

로봇의 정밀도 향상을 위한 평행식 구조의 정밀로봇 설계

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The Optimum Design of A 6 D.O.F Fully-Parallel Micromanipulator for Enhanced Robot Accuracy

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ABSTRACT

이 논문에서는 정밀로봇 설계에 관한 여러 사안을 다루었다. 정밀로봇이란 미세한 오차와 정밀한 제어로 기존 로봇의 정밀도 향상을 위한 작고, 정밀한 운동범위를 갖춘 로봇이다. 원하는 운동범위나 효과적인 힘 전달률, 최소한 작은 힘으로의 동작을 수행하기 위한 최적의 기구학적 변수를 컴퓨터 시뮬레이션을 통하여 구현하고자 한다. CAD/CAM 시스템을 이용한 합성, 해석 및 제작을 위한 정보가 만들어질 수 있으며 최대 휨 및 응력해석을 통하여 최종적인 검증 및 설계 변경을 위한 자료로서 사용될 수 있을 것이다.

Key Words : Parallel(평행식 구조), Micromanipulator(정밀로봇), Robot Accuracy(로봇정밀도), Analysis(해석), Synthesis(합성)

1. INTROUCTION

The idea behind the Stewart platform⁽¹⁾ type of micromanipulator (Fig.1) is to design and build a small amplitude, high resolution parallel mechanism which yields high structural rigidity and balanced load distributions as compared to a serial mechanism.

The resulting compact small device can be mounted on any existing macro robot between the wrist and end effector to provide fine adjustments for precise error compensation and delicate force control through general spatial 6 D.O.F motion. Therefore, the proposed system

will superimpose high resolution micromanipulator control over the large motion range of macromanipulator module.

Thus, it permits not only real time compensation for high speed but also very accurate positioning for fine motion. The actuators of the large robot could be specially designed for high speed and flexibility, while the micromanipulator actuators would be specially designed for accurate fine motion. Conventional robots have two major limitations, lack of speed and positioning errors.

No robot is precise under large payload or at high speeds. If a robot is made stiffer for

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increased accuracy, speed is usually inversely affected. Limited accuracy is due to inertia effects, static friction and backlash in the joint, link bending deflections, and finite encoder resolution, etc.

Most industrial robots have been designed to carry large payloads in a relatively large workspace and their weight, size, and cost is also usually large. As a result, the resolution is not fine enough for high precision tasks which require positional error compensation at scales much smaller than the coarse resolution of the robot. In addition, robots do not operate in terms of a real time dynamic model to compensate for deformations resulting from external loads.

Since the large robot system is highly nonlinear, its operation would be difficult to treat dynamically in real time. However, due to the small motion range of the proposed micromanipulator system, it is highly linear and can easily compensate for system inaccuracies in real time.

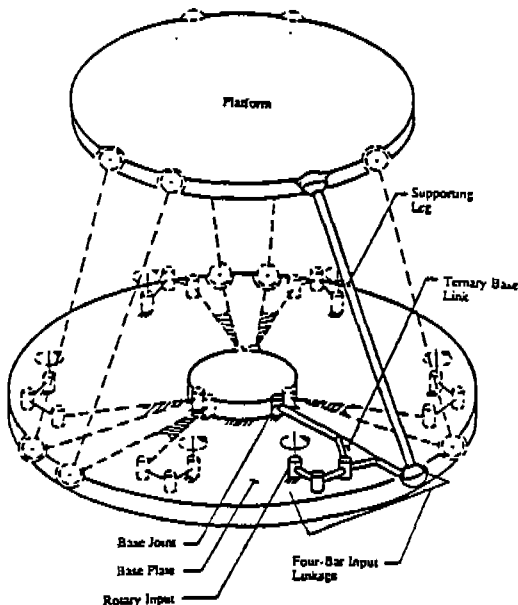


Fig.1 Kinematic representation of the micromanipulator

This means that the large motion could be compensated for by the small scale motion of the micromanipulator module.

2. DESIGN OBJECTIVES

As robotic functions become more and more complicated, and the range of applications is widened, optimal robot designs will be needed to minimize cost while maximizing performance. The design process seeks to define the basic parameters of the robot which are more or less closely associated with the mechanical structure. Those parameters to be considered are payload, mobility, workspace, compactness, repeatability, accuracy, agility, structural stiffness, damping coefficients, natural frequencies, and finally, economic factors like cost, reliability, maintainability, etc. The detailed design specifications were developed for the design of a micromanipulator.

