# Damage Estimation of Bridge Structures Using System Identification

## 동특성추정법을 이용한 교량구조물의 손상도 추정

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Abstract

A method to estimate damage of bridge structures is developed using system idetification approach. Dynamic behavior of damaged structures is represented by a non-linear hysteretic moment model. Structural properties can be evaluated through system identification. To incorporate variability of the structural properties and uncertainties of structural response, damage is represented as random quantities. Numerical example is shown for the bridge structure under different ground excitation.

요 약

본 논문에서는 동특성추정법(system identification)을 이용한 교량구조의 손상정도를 평가하는 방법이 제시되었다. 손상된 구조물의 비선형 동적거동은 휨모우멘트와 곡률반경의 시간이력 관계로써 표시하였으며, 구조물의 성질을 나타내는 특성계수는 실측자료를 이용한 동특성추정법을 사용하여 추정하였다. 또한, 구조물의 성질의 다양성과 응답의 불확실성을 고려하여 손상은 확률변수로 나타내었으며, 예제해석으로는 곡선교의 손상도추정이 수행되었다.

### Introduction

Considerable damage has been observed in many bridge structures during recent earthquakes. Damage, however, is still determined by intuition, experience and judgement of engineers or as a function of simple quantities, such as maximum deformation or

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change of stiffness. Considering that damage is a non-linear function of the excitation, a more systematic approach is essential in damage estimation, which includes non-linear characteristics of structure.

In this paper, a method for damage estimation incorporating system identification is suggested. The structural response is analyzed by random vibration for earthquake excitations and uncertainties related with structural properties are determined with system identification. In particular, the extended Kalman filtering algorithm is used to identify the parameters related with structural properties. For this purpose, a hysteretic model is developed to describe non-linear behavior of a moment-resisting frame. The optimal state of a system can be obtained through filtering using the measured data of excitation and response. With these identified parametrs, a structure is analyzed and damage is assessed at locations where plastic hinges are expected to occur. Damage is represented in terms of damage indices of the hinges from which the overall damage index of a bridge is determined.

In this paper, the proposed method is applied to an curved girder bridge as a numerical example. The structural properties were determined from the identified parameters ; degradation of properties relative to those of the undamaged structure is included. The calculated damage index is compared with the observed damage. In particular, fragility curves are obtained for different ground intensies. The present method can be used with a simple test on a bridge to give useful information for decision to bridge maintenance and rehabilitation of bridge structures.

## Damage Model for Bridge Strctures

### Discretization of Bridge for Damage Analysis

Bridges are usually constructed of flexural members or as frame structures. Therefore, moment-curvature relations govern the non-linear behavior of a structure. Hence, the structures are modeled as moment-resisting frames for damage assessment purpose. Moreover, damages may occur anywhere along a member of the structure. However, for mathematical simplicity, damages may be idealized as concentrated at the appropriate nodes. Accordingly, a discretized model for bridge damage analysis can be represented from behavior at nodes as shown in Fig. 1.

For damage analysis, it is assumed that plastic hinges occur at each node under strong exciations, whereas remaining parts of beam elements are still elastic. As a result, damages are calculated at locations where potential hinges may occur. For the bridge shown in Fig.1, three nodes are defined at each beam-column joint, because different

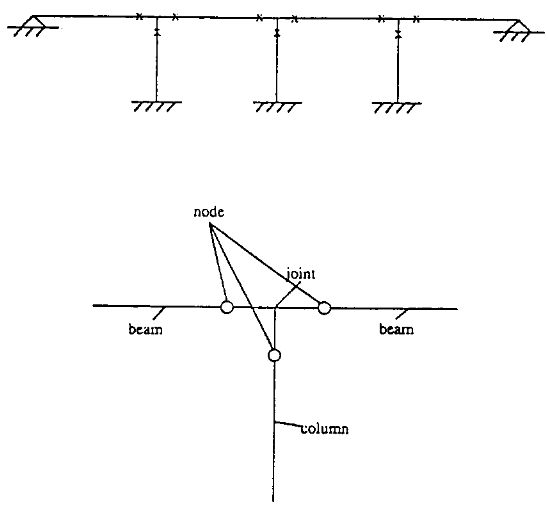



Fig. 1 Bridge Model for Damage Analysis

hystereses may develop at those locations. With this observation, damages can be expressed at these nodes.

