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기하학적 공차의 정밀측정을 위한 컴퓨터 모듈개발

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Development of Computer Modules for Precision Measurement of Geometric Tolerances

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ABSTRACT

제품의 기하학적 칫수 공차는 생산제품의 품질관리에 매우 중요하며, 측정기기의 발달과 함께, 컴퓨터를 이용한 기하학적 공차 해석의 수요와 경향은 늘고 있는 추세이다. 본 논문에서는, ISO의 공차 정의에 기초한 공차들(진직도공차, 편평도공차, 진원도공차, 평행도공차, 직각도공차, 윤곽공차)을 해석할 수 있는 컴퓨터 모듈이 개발되었다. 특히 윤곽공차의 경우 측정프로브의 반경의 효과가 배제되는 알고리즘을 개발하였다. 개발된 컴퓨터 모듈을 실제적인 측정작업에 적용하였으며, 고정도와 고효율성을 보임을 입증하였다.

Key Words : Precision Measurement , Geometric Tolerances, Computer

1. Introduction

Geometric tolerances in dimensional measurement for products play important roles in manufactruing now a days, as better accuracy and precision are required for the machined parts.

With the advent of computer integrated measurement equipments, there are increasing trends in computer aids in engineering metrology, for example, computer assisted coordinate measuring machines.

Several types of geometric tolerances are specified in the ISO[5], and the form and orientation tolerances are commonly used in the manufacturing.

There were several numerical methods for claculating the geometric tolerance : the least squares technique was conventionally adopted for the calculation of straightness and flatness tolerance, [e · g · 8]. Murthy and Abdin[6] used numerical methods such as simplex search to obtain precise results for the minimum separation of straightness, roundness, and flatness. Chetwynd[3] applied the linear programming techniques such as stiefel exchange algorithm and revised simplex for the straightress, flatness, and roundness. Burdekin and Pahk[2] proposed the enclose tilt technique, which can calculate the flatness and straightness in terms of minimum separation, as the standards define. Menq[9] proposed an iterative methods based on the least squares approach for the evaluation of the profile tolerance.

In this research, a comprehensive module has been proposed, where the basic geometric

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features can be evaluated with multiple options such as the conventional least squares technique and the minmum separation technique. An efficient algorithm has been developed for the profile·tolerance considering the effect of probe radius.

The developed module has been applied to measurement tasks and showed efficiency with high precision.

2. Geometric Tolerances

There are several types of geometrical tolerances in ISO[5], and the types of ISO tolerance is shown in table 1. A geometrical tolerance applied to a fearure defines the tolerance zone within which the feature is to be contained. There are four basic types of geometrical tolerances : form, orientation, location, and run-out. These four basic types of tolerances can also be classified into two major groups by their relationships to datum : related features and single features. The single features are defined by the single datum, and the related features are defined by the relationship between the features. Form tolerances state how far the actual features are permitted to vary from the designed nominal form, and consist of standard form features and non standard form features. Standard form features include lines, planes, circles, and cylinders.

Non-standard form features include curves and surfaces. Thus the corresponding form tolerances are defined as straightness, flatness, circularity(roundness), sphericity, cylindricity, and profile of a curve or surfaces. The form tolerance are described by tolerance zones which set the limits of the extreme boundaries of features. Orientation' tolertation specify the geometrical relationships to datums. Three types of orientation tolerances are parallelism, perpendicularity, and angularity. Location tolerances state the permissible variation in the specified position of a feature in relation to some other features or datum : true position concentricity, coaxility, and symmetry. Run-out tolerance is the deviation from the perfect form of a part surface of revolution directed by full rotation of the part on a datum axis. In this paper, form tolerance and some of orientation tolerances are considered, which are frequently used in practical measurement works.

3. Tolerance Evaluation

3.1 Straightness tolerance

The straightness tolerance is defined by a distance, t, between two parallel lines containing the measured line feature as shown in fig. 1. Mathematically, the straightness tolerance can be calculated from the deviation of the measured line feature from the best fit straight line. The least squares technique can be applied as an approximate technique, and the Simplex search algorithm also can be used for the minimum separation.