- 6 D.O.F motion
- Payload capacity(10 lb to 100 lb)
- Motion range(+0.1" to -0.1", +2 degree to -2 degree)
- Size(depends on end effector diameter)
- High resolution(1 part in 1000)
- Bandwidth(greater than 50 HZ)
- No backlash
- Minimal friction
- Minimum weight
- Compactness
- Linearity
- Repeatability
- Rigidity

3. MODEL REPRESENTATION AND PARAMETERS

3.1 Kinematic model description of upper structure

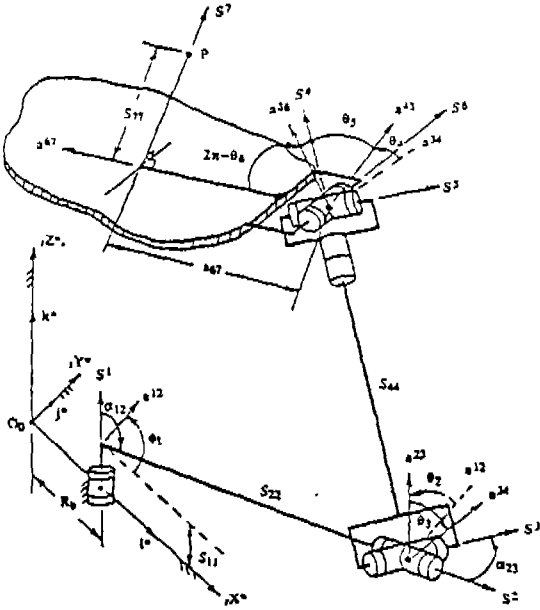


Fig.2 Kinematic representation of upper structure

3.2 Kinematic model description of lower 4 bar linkage

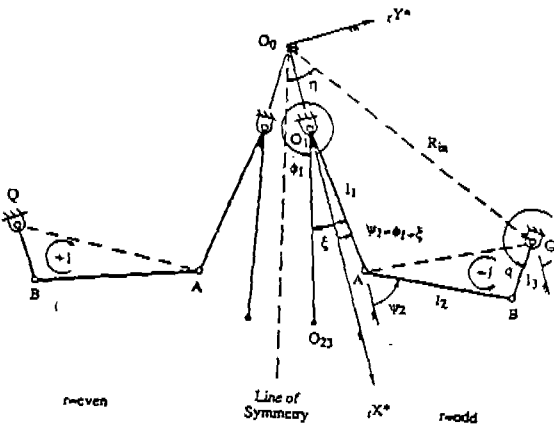


Fig.3 Kinematic representation of lower 4 bar linkage

3.3 Parameters to be optimized

Link dimensions :

$S_{11} = 0$ (inches)

S_{22} = normalized reference value

S_{44} = supporting leg

$S_{77} = 0$

L_1 = output link 4 bar

L_2 = coupler link in 4 bar

L_3 = input link 4 bar

Angular parameters (degrees)

λ_0 = base joint connection angle

γ_0 = platform connection angle

ξ = 4 bar linkage angle

ζ = motor position angle

Radial parameters (inches)

R_p = radius of the platform (inches)

R_b = radius of the base

R_{in} = radius of the motor position

4. DESIGN PROCESS

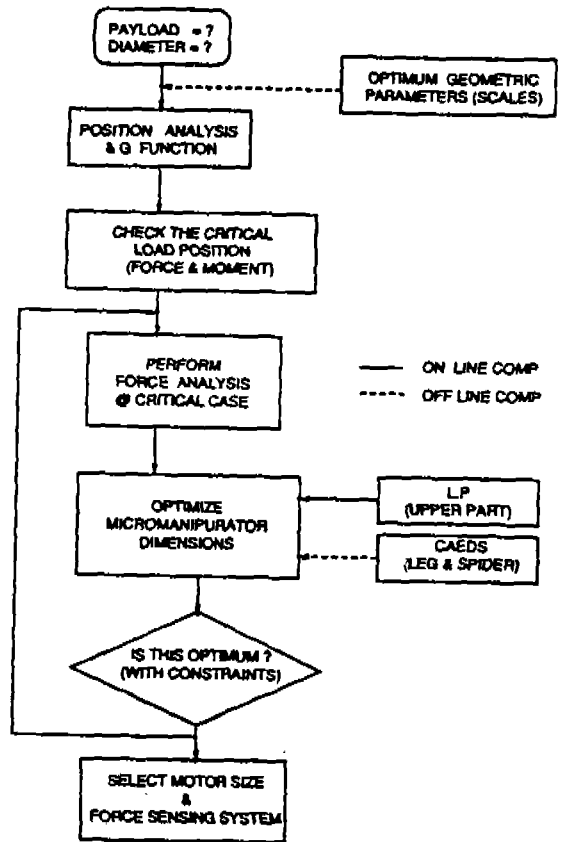


Fig.4 Flowchart of the design process

The flowchart of the design process shown in Figure. 4 includes the systematic steps needed to determine the optimal system geometry and dimensions of the detailed hardware design required for performing a given task. The goal is to use powerful design tools which address local and global design issues.

4.1 Kinematic & dynamic modeling analysis

The fundamental concepts in the general kinematic and dynamic model formulation are the Kinematic Influence Coefficient(KIC) (2, 3) and the generalized coordinate transformation(4). The idea of KIC is based on the separation of time dependent and position dependent functions. Further developments extended this approach to the analysis of serial manipulators(5, 6), and hybrid parallel/serial manipulators(7).