Models for Structural Damage

Damage of the reinforced concrete member using moment-curvature relation may be expressed as[10]

$$Dr=\frac{\phi_M}{\phi_U}+\frac{\beta_E}{M_y\phi_U}\int dE \qquad (1)$$

where, $\phi_M$ = maximum response curvature under an earthquake : $\phi_U$ = the ultimate curvature capacity under monotonic loading ; $M_y$ = the calculated yield moment ; dE=the incremental dissipated hysteretic energy ; $\beta_E$ is a non-negative constant and a function of steel ratio, axial force and the stirrup ratio given as

$$\beta_E=[0.37n_0+0.36(k_p-0.2)^2]0.9^{\rho\omega} \qquad (2)$$

in which, $k_p$ is the normalized steel ratio and given as $\frac{\rho_t f_y}{0.85f'c}$;and $\eta_o$ is the normalized axial stress given by $\frac{N}{bdf'c}$. In this paper, $\rho\omega$ is defined as the stirrip ratio ; N is the axial force ; $\rho_t$ is the ratio of tension steel to beam section ; b is the width of a beam ; d is the effective depth of a beam ; $f_y$ is the yield strength of steel and f'c is the compressive strength of concrete[2].

Damage of a reinforced member can be described including structural capacity as,

$$D_i=\frac{D_r}{D_a} \qquad (3)$$

where, $D_i$=the damage index ratio for node-i inside the structure ; $D_r$=the structural damage ; $D_a$=the ultimate damage capacity with $\overline{D}_a$=1.0 and $\sigma_{Da}$=0.54.

To incorporate the uncertainties in the random response, the maximum curvature and hysteretic energy are represented as random quantities. In such a case, the mean and variance of damage at each node can be calculated as[2]

$$D_i=(1+\sigma^2_D)\left(\frac{\phi_M}{\phi_U}+\frac{\beta_E}{M_y\phi_U}\int_0^t dE\right) \qquad (4)$$

$$Var[Di]=\sigma^2{}_D D^2{}_r+\frac{1}{\phi^2{}_U}Var[\phi_M]+\left(\frac{\beta_E}{M_y\phi_U}\right)^2 Var\left[\int_0^t dE\right] \qquad (5)$$

Response Statistics

To calculate the damage of the nodes, the mean and variance of the respective maximum curvatures and dissipated hysteretic eneriges are necessary. For this purpose, the followings are required : a non-linear model for bridge structure, a ground motion model, and a method for a response analysis.

(1) Non-linear Model

The equation of motion to include non-linear restoring force can be written as

$$[M]\{X\}+[C]\{\dot{X}\}+\{F_{NL}\}=-[M]\{J\}\ddot{x}_g \qquad (6)$$

where, [M] is a mass matrix ; [K] is a stiffness matrix ; {J} is a direction vector ; $\ddot{x}_g$ is the ground acceleration. Non-linear force vector, $\{F_{NL}\}$, is defined from the moments at nodes to incorporate the damae model and given as,

$$\{F_{NL}\}=[K]\{X\}+(1.0-\alpha)[T][K_{el}](\{z\}-\{\phi\}) \qquad (7)$$

where, [T] is a matrix transforming moments in local coordinates to forces in global coordinates, and $[K_{el}]$ is the element stiffness matrix. Observe that a case with $\alpha=1.0$ in Eq. 7 indicates the linear restoring force. Non-linear forces are calculated from the locations where plastic hinges are expected to occur. The curvature at each node, $\{\phi\}$, can be expressed from a modal displacement vector, and hysteretic component, z, can be described[10]as

$$z=\frac{A\phi-\nu\{\beta|\phi||z|^{n-1}z+\gamma\phi|z|^{n}\}}{\eta} \tag{8}$$

where, $\alpha,\beta,\gamma$ and n are constants related to the hysteretic restoring force characteristics ; A, $\nu$ and $\eta$ are parameters related to the degradation and function of dissipated energy.

In this study, the state vector approach is used to solve the equations of motion, and the mode superposition is adopted to reduce the number of variables. In such a case, the equations of motion can be rewritten as

$$\{\ddot{W}\}=[-2\xi\omega]\{W\}-[\omega^2]\{W\}-(1-\alpha)\frac{[\phi^T][T]}{[\phi]^T[M][\phi]}[k_{el}]\,(\{z\}-\{\phi\})-\{\Gamma\}\ddot{x}_g \tag{9}$$

where, $\{\Gamma\}$ is modal participation vector ; $\xi$ is damping ratio ; $\omega$ is the natural frequency of the structure ; $\{W\}$, $\{\dot{W}\}$ and $\{\ddot{W}\}$ are modal displacement, velocity and acceleration, respectively.