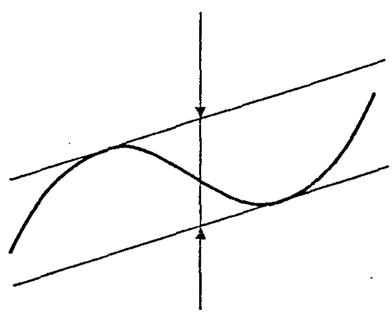

Fig. 1 Straightness tolerance

Least squares technique

Let(Xi, Yi) be the measured data of line feature, where Yi is height, and Xi is nominal coordinate at the ith point along a measurement direction. A best fit slope(A), and offset(B) can be derived from the variational principle such that $Y=AX+B$ can be the best fit straight line for the measured data. The sum of squares of errors, E, is

$$E=\sum(Y_i-AX_i-B)^2, \quad i=1, 2 \cdots N$$

where N is the total number of nominal coordinates.

In order to have minimum E, the variational principle can be applied such that the increment ΔE become zero. Thus, the partial derivative of E with respect to A, B become zero. That is,

$$\partial E/\partial A=2\sum(Y_i-AX_i-B)(-X_i)=0$$

$$\partial E/\partial B=2\sum(Y_i-AX_i-B)(-1)=0$$

The unknown, A, B can be evaluated from the Gaussian elimination technique.

Therefore the straightness error, dY_i, is

$$dY_i=Y_i-AX_i-B$$

and straightness tolerance$=\max(dY_i)-\min(dY_i)$ (1)

Simplex Search Technique

The straightness calculation can be performed by the mathematical linear programming, and the Simplex search method is applied in this paper.

Two parallel lines containing the measured line feature(X_i, Y_i) can be expressed as follows.

$$Y=AX+B$$

$$Y=AX+B'$$

The mathematical model for this case is

MINIMIZE $B-B'$

subject to $AX_i+B' \leqq = Y_i$

$AX_i+B \geqq = Y_i$

$i=1, 2 \cdots N$

This is the dual formulation and the coefficients A, B as well as the objective function $B-B'$ can be calculated using the Simplex search algorithm. The flow chart for the simplex search algorithm is given in the appendix. The straightness tolerance can be calculated as $B-B'$. Thus

Straightness Tolerance$=B-B'$ (2)

2.2 Flatness tolerance

The flatness tolerance is defined by a distance, t, apart by two parallel planes containing the measured feature as shown in fig.2, and it is planar expansion of the straightness tolerance. Similarly, the least squrares technique and the Simplex search algorithm for the minimum separation can be applied.

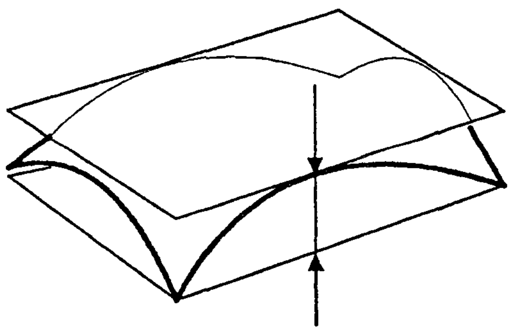

Fig.2 Flatness tolerance

Least squares technique

Assume that the measured data are on the XY plane, Z_i is measured height data at(X_i, Y_i) location on the XY plane. The best fit plane is to be evaluated from the measured datum, thus the flatness error can be calculated as the deviation of the measured height data with respect to the best fit plane.

Let $Z=AX+BY+C$ be the best fit plane, the coefficients, A, B, C can be calculated as follows.

The sum of squares of the deviation, E, is

$$E=\sum(Z_i-AX_i-BY_i-C)^2$$

The coefficients, A, B, C can be claculated from the variational principle.

$\partial E/\partial A=\sum 2(Zi-AXi-BYi-C)(-Xi)=0$

$\partial E/\partial B=\sum 2(Zi-AXi-BYi-C)(-Yi)=0$

$\partial E/\partial C=\sum 2(Zi-AXi-BYi-C)(-1)=0$

Thus the flatness deviation, dZi, is

$dZi=Zi-AXi-BYi-C$

and the flatness tolerance is the difference between the maximum and the minmum of the flatness deviation, dZi.

$$\text{Flatness tolerance}=\max(dZi)-\min(dZi) \qquad (3)$$

Simplex Search

The Simplex search also can be applied to the case of flatness evaluation. Consider two parallel planes containing the whole measured datum. When the upper plane is $Z=AX+BY+U$, and the lower plane is $Z=AX+BY+L$, the mathematical linear programming model can be defined as follows.