Feasibility study of a platform type of robotic manipulator from a kinematic viewpoint has been performed(8, 9). With the use of these ideas, a kinematic and dynamic model formulation referenced to the actuator coordinates for the fully parallel 6 D. O. F micromanipulator and a kinematic representations of this mechanism are represented in the literature(10). This analytical foundation will guide the design, manufacture, implementation, and control(11, 12) of this unique micromanipulator. The computer simulation based on these kinematic and dynamic modeling formulations will be used to determine optimal geometry for the micromanipulator. The author will briefly mention the basic kinematic and dynamic equations required in the modeling.

Using the chain rule, the first order time derivative of \underline{u} is

$$\begin{aligned} \frac{d\underline{u}}{dt} &= \frac{\partial \underline{u}}{\partial \underline{\phi}} \frac{\partial \underline{\phi}}{\partial t} \\ &= \frac{\partial \underline{u}}{\partial \underline{\phi}} \dot{\underline{\phi}} \end{aligned} \tag{4.1.1}$$

By defining the first and second order kinematic influence coefficients, which will be referred to as the G-function and H-function respectively, as

$$[G_{\bullet\bullet}^u] = \frac{\partial \underline{u}}{\partial \underline{\phi}} \quad [H_{\bullet\bullet\bullet}^u] = \left(\frac{\partial}{\partial \underline{\phi}} [G_{\bullet\bullet}^u] \right) = \left(\frac{\partial}{\partial \underline{\phi}} \frac{\partial \underline{u}}{\partial \underline{\phi}} \right) \tag{4.1.2}$$

then

$$\dot{\underline{u}} = [G_{\bullet\bullet}^u] \dot{\underline{\phi}} \tag{4.1.3}$$

and

$$\ddot{\underline{u}} = [G_{\bullet\bullet}^u] \ddot{\underline{\phi}} + \dot{\underline{\phi}}^T [H_{\bullet\bullet\bullet}^u] \dot{\underline{\phi}} \tag{4.1.4}$$

The dynamic equation is derived from the principles of virtual work and d'Alembert with previously obtained kinematic influence coefficient.

$$\underline{T}_{\bullet\bullet}^L = \sum_{j=1}^M \left\{ [{}^jG_{\bullet\bullet}^f]^T {}^j f^L + [G_{\bullet\bullet}^m]^T \underline{m}^{jK} \right\} \tag{4.1.5}$$

where $\underline{T}_{\bullet\bullet}^L$ is defined as the effective input loads.

The effective inertial load is

$$\underline{T}_{\bullet\bullet}^I = [I_{\bullet\bullet\bullet}^I] \ddot{\underline{\phi}} + \dot{\underline{\phi}}^T [P_{\bullet\bullet\bullet}^I] \dot{\underline{\phi}} \tag{4.1.6}$$

where

$$[I_{\bullet\bullet\bullet}^I] = \sum_{j=1}^M \left\{ M_{jK} [{}^jG_{\bullet\bullet}^c]^T [{}^jG_{\bullet\bullet}^c] + [G_{\bullet\bullet}^m]^T [\Pi^{jK}] [G_{\bullet\bullet}^m] \right\}$$

and

$$\begin{aligned} [P_{\bullet\bullet\bullet}^I] &= \sum_{j=1}^M \left\{ M_{jK} [{}^jG_{\bullet\bullet}^c]^T \cdot [{}^jH_{\bullet\bullet\bullet}^c] + [G_{\bullet\bullet}^m]^T [\Pi^{jK}] \right. \\ &\quad \left. \cdot [H_{\bullet\bullet\bullet}^m] + [G_{\bullet\bullet}^m]^T \left([G_{\bullet\bullet}^c]^T \cdot [\Xi^{jK}] \right) [G_{\bullet\bullet}^m] \right\} \end{aligned}$$

Having determined the equations for the applied and effective inertial loads, the general dynamic equation can be expressed as

$$\begin{aligned}
 \mathbf{I}_* &= \mathbf{I}_*^i - \mathbf{I}_*^l \\
 &= [\mathbf{I}_{**}] \ddot{\mathbf{q}} + \dot{\mathbf{q}}^T [\mathbf{P}_{**}] \dot{\mathbf{q}} \\
 &= \sum_{j=1}^M \left\{ [{}^j\mathbf{G}_*^T] {}^j\mathbf{f}_* + [{}^j\mathbf{G}_*^T] \mathbf{m}^{j*} \right\} \quad (4.1.7)
 \end{aligned}$$

4.2 Find optimum geometric parameters

First, one needs to fully understand how the basic parameters affect the motion range^(13, 14, 15), transmission ratio⁽¹⁶⁾, manipulability⁽¹⁷⁾, dexterity⁽¹⁸⁾, singularity, and agility. By using computer simulation based on the kinematic and dynamic formulations, one can determine specifications of the optimal system geometry satisfying a set of design objectives. Possible design objectives are: desired motion range, effective force and velocity transmission ratio, joint bending load, dexterity, agility, manipulability, and minimal variations in the elements of the modeling coefficients. There are 14 parameters to be optimized for this mechanism. The basic idea behind this is the generalized eigenvalue analysis (Appendix) of the system Jacobian matrix. These concepts are applied to set up optimal system geometry parameters for the micromanipulator by finding the maximum and minimum bounds of the transmission ratio⁽¹⁹⁾. Secondly, to maximize the motion range of the platform, one needs to synthesize the optimum 4 bar linkage considering the input and output angle ratio of this mechanism. A design tool has been developed and applied in a synthesis of the 4 bar linkage mechanism subject to constraints. The optimum set of geometric parameters and joint angle have been identified.