(2) Modeling of Ground Motion

Earthquake motions are modeled as a zero mean filtered Gaussian shot noise with a Kanai-Tajimi spectrum. To model the non-stationarity in groung motion, its intensity is modulated by Ang and Amin type envelope function[1].

(3) Random Vibration analysis

Random vibration analysis is performed by solving following differential equation[3],

$$\frac{d}{dt}S=GS+SG^T+B \tag{10}$$

where, $S=E[y(t)(t)^T]$

$I(t)$=intensity fuction of the earthquake ground acceleration

And,

$$\frac{d}{dt}y=GY+F \tag{11}$$

in which,

$$y=\{x_g,\ \ddot{x}_g,\ w,\ \dot{w},\ z\}^T$$
$$F=\{0,\ \ddot{x}_g,\ 0,\ 0,\ 0\} \tag{12}$$

and acceleration of the ground, respectively ; w and $\dot{w}$ are modal displacement and velocity of a structure.

(4) Statistics of Maximum Curvature and Dissipated Energy

The mean and variance of maximum curvature may be obtained assuming nonstationary peaks as Weibull distribution[6]. Mean hysteritic energy is obtained solving Eq. 10 and c.o.v. of the hysteretic energy can be approximated as 0.2 after 20 seconds. [8]

Global Damage Index

The global damage is also represented as the mean and variance, since the global damage is obtained from the mean and variance of the damage index at each node as shown in Eqs. 4 and 5. As a result, the event that the global damage is greater than a damage level d is combination of the cases where the damage indices at nodes are greater than the level d and represented as

$$(D_T>d)=U(D_i>d) \tag{13}$$

where, $D_T$ is the global damage of the structure ; $D_i$ is the damage index at node i, and d is a prescribed damage level.

The mean and variance of $D_T$ can not be calculated directly from the summation of $D_i$ since the damage of each node is correlated. Consequently, the damage of the global damage is calculated in a probablistic concept, and this probability will be converted to the mean and variance of the global damage. Using Eq. 13, the probability of global damage exceeding the damage level d is then expressed as

$$P(D_T>d)=P[U(D_i>d)] \tag{14}$$

The second order bounds of the probability of exceeding a damage level d can be written as

$$P(Ei)+\sum_{i=2}^{k} \max[\{P(Ei) - \sum_{j=1}^{i-1} \max [\{P(E_iE_j) ;0]0 \le P(D_T>d)$$

$$P(D_T>d) \le \sum_{i=1}^{k} P(E_i) - \sum_{i=2 j<i}^{k} \max P(E_iE_j) \tag{15}$$

where,

$$P(E_i)=P(D_i>d) \tag{16}$$

in which i denotes the ith node where plastic hinge is expected to occur and k is the number of nodes.

To calculate $P(E_i)$, the performance function is defined considering maximum curvature and hysteretic energy. Using the damage model defined in Eqs. 1 and 3, the performance function can be written as

$$g_i(X)=\frac{aX_i+bY_i-d}{D} \tag{17}$$

where, $a=\frac{1}{\phi_U}$, $b=\frac{\beta_E}{M_y\phi_U}$, $X_i=\phi_{Mi}$, $Y_i=\int dE_i$, $D=D_a$ and d is damage level.

Safety index for each node is calculated using Eq.17 and the correlation factors are also obtained. Assuming log-normal dirstribution for the global damage, parameters in log-normal distribution function can be determined using the probabilities exceeding each damage level d and these median indicates global damage of the structure.

## NUMERICAL EXAMPLE

In this study, Highway 5/14 overcrossing [9] in the U.S.A. is selected as numerical example. This bridge is a typical overcrossing with piers and curved girders. The structure collapsed during 1971 San Fernando earthquake and the behavior of this bridge was examined by 1/30 scale model study.