MINMMIZE U-L

Subject to $AXi+BYi-U \leqq -Zi$

$AXi+BYi-L \geqq -Zi$

for $i=1, 2\cdots N$

This model can be solved by the Simplex search, thus the coefficients, A, B, C, U, L can be successfully assessed.

The flatness tolerance can be obtained as the difference the U and L, thus,

$$\text{Flatness Tolerance}=U-L \qquad (4)$$

3.3 Parallelism tolerance

The parallelism tolerance is defined as the tolerance zone limited by two parallel planes which is parallel to the reference plane as shown in fig. 3.

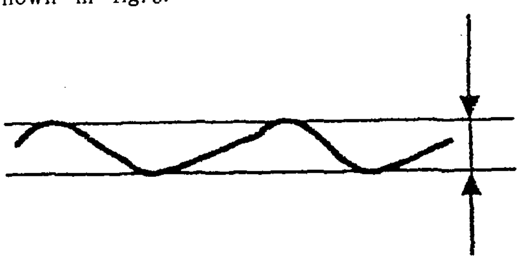

Fig. 3 Parallelism tolerance

In order to evaluate the parallelism tolerance, the reference plane datum are to be given. The referance plane can be constructed based on the best fit planes for the measured feature, and either the least squares technique or the simplex search are also used for the reference plane.

Let $Ar\ X+Br\ Y+Cr\ Z+Dr=0$ be the plane equation, and (Xi, Yi, Zi) be the measured data for the parallelism feature. The algebraic distance, D, between the reference plane and the measured data (Xi, Yi, Zi), is

$$D=(Ar\ Xi+Br\ Yi+Cr\ Zi+Dr)/\sqrt{(Ar^2+Br^2+Cr^2)}$$

The parallelism tolerance, PT, is the difference between the maximum and the minimum algebraic distance. Thus,

Parallelism tolerance,

$$PT=D\ \max\ D-\min\ D \qquad (5)$$

3.4 Squareness tolerance

The squareness error is in general the out of squareness angle (90 degree), and the squareness tolerance is the tolerance zone limited by two perpendicular planes to the given reference plane, where the two perpendicular planes contain the whole mesured feature as shown in fig. 4.

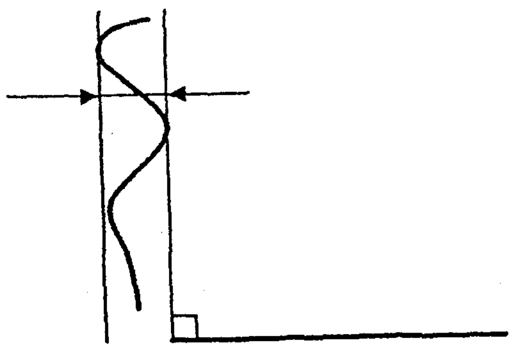

Fig. 4 Squareness tolerance

There can be several possible ways for the squareness evaluation, and the measured datum are projected onto the perpendicular plane in this research.

In order to evaluate the squareness tolerance, the reference plane datum are to be given, and let $A_r X+B_r Y+C_r Z+D_r=0$ be the best fit reference plane from the measured datum, and $A X+B Y+C Z+D=0$ be the best fit plane for the measured plane datum for squareness. Then a projection plane which is perpendicular to the measured plane and the reference plane can be constructed, where the measured datum are to be projected onto that plane as in fig. 5. In fig. 5, let e_M, e_R be the normal vectors to the measured plane and the reference plane, respectively. A normal vector e_N which is normal to the e_M, e_R vectors, and it is the normal vector to the projection plane.

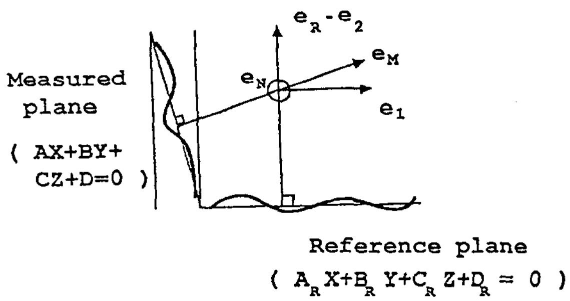

Fig. 5 Determination of the projection plane for squareness tolerance

The e_N vector can be obtained from the vector product of the two normal vector to the measured datum. Thus, e_N vector is

$$e_N=e_M \times e_R=(A,\ B,\ C)\times(Ar,\ Br,\ Cr)$$

The projection plane can be defined by the normal vector, e_N;and the base vectors, e1, e2, can be.