4.3 Force analysis

The objectives of the force analysis are to determine the required input, constraint reaction

forces, the internal stresses in components, bearing loads, and evaluation of the deformation in critical members of the system. A clear understanding of the forces acting in the system allows the designer to create an optimum shape of the components as well as to select appropriate materials and dimensions of those members to comply with the performance specifications. To do this, the module is divided into two parts which are the upper structure and the lower 4bar linkage.

A. Upper structure

The difficulty of force analysis of the Stewart type parallel mechanism is to solve the constraint reaction of the six legs, which connect the platform and base. We can assume the moment components are zero and consider only three unknown force components. This means that the axial load computed are sufficient to support the static applied loads and main source of loading (Fig. 5). This assumption is compatible with the physical reality of a relatively small servo motor and a close fit between the crank

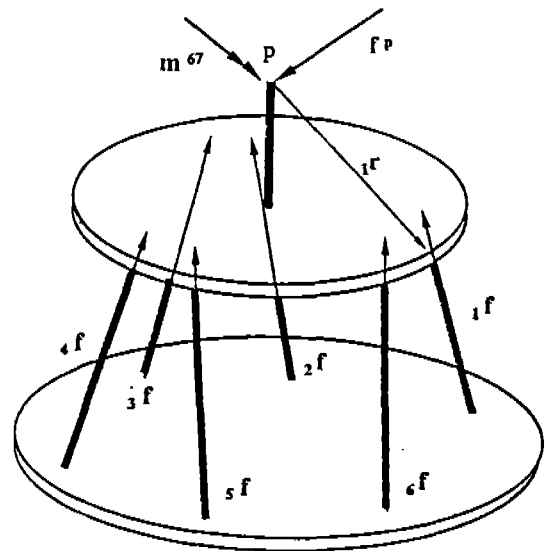


Fig. 5 Load vector applied to the platform

ends and the stiff base plates.

$$(i). \sum \mathbf{F} = 0$$

$${}_r \mathbf{f}^4 = {}_r \mathbf{f}^4 {}_r \mathbf{S}^4 \quad ; r = 1, 2, \dots, 6$$

$$\mathbf{F}^p = \sum_{r=1}^6 {}_r \mathbf{f}^4 {}_r \mathbf{S}^4$$

$$\mathbf{F}^p = \begin{bmatrix} {}_1 \mathbf{S}^4 & {}_2 \mathbf{S}^4 & {}_3 \mathbf{S}^4 & \dots & {}_6 \mathbf{S}^4 \end{bmatrix} \begin{Bmatrix} {}_1 \mathbf{f}^4 \\ {}_2 \mathbf{f}^4 \\ \vdots \\ {}_6 \mathbf{f}^4 \end{Bmatrix} \quad (4.3.1)$$

$$(ii). \sum \mathbf{M} = 0$$

$${}_r \mathbf{m}^{67} = {}_r \mathbf{r} \times {}_r \mathbf{f}^4$$

$$= - (a_{67r} \mathbf{a}^{67} + s_{77} \mathbf{k}) \times {}_r \mathbf{f}^4$$

$$= {}_r \mathbf{S}^4 \times (a_{67r} \mathbf{a}^{67} + s_{77} \mathbf{k}) ({}_r \mathbf{f}^4)$$

$$\mathbf{M}^{67} = \sum_{r=1}^6 {}_r \mathbf{S}^4 \times (a_{67r} \mathbf{a}^{67} + s_{77} \mathbf{k}) ({}_r \mathbf{f}^4)$$

$$\mathbf{M}^{67} = \begin{bmatrix} {}_1 \mathbf{S}^4 \times {}_1 \mathbf{r} & | & {}_2 \mathbf{S}^4 \times {}_2 \mathbf{r} & | \\ \dots & & \dots & & \dots \\ \dots & & {}_6 \mathbf{S}^4 \times {}_6 \mathbf{r} & & \dots \end{bmatrix} \begin{Bmatrix} {}_1 \mathbf{f}^4 \\ {}_2 \mathbf{f}^4 \\ \vdots \\ {}_6 \mathbf{f}^4 \end{Bmatrix} \quad (4.3.2)$$

$$\begin{Bmatrix} \mathbf{F}^p \\ \mathbf{M}^{67} \end{Bmatrix} = \begin{bmatrix} {}_1 \mathbf{S}^4 & | & {}_2 \mathbf{S}^4 & | \\ {}_1 \mathbf{S}^4 \times {}_1 \mathbf{r} & | & {}_2 \mathbf{S}^4 \times {}_2 \mathbf{r} & | \\ \dots & & \dots & & \dots \\ \dots & & {}_6 \mathbf{S}^4 & & \dots \\ \dots & & {}_6 \mathbf{S}^4 \times {}_6 \mathbf{r} & & \dots \end{bmatrix} \begin{Bmatrix} {}_1 \mathbf{f}^4 \\ {}_2 \mathbf{f}^4 \\ \vdots \\ {}_6 \mathbf{f}^4 \end{Bmatrix} \quad (4.3.3)$$