Table 1. Description of Model and Real Structure

| | real structure | model |
|---|---|---|
| Total Length | 636ft | 254.5inch |
| Radius of Curvature | 270ft | 108inch |
| Column Height | 90ft | 36inch |
| Deck Section | 30ft×7ft | 8.5inx2.5in |
| Column Section | 10ftx5ft | 4inx2in |

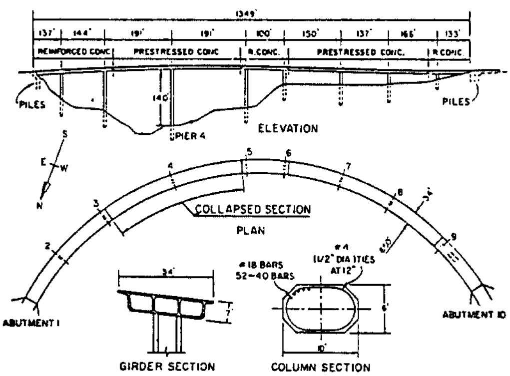
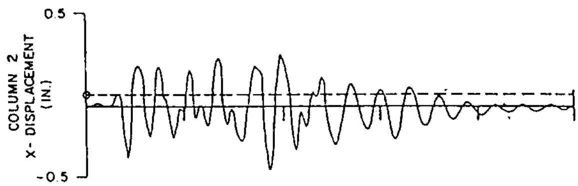



Fig. 2 Description of Highway 5/ 14 Overcrossing

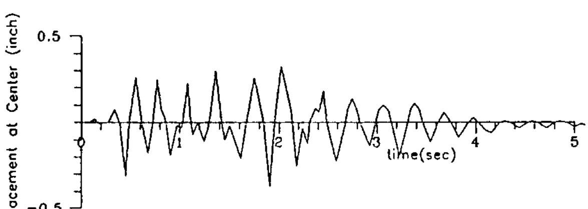
The dimensions of the original and model structure are shown in Table 1 and the description of original structure is shown in Fig.2 Structural properties of model structure are obtained through the extended Kalman filtering[5] using measured time histories, and converted to the original structure. The results of measured and identified time histories of model structure is shown in Fig. 3. With the result of system identification on the model structure, the properties of original structure needed for damage estimation are summarized in Table 2.

Table 2. Structural Properties of Highway 5/ 14 Overcrossing

| $\alpha$ | $\beta$ | $\gamma$ | $\xi$ | $\omega$ | $\xi_g$ | $\omega_g$ |
|---|---|---|---|---|---|---|
| 0.05 | $6.09 \times 10^6$ | $-2.03 \times 10^6$ | 0.10 | 1.15 | 0.8 | 15.08 |




(a) Measured Displacement



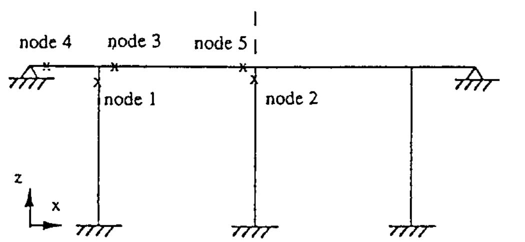

(b) Regenerated Displacement from Identified Parameters

Fig. 3 Time Histories of Response

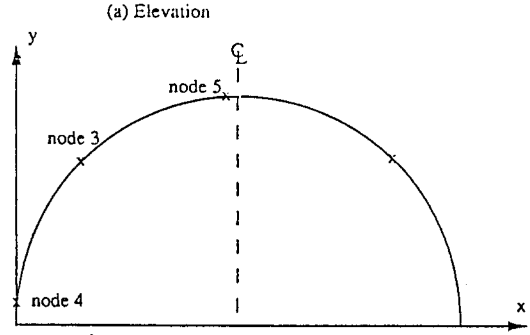
Damage is calculated from the expected maximum curvature and dissipated energy at the locations as shown in Fig.4, and the damage at the corresponding nodes are summarized in Table 3. Global damage of the structure is calculated from combination of damage at the nodes and the result is summarized in Table 4. The coefficient of variance for the global damage is found to be fairly constant and given as approximately 0.62. For this uncertainty, 0.54 comes from the variability of the structural properties and the rest can be attributed to the randomness in the structural response. This structure collapsed around 0.87g of ground excitation. The main reason for the collapse is considered to be damage concentration to the top of center pier(node 2 in Fig. 4).