also calculated for the plane. The e2 vector can be assigned as the unit vector parallel to the e_R, as the e_R vector is the normal to the reference plane. The e1 vector can be calculated as the vector product of the e_R and the e_N vector, thus,

$$e2=eR/|eR|,\quad e1=(e2\times e_N/|e2\times e_N|)$$

Now the measured datum are projected onto the projection plane, the projected coordinate (Axi, Ayi) after projection is

$$Axi=(Xi,\ Yi,\ Zi)\cdot e1$$
$$Ayi=(Xi,\ Yi,\ Zi)\cdot e2$$

As the projection plane is perpendicular to the reference plane, the maximum difference in the Axi will give the squareness tolerance. That is,

$$\text{Squareness tolerance} = \max(Axi) - \min(Axi) \quad (6)$$

3.5 Circularity (Roundness) tolerance

The circularity tolerance is defined as the tolerance zone limited by two concentric circles a distance, t, apart as shown in fig. 6.

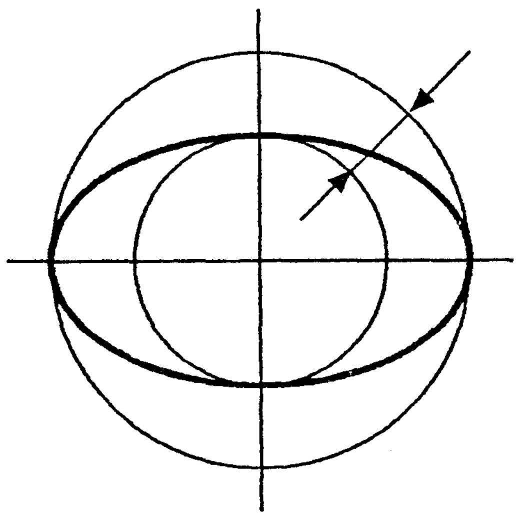

Fig. 6 Circularity (roundness) tolerance

There are usually 4 methods[1] for evaluation of the circularity : least squares circle (LSC), minimim zone circle (MZC), minimum circumscribed circle (MCC), and maximum inscribed circle(MIC). The LSC is that the sum of the squares of the departure be minimum, and the MZC is that two concentric enclosing the datum have the least radial separation. The MCC is the smallest possible circle of shaft that can be fitted within a hole, and the MIC is the largest possible circle that can be fitted around a shaft. In this paper, the LSC and MZC are considered, which are generally used in most of practical roundness measurement cases.

Least Squares Circle(LSC)

Let(Xi, Yi) be the measured datum, and (a, b), R be the center coordinate and the radius to be evaluated. The sum of squares of departure, E, is

$$E = \sum (R^2 - (Xi-a)^2 - (Yi-b)^2)^2$$
$$= \sum (R^2 - Xi^2 - Yi^2 - a^2 - b^2 + 2aXi + 2bYi)^2$$

If we replace $R^2 - a^2 - b^2$ as C, 2a as A, and 2b as B, then the above equation is

$$E = \sum (C + AXi + BYi - Xi^2 - Yi^2)^2$$

applying the variational principle,

$$\partial E/\partial A = 2\sum (C + AXi + BYi - Xi^2 - Yi^2)(Xi) = 0$$
$$\partial E/\partial B = 2\sum (C + AXi + BYi - Xi^2 - Yi^2)(Yi) = 0$$
$$\partial E/\partial C = 2\sum (C + AXi + BYi - Xi^2 - Yi^2) = 0$$

The unknowns, A, B, C can be evaluated by the Gauss elimination technique.

Thus the circular departure, DRi, is

$$DRi = Ri - R = \sqrt{((Xi-a)^2 + (Yi-b)^2} - R$$

and the circularity tolerance, CT, is

$$CT = \max(DRi) - \min(DRi)$$

Minimum Zone Circle(MZC)

The minimum zone circle can be obtained from linear programming technique, and let(a, b) be the center coordinate, and R1, R2($R1 < R2$) be the radii of two concentric circles enclosing the measured datum, (Xi, Yi). The mathematical programming for the problem is,

MINIMIZE R2 - R1

subject to $(Xi-a)^2 + (Yi-b)^2 \leqq R2^2$

$(Xi-a)^2 + (Yi-b)^2 \geqq R1^2$

for i=1, 2 ⋯ N

The above problem is, however, nonlinear programming, thus needs linear approximation or non-linear techniques.