$$\begin{Bmatrix} {}_1 \mathbf{f}^4 \\ {}_2 \mathbf{f}^4 \\ \vdots \\ {}_6 \mathbf{f}^4 \end{Bmatrix} = \begin{bmatrix} {}_1 \mathbf{S}^4 & | & {}_2 \mathbf{S}^4 & | \\ {}_1 \mathbf{S}^4 \times {}_1 \mathbf{r} & | & {}_2 \mathbf{S}^4 \times {}_2 \mathbf{r} & | \\ \dots & & \dots & & \dots \\ \dots & & {}_6 \mathbf{S}^4 & & \dots \\ \dots & & {}_6 \mathbf{S}^4 \times {}_6 \mathbf{r} & & \dots \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{F}^p \\ \mathbf{M}^{67} \end{Bmatrix} \quad (4.3.4)$$

B. Lower four bar linkage

From previous force analysis of the leg in the upper structure, one can calculate the unknown reaction forces to establish the force balance. That reaction force is directly related to the required input torque at the motor. The force transmitted from the leg is decomposed into vertical and horizontal components. The horizontal force component is equivalent to the external force and 6 unknown forces at the link joint as well as required torque input can be obtained by solving 6 linear equations.

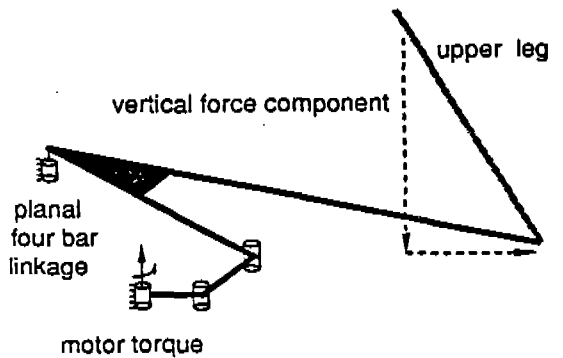


Fig.6 Force vector applied to the 4 bar linkage

4.4 Flexural member design

A. Optimum dimension of component

Special attention must be paid to design critical flexural components to have low stiffness, low inertia, and no backlash while maintaining a

structurally sound system. Here, the concept is to determine and optimize the dimensions of the necked down flexural joint part for specified deflection and predetermined link length is described. The flexural hinge to be designed is shown in the Figure.7

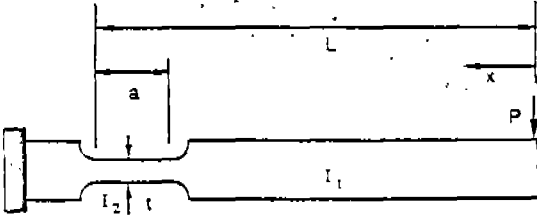


Fig.7 Flexural joint

The member consists of a beam of length 'L' in which a portion (length 'a') has a reduced cross section. The portion of the member between the load and the hinge has a moment of inertia I₁, which is much larger than the moment of inertia I₂ of the hinge portion of the beam. It is required to find the deflection of the end of the beam under load P.

Castigliano's theorem will be used to determine the deflection for the pure bending stress, the theorem stated as follows :

$$\delta = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial P} dx \quad (4.4.1)$$

The bending moment M is given by

$$M = PX \quad (4.4.2)$$

So,

$$\begin{aligned} \delta &= \int_0^{L-a} \frac{M}{EI_1} \frac{\partial M}{\partial P} dx + \int_{L-a}^L \frac{M}{EI_2} \frac{\partial M}{\partial P} dx \\ &= \int_{L-a}^L \frac{M}{EI_2} \frac{\partial M}{\partial P} dx \quad , \text{ with } I_1 \gg I_2 \end{aligned}$$

$$\begin{aligned} &= \int_{L-a}^L \frac{PX^2}{EI_2} dx \\ &= \left[\frac{PX^3}{3EI_2} \right]_{L-a}^L = \frac{P}{3EI_2} [L^3 - (L-a)^3] \\ &= \frac{P}{3EI_2} [3L^2a - 3a^2L + a^3] \quad (4.4.3) \end{aligned}$$

The stress in the flexural hinge can be determined from the flexure formula :

$$\begin{aligned} \sigma &= \frac{Mc}{I_2} \\ \sigma_{\max} &= \frac{PLc}{I_2} = \frac{PLt}{2I_2} \quad , \quad I_2 = \frac{PLc}{\sigma} = \frac{PLt}{2\sigma} \\ \text{where } M &= PL, c = t/2 \quad (4.4.4) \end{aligned}$$

Substitute this equation into the deflection equation to yield :

$$\delta = \frac{2\sigma}{3ELt} (3L^2a - 3a^2L + a^3) \quad (4.4.5)$$

To put in dimensionless form, divide both sides by L.

$$\begin{aligned} \frac{\delta}{L} &= 2 \left(\frac{\sigma}{E} \right) \left[\frac{a}{t} - \left(\frac{a}{L} \right) \left(\frac{a}{t} \right) + \frac{1}{3} \left(\frac{a}{L} \right)^2 \left(\frac{a}{t} \right) \right] \\ \frac{\delta}{L} &= 2 \left(\frac{\sigma}{E} \right) \left(\frac{a}{t} \right) \left[1 - \left(\frac{a}{L} \right) + \frac{1}{3} \left(\frac{a}{L} \right)^2 \right] \quad (4.4.6) \end{aligned}$$

This dimensionless form allows us to plot contours for determining link dimensions for specified deflections and L. Therefore, this equation gives δ/L as a function of a/t and a/L for any constant value of $\sigma/E = 0.001$. Figure.8 shows the chart for determining link dimensions.