Table3. Calculated Damage Index for Each Node(Mean)

| node | 1/6g | 1/3g | 1/2g | 2/3g | 5/6h | 1g |
|---|---|---|---|---|---|---|
| 1 | 0.0107 | 0.0428 | 0.0963 | 0.1714 | 0.2680 | 0.3856 |
| 2 | 0.0361 | 0.1446 | 0.3257 | 0.5794 | 0.9059 | 1.3032 |
| 3 | 0.0090 | 0.0361 | 0.0812 | 0.1445 | 0.2259 | 0.3249 |
| 4 | 0.0050 | 0.0200 | 0.0449 | 0.0800 | 0.1250 | 0.1798 |
| 5 | 0.0074 | 0.0298 | 0.0671 | 0.1193 | 0.1866 | 0.2684 |




(a) Elevation




(b) Plan

Fig. 4 Scaled Model of Highway 5/ 14 Overcrossing

In accordance with this observation, the structure is considered to collapse at approximately gobal damage index of 0.8. The probability of exceeding different damage level d is shown in Fig. 5. From this cumulative probability, global damage of this structure is

Table 4. Global Damage Statistics under Different Intensities

| | 1/6g | 1/3g | 1/2g | 2/3g | 5.6g | 1g |
|---|---|---|---|---|---|---|
| Global Damage | 0.0307 | 0.122 | 0.277 | 0.493 | 0.770 | 1.11 |
| C.O.V | 0.620 | 0.620 | 0.620 | 0.620 | 0.621 | 0.621 |

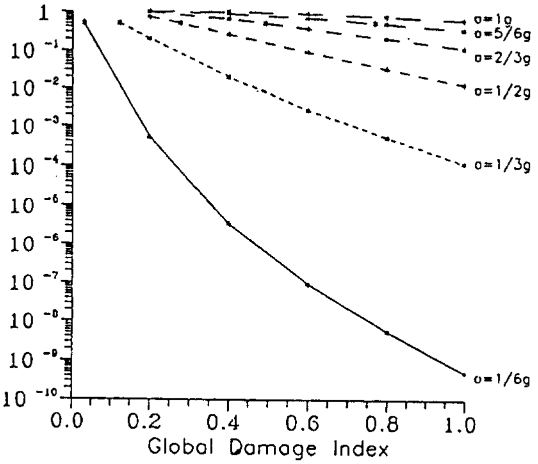



Fig. 5 Probability of Exceeding a Damage Level for Different Intensities

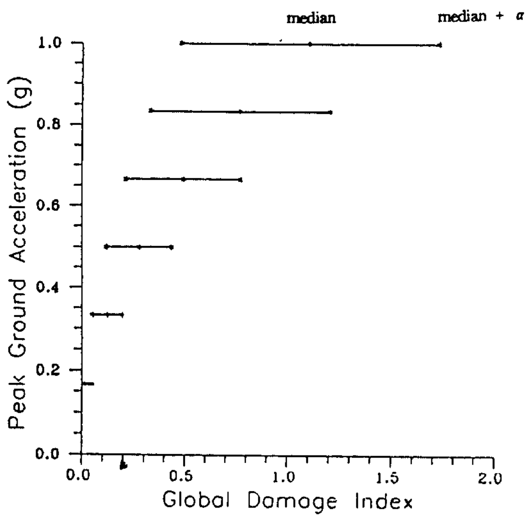



Fig. 6 Global Damage Index

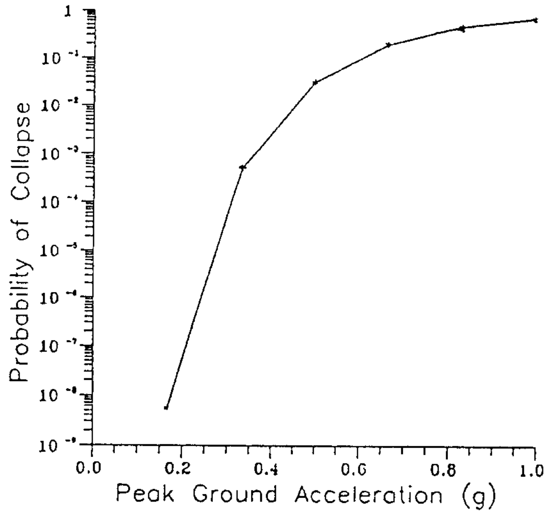



Fig. 7 Fragility Curve

calculated and the result is also shown in Fig. 6. The probability of collapse is assumed as the probability exceeding damage level 0.8 based on the collapse of 5/14 Highway Overcrossing and the damaged building structures in Park, Ang and Wen.[2]

## CONCLUSION

In this study, a method for damage estimation of bridge structure is developed using the identified structural properties. Numerical example indicates that the bridge structure collapsed at approximately damage index of 0.8. In particular, the example structure is considered to be collapsed because of damage concentration to the specific location. To increase resistance to repeated loadings, the structure has to be strengthened to avoid such damage concentration. This proposed method can be used with a simple test on a bridge to give useful information for decision related to maintenance and rehabilitation of bridge structures.

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