If we set $R2^2 - a^2 - b^2 = A$, $R1^2 - a^2 - b^2 = B$, and the object function R2 - R1 is replaced by $R2^2 - R1^2$, then the problem is changed to the linear programming.

That is,

MINIMIZE A - B

subject to $-2aXi - 2bYi - A \leqq -Xi^2 - Yi^2$

$-2aXi - 2bYi - B \geqq -Xi^2 - Yi^2$

i=1, 2 ⋯ N

which is the typical linear problem and can be solved by the Simplex method. The radial departure and the circularity(roundness) tolerance can be evaluated.

3.6 Cylindricity tolerance

The cylindricity tolerance is defined as the tolerance zone limited by two coaxial cylinders a distance, t, apart as shown in fig. 7. In this paper, the cylindricity tolerance is assessed by the least sauares technique.

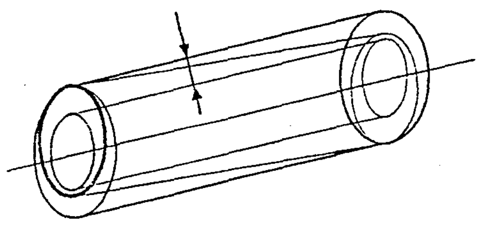

Fig. 7 Cylindricity tolerance

Least squares technique

Let the measured datum be (Xi, Yi, Zi), (a, b, c) be the center coordinate of the lower circle of the cylinder, and (l, m, n) be the unit vector defining the direction of cylinder.

The equation of the straight line of the

Central axis : $(X-a)/l=(Y-b)/m=(Z-c)/n$ and the radial distance, Ri, from the central axis to the measured datum (Xi, Yi, Zi) can be defined as

$$Ri=\sqrt{((n(Xi-a)-l(Zi-c))^2+(m(Xi-a)-l(Yi-b))^2+(n(Yi-b)-m(Zi-c))^2}$$

The sum of squares of the departure in the radial direction can be expressed as

$$E=\sum(Ri^2-R^2)^2$$

applying the variational principle

$$\partial E/\partial a=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial a=0$$
$$\partial E/\partial b=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial b=0$$
$$\partial E/\partial c=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial c=0$$
$$\partial E/\partial l=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial l=0$$
$$\partial E/\partial m=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial m=0$$
$$\partial E/\partial n=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial n=0$$
$$\partial E/\partial R=2\sum(Ri^2-R^2)\quad\partial(Ri^2-R^2)/\partial R=0$$

This is the nonlinear equation, and can be solved by the steepest descent algorithm for the unknowns, a, b, c, l, m, n, and R.

Thus the cylindricity departure dRi becomes

$Ri-R$

and the cylindricity tolerance is the difference between the maximum and the minimum of the departure, that is,

$$\text{cylindricity tolerance} =\max(dRi)-\min(dRi) \qquad (8)$$

3.7 Profile tolerance

The profile tolerance for surface is defined by the tolerance zone which is limited by two surfaces enveloping spheres of diameter, t, the centers of which are situated on a surface having the true geometrical form, as shown in Fig. 8.

In this paper, the reference profile is given by the CAD defined sculptured surface, and thus the surface of true geometrical form can be replaced by the CAD surface, and the measured data are assumed to be obtained from coordinate measuring equipments such as coordinate measuring machines(CMMS), which is very common for the profile measurement.

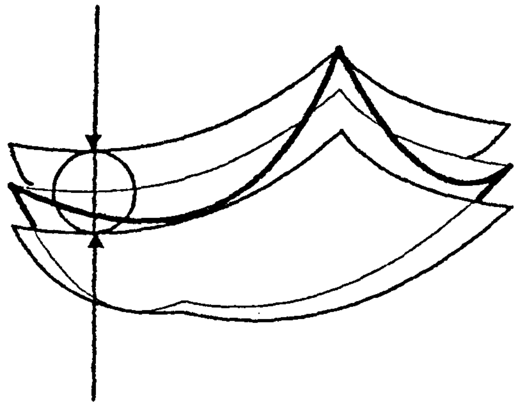

Fig. 8 Profile tolerance of surface

Mathematical description of sculptured surface

There are several ways for describing sculptured surfaces mathematically, and it is very common to construct a sculptured surface with several patches of cubic polynomial surfaces. The conventional parametric polynomial surface is implemented in this system. Let u, v_x be the principal parameter defining the principal direction in a surface patch, then the point data on the surface, P(u, v) can be described in the standard parametric surface.