For instance,
for steel, $E = 30 \times 10^6$ psi (Young's modulus)
 $\sigma = 30,000$ psi (strength)

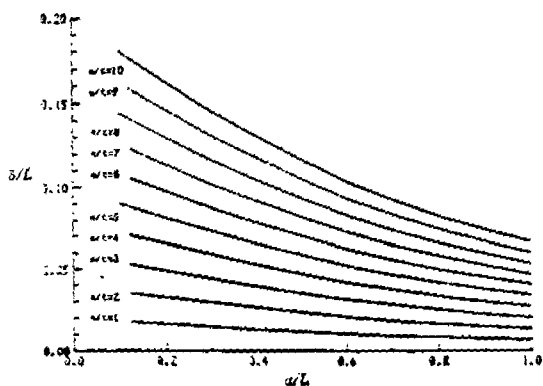


Fig. 8 Chart for determining link dimensions

Given $\delta=0.1''$, we can calculate δ/L . There we can choose various a/L resulting in different a/t 's from chart. This allows us to choose a particular thickness t for the given parameters.

B. Optimum cross section by Linear Programming

By finding the optimum cross section of the legs, we can minimize total weight, minimize inertial effects, and increase the payload. Optimization is carried out by using linear programming⁽²⁰⁾ under the design constraints which are displacement limitations, allowable axial stress in leg, avoid buckling, and positive cross section area.

i.e Minimize objective function

$$Z = \text{TOTAL WEIGHT} = F(X) \quad (4.4.7)$$

Subject to design constraints

$$G_j \geq 0 \quad j = 1, 2, \dots, m$$

m =number of constraints

x =cross section area

OBJECTIVE FUNCTION
MINIMIZE:

$$\sum_{i=1}^m \rho_i L_i X_i$$

CONSTRAINTS.

$$\sum_{i=1}^n \frac{P_{ij} L_i}{E_i X_i} \leq \delta_{jk} \quad \text{displacement limitation}$$

$$\sigma_{si} \leq \frac{P_{ij}}{X_i} \leq \sigma_{si} \quad \text{axial stress}$$

$$\sigma_{bi} \leq \frac{\pi^2 E_i I_i}{L_i^2 X_i} \quad \text{buckling stress}$$

$$\bar{X}_i \leq X_i$$

4.5 Finite element application for analysis

Final checking or modification to insure meeting the design criteria is made based on the local maximum stress and deflection analysis. The component models are analyzed using Computer Aided Engineering Design System (CAEDS) which is an integrated finite element solver, composed of the finite element analysis package and pre, post processing package*. Here, the two important parts in the micromanipulator design, the supporting leg (Fig. 9-12) and the planar four bar input linkage mechanism (Fig. 13-16) are analyzed. The results of the components analysis can then be interactively displayed using the Graphics system serving as a postprocessor.

5. CONCLUSION

The idea of the simulation, computer aided design, optimization theory, and finite element method is being used in designing the 6 D. O. F. fully parallel micromanipulator robot. Design objectives and the detailed design process which shows the systematic steps needed to determine the optimal system geometry and dimensions of the components to comply with

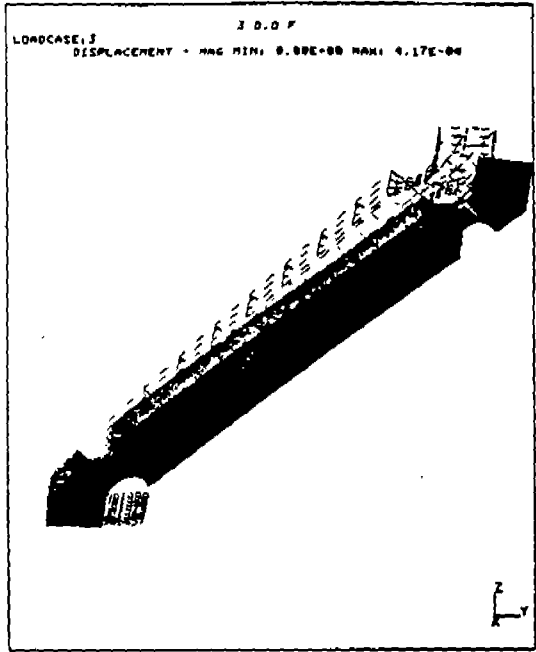
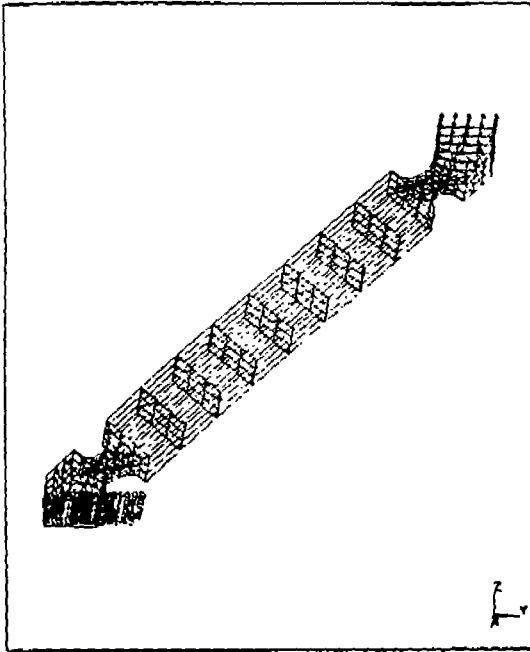


