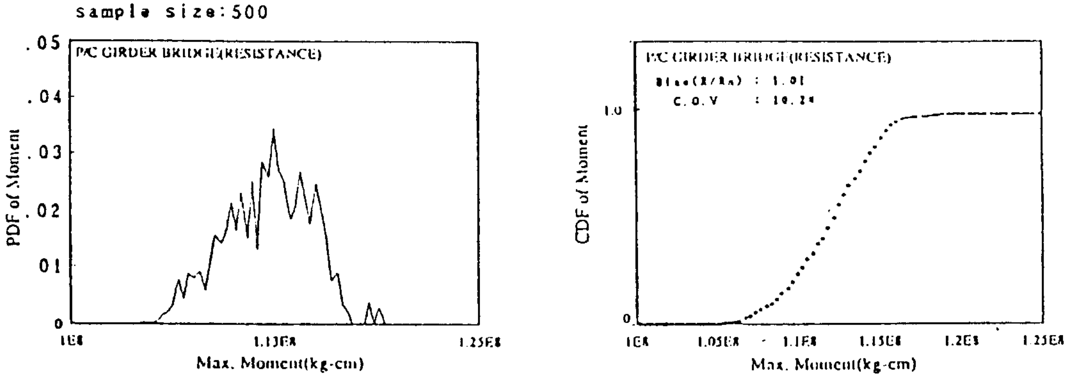
Realistic safety evaluation of bridge



(a) Probability density function

(b) Cumulative distribution function

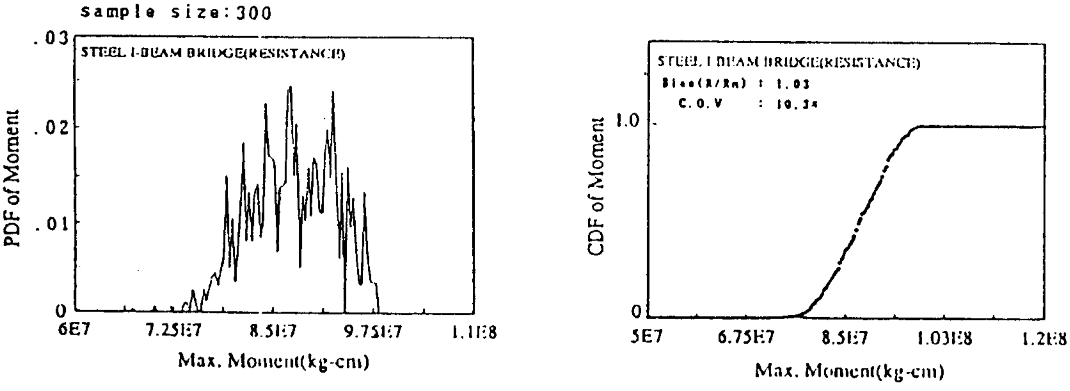
Fig. 4 Probabilistic characteristics of generated resisting moments (R.C Slab Bridge)



(a) Probability density function

(b) Cumulative distribution function

Fig. 5 Probabilistic characteristics of generated resisting moments (P/ C Girder Bridge)



(a) Probability density function

(b) Cumulative distribution function

Fig. 6 Probabilistic characteristics of generated resisting moments (Steel I-Beam Bridge) structures requires reasonable probability characteristics for dead load effect and resistances. Probabilistic characteristics of dead load effect and resistance variables for bridge members can be described with mean value and coefficient of variation calculated using Monte Carlo Simulation technique based on data base for basic random variables. The procedure has been programmed and calculations were carried out by the computer. Information on statistics of basic random variables, namely, properties of concrete, reinforcing steel bars and prestressing steels, are furnished based on actual field data. Some typical values of probabilistic characteristics are given for each bridge type. The present method allows more realistic evaluation of the safety of bridge structures and can be efficiently employed in actual structures.

REFERENCES

1. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 499, 1989, 12.
2. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 450, 1984, 12.
3. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 454, 1985, 12.
4. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 487, 1988, 12.
5. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 499, 1989, 12.
6. "Safety Evaluation of Existing Bridges", National Construction Research Institute, Ministry of Construction, No. 519, 1991, 12.
7. 김동규, 장승필외 7인., "TMCP 극후강재의 성능평가에 관한 실험적 연구 (I)", 한국강구조학회지, 제3권 제3호, pp50−58, 1991.
8. 오병환., "프리스트레스 스트랜드 콘의 슬립특성 연구", 삼진개발 연구보고서, 서울대학교 공학연구소, 1991.
9. Oh, B.H., Chung, C.H., "Development of Reliability-Based Design Code and Realistic Anaylsis Methods of Structures Based on Probabilistic Reliability Theory", Final Report, 1991(in Korean).
10. 오병환., 정철헌., 한승환., "철근 및 프리스트레스 콘크리트 합성형 교량의 안전도 평가기법에 관한 연구", 1991년도 대한토목학회 구조위원회 학술발표회 개요집, 1991, 10, pp.192−195.
11. 한국도로공사, "서울-부산간 교량파손원인조사 및 보수공법연구 (종합보고서)", 대한콘설탄트, 1983.
12. Korea Institute of Construction Technology., "Reliability Eva-luation of Structures", KICT 89-SE-1, Final Report, 1989. 12.
13. Kim, S.H., and Bae, K.W., "Statistical Study on Compressive Strength of Field-Cast Concrete", Journal of The KOREA Concrete Institute, 1989.12, pp.93−100.
14. Siddall., James. N., "Probabilistic Engineering Design, Priciples and Applications", Marcel Dekker, Inc. , 1983.

(접수일자 : 1993. 5. 24)