$$P(u,\ v)=\sum\sum Aij(u^i)(v^j) \qquad (9)$$

The point data P(u, v) consist of X(u, v), Y(u, v), and Z(u, v);and an example of the coefficients, Aij, are given on the table 2, where the first column give those for the X component, the second and third column give for the Y, Z components, respectively.

Form Error Evaluation for Sculptured Surface

The error calculation based on the comparison of the measured data to the CAD defined profile data. There are two types of error in the sculptured surface as shown in fig. 9 : (1) the

Table 1 Types of tolerances in ISO

Features and tolerances		Toleranced characteristics
Single features	Form tolerances	Straightness Flatness Circularity Cyindricity
Single or related features		Profile of any line Profile of any surface
Related features	Orientation tolerances	Parallelism Perpendicularity Angularity
	Location tolerances	Position Concentricity and Coaxiality Symmetry
	Runout tolerances	Circular run-out Total run-out

Table 2 Typical example of coefficients defining a sculptured surface

X	Y	Z	
0.00000	0.00000	-10.000	A_{00}
0.00000	100.00000	106.670	A_{01}
0.00000	0.00000	-320.000	A_{10}
0.00000	0.00000	213.333	A_{11}
0.00000	0.00000	0.000	A_{20}
100.00000	0.00000	0.000	A_{21}
0.00000	0.00000	-213.333	A_{30}
0.00000	0.00000	640.000	A_{31}
0.00000	0.00000	-426.670	A_{40}
0.00000	0.00000	0.000	A_{41}

form error which gives the profile tolerance (2) the offset involved in tilting or translation of the sculptured surface with respect to a reference datum. An efficient algorithm has been implemented in this system to evaluate the form error and the offset.

Consider Pi be the measured coordinate data (center coordinate of the measurement probe of CMM);and Qi, Ri be the CAD coordinate data and the normal vector at Qi point, such that the (Qi+Ri) be the corresponding CAD coordinate to the measured data, Pi. Assuming T as the transformation matrix between the CMM coordinate and the CAD coordinate, the maximum Euclidean distance, D, between the Pi and the (Qi+Ri) is

$$D = \max | T^{-1} * Pi - (Qi+Ri) |, \quad \text{for } i=1..N \tag{10}$$

An optimal transformation matrix can be found such that the maximum Euclidean distance, D, be minimum. Thus the form error, that is, the profile tolerance, is

$$\text{Form error(profile tolerance)} = \min D \tag{11}$$

where the optimal transformation matrix gives the offset.

The relationship between the Pi, Qi, T matrix, and Ri are shown in fig. 9a, and the form error and the offset are conceptually shown in fig. 9b.

An error calculation algorithm has been developed uniquely in the research : The corresponding CAD coordinate, Qi, and the