Fig.9 Finite element model developed with CAEDS

Fig.10 Deflection analysis of the leg

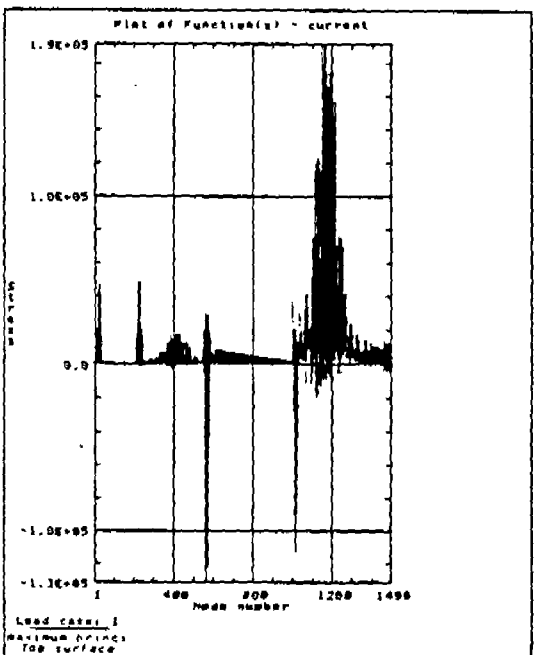
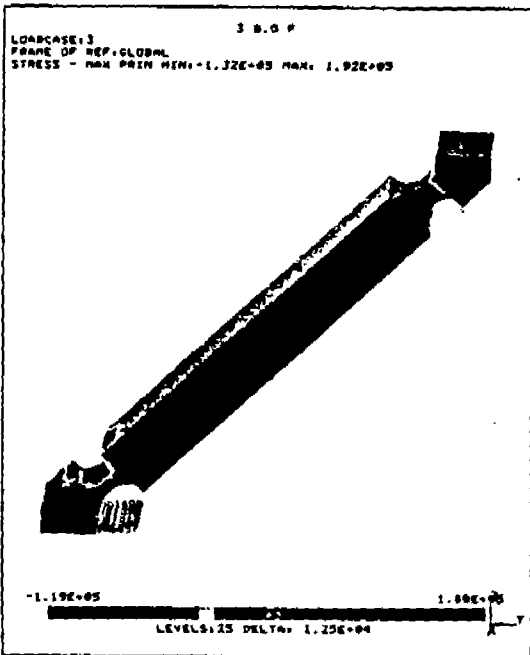