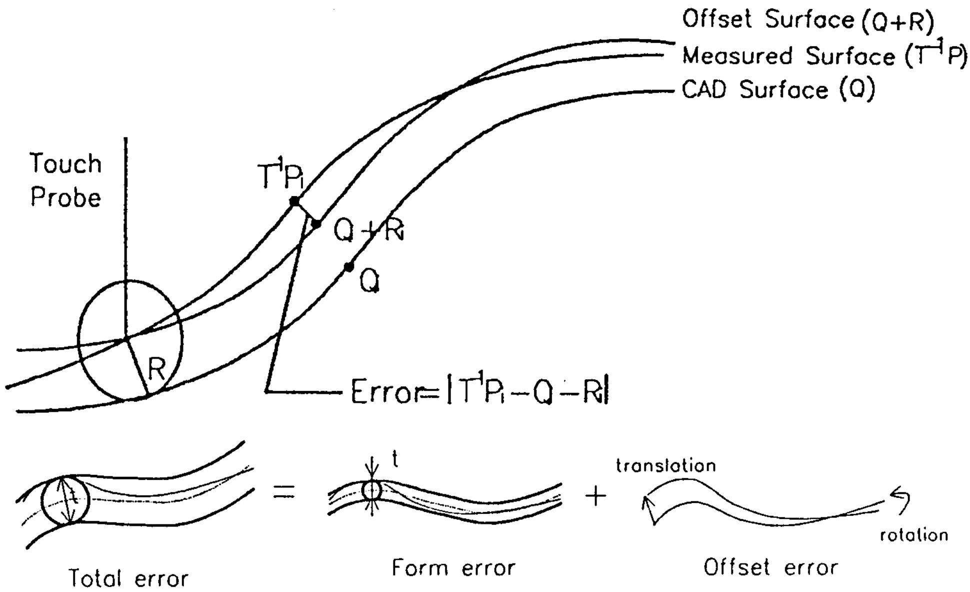

Fig. 9 a : Error evaluation for sculptured surface
b : Form error and offset for sculptured surface

normal vector, Ri, can be found as the minimum distance point by the subdivision technique, where recursive subdivisions are performed on the suclptured surface until the specified tolerance is met. After Qi and Ri are found, the new relationship between the Pi and the (Qi+Ri) can be constructed based on the least squares approach. Thus the transformation matrix, T, is newly defined. Then the above procedures are repeated until the maximum Euclidean distance, D, converges to a minimum value. Fig. 10 shows the flow chart for the developed algorithm for the profile tolerance calculation.

4. System implementation and practical appliction

The computer modules for assessing geometric tolerance have been implemented, and installed around a micro computer. The developed computer system has been applied to the measured datum, in order to assess the validity and efficiency.

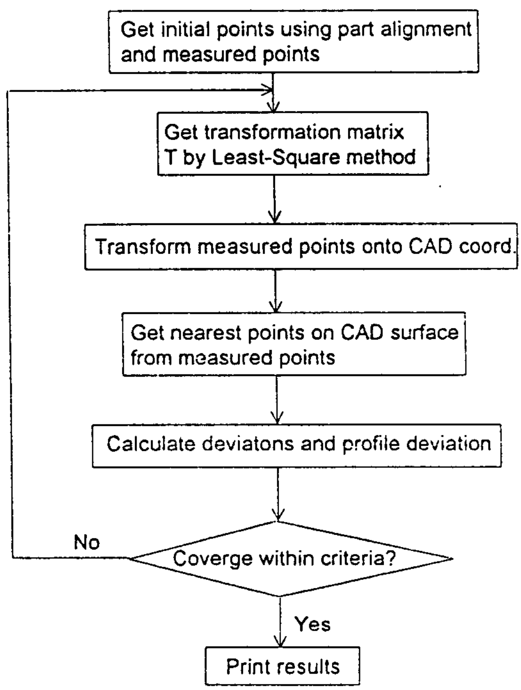

Fig. 10 Flow chart for the form error calculation

Table 3 Straightness tolerance analysis data

X	Y	Deviation (Least square)	Deviation (Minimum zone)
100.247	203.269	10.416	6.500
113.250	203.248	-11.000	-14.500
122.249	203.255	-4.288	-7.500
131.260	203.256	-3.576	-6.500
143.240	203.263	3.040	0.500
151.252	203.279	18.784	16.500
163.227	203.275	14.401	12.500
171.286	203.253	-7.857	-9.500
189.287	203.274	12.567	11.500
197.269	203.251	-10.689	-11.500
205.251	203.257	-4.944	-5.500
215.282	203.259	-3.265	-3.500
225.270	203.251	-11.585	-11.500
235.266	203.246	-16.905	-16.500
245.248	203.278	14.776	15.500
255.261	203.265	1.456	2.500
265.251	203.249	-14.864	-13.500
275.262	203.259	-5.184	-3.500
285.278	203.263	-1.505	0.500
295.251	203.266	1.176	3.500
305.242	203.268	2.856	5.500
318.271	203.267	1.439	4.500
321.257	203.269	3.344	6.500
333.278	203.265	-1.041	2.500
349.245	203.279	12.448	16.500