Fig.11 Element stress analysis of the leg

Fig.12 Node stress analysis of the leg

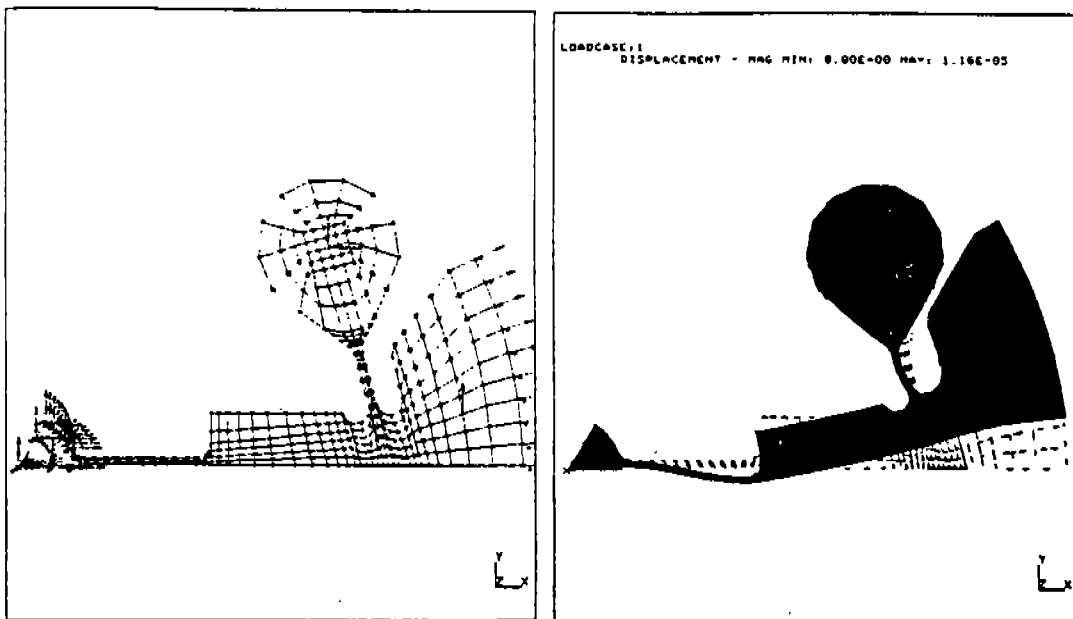


Fig. 13 Finite element model developed with CAEDS

Fig. 14 Deflection analysis of the 4 bar link

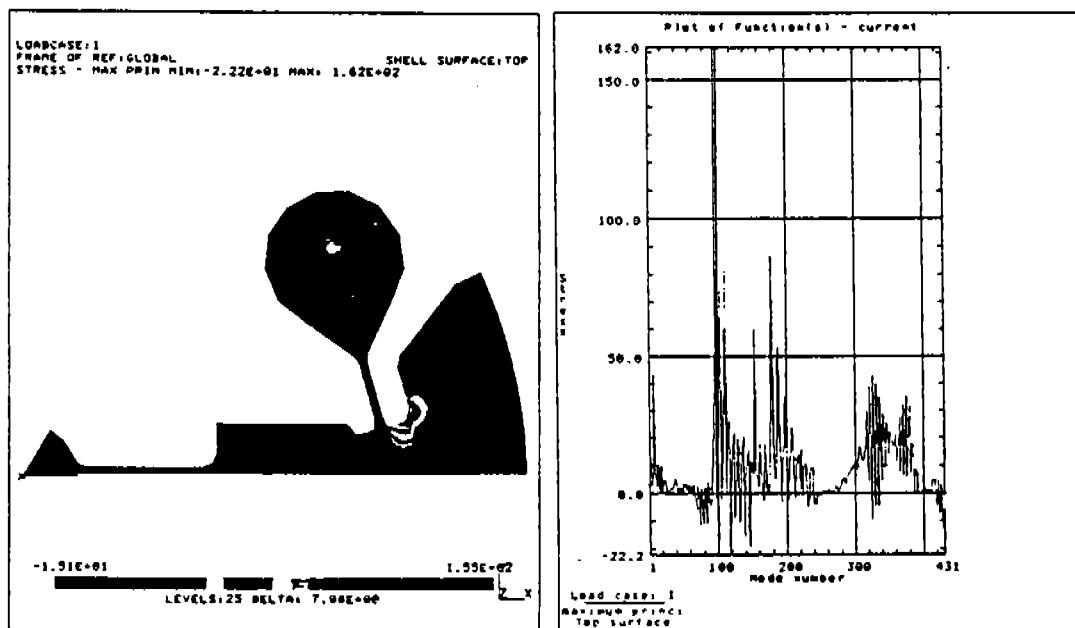


Fig. 15 Element stress analysis of the 4 bar link

Fig. 16 Node stress analysis of the 4 bar link

performance specifications are analyzed. The use of those integrated tools permits an increase in the productivity of the designer and a more through engineering analysis.

Appendix

It is useful to define the ratio of the Euclidean norm (vector 2 norm) of the generalized input load to that of a generalized output load.

$$\tau = \frac{\|T_i\|}{\|T_o\|} \quad (\text{a.1})$$

then,

$$\|T_i\| = (T_i^T T_i)^{1/2}$$

$$\|T_o\| = (T_o^T T_o)^{1/2}$$

$$T_i = [G]^T T_o.$$

Therefore,

$$\tau = \frac{\begin{bmatrix} T_i^T & T_i \\ T_o^T & T_o \end{bmatrix}^{1/2}}{\begin{bmatrix} T_o^T [G] [G]^T T_o \\ T_o^T & T_o \end{bmatrix}^{1/2}} \quad (\text{a.2})$$

The expression inside the square root is known as the Rayleigh quotient⁽¹⁹⁾, i.e. the eigenvalues λ of the matrix product $[G][G]^T$ can be used to describe the global condition of the system. Then, the ratio of the vector 2 norm of the input to output load is bounded by the square root of λ_{\min} and λ_{\max} ,

$$(\lambda_{\min})^{1/2} \leq \frac{\|T_i\|}{\|T_o\|} \leq (\lambda_{\max})^{1/2} \quad (\text{a.3})$$

$$\|T_o\| (\lambda_{\min})^{1/2} \leq \|T_i\| \leq \|T_o\| (\lambda_{\max})^{1/2}. \quad (\text{a.4})$$

An analogous result can be derived for velocity using the fact that $[G][G]^T$ is symmetric. But there is a contradiction not only between

force and velocity from the compatibility point of view, but also between respective magnitudes from the control point of view. Awareness of this contradiction allows the designer to make a satisfactory balance or tradeoff between these two values.

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