STRAIGHTNESS TOLERANCE
Least square method : 35.6um
Minimum zone method : 33.0um

Table 4 Flatness tolerance analysis data

X	Y	Z	Deviation (Least square)	Deviation (Minimum zone)
84.000	84.000	91.554	1.080	-0.750
84.000	68.000	91.548	-4.140	-6.250
84.000	52.000	91.560	8.640	6.250
84.000	36.000	91.553	2.420	-0.250
84.000	20.000	91.547	-2.800	-5.750
68.000	20.000	91.548	-1.020	-3.250
68.000	36.000	91.546	-3.800	-5.750
68.000	52.000	91.552	1.420	-0.250
68.000	68.000	91.551	-0.360	-1.750
68.000	84.000	91.552	-0.140	1.250
52.000	84.000	91.553	1.640	1.250
52.000	68.000	91.546	-4.580	-5.250
52.000	52.000	91.548	-1.800	-2.750
52.000	36.000	91.546	-3.020	-4.250
52.000	20.000	91.549	0.760	-0.750
36.000	20.000	91.551	3.540	2.750
36.000	36.000	91.552	3.760	3.250
36.000	52.000	91.543	-6.020	-6.250
36.000	68.000	91.556	6.200	6.250
36.000	84.000	91.548	-2.580	-2.250
20.000	84.000	91.550	0.200	1.250
20.000	68.000	91.552	2.980	3.750
20.000	52.000	91.547	-1.240	-0.750
20.000	36.000	91.545	-2.460	-2.250
20.000	20.000	91.548	1.320	1.250

FLATNESS TOLERANCE
Least square method : 14.7um
Minimum zone method : 12.5um

Table 3 shows the straightness analysis data, where the (X, Y) datum are from practical measurement of an edge using a CMM. The first, second give (Xi, Yi) datum, and the third, fourth column give the straightness departure based on the least squares technique, and the minimum zone technique, respectively. The straightness tolerance is 35.6um, 33.0um respectively.

Table 4 shows the flatness tolerance analysis data, where the first 3 column give (X, Y, Z) measured data from practical measurement of a granite block using a CMM. The data of flatness departure are given on the third, fourth column, based on the least squares technique, and the minimum zone technique, respectively. The flatness tolerance were evaluated as 14.6um, 12.5um, respectively.

Table 5 shows the parallelism tolerance analysis data, where the first three columns give the data for the reference plane, and the next three columns give the data for the parallelism measurement. The last columns gives the parallelism deviation, and the parallelism tolerance was evaluated as 15.7um.

Table 6 shows the squareness tolerance analysis data, where the first three columns give the datum for the reference plane, and the last three columns give the measured data for squareness measurement. The last column gives the squareness deviation, and the squareness tolerance was evaluated as 14.7um.

Table 7 shows the circularity(roundness) analysis data, where the first 3 columns give the measured (X, Y) data, which are from practical measurement for a hole with a CMM. The least squares technique and the minmum zone technique are applied for the circularity evaluation. The third and fourth columns give the roundness departure based on the least

Table 5 Parallelism tolerance analysis data

〈PARALLELISM ERROR ANALYSIS〉

Reference plane : Z=0.0000 X+0.0000 Y+91.5447

Ref. X	Ref. Y	Ref. Z	X	Y	Z	Deviation
84.000	84.000	91.554	84.000	84.000	163.255	0.000
84.000	68.000	91.548	84.000	68.000	163.248	6.220
84.000	52.000	91.560	84.000	52.000	163.261	-7.560
84.000	36.000	91.553	84.000	36.000	163.253	-0.340
84.000	20.000	91.547	84.000	20.000	163.247	4.880
68.000	20.000	91.548	68.000	20.000	163.248	3.100
68.000	36.000	91.546	68.000	36.000	163.245	6.880
68.000	52.000	91.552	68.000	52.000	163.252	0.660
68.000	68.000	91.551	68.000	68.000	163.251	2.440
68.000	84.000	91.552	68.000	84.000	163.252	2.220
52.000	84.000	91.553	52.000	84.000	163.254	-0.560
52.000	68.000	91.546	52.000	68.000	163.246	6.660
52.000	52.000	91.548	52.000	52.000	163.248	3.880
52.000	36.000	91.546	52.000	36.000	163.246	5.100
52.000	20.000	91.549	52.000	20.000	163.248	2.320
36.000	20.000	91.551	36.000	20.000	163.251	-1.460
36.000	36.000	91.552	36.000	36.000	163.252	-1.680
36.000	52.000	91.543	36.000	52.000	163.243	8.100
36.000	68.000	91.556	36.000	68.000	163.257	-5.120
36.000	84.000	91.548	36.000	84.000	163.248	4.660
20.000	84.000	91.550	20.000	84.000	163.250	1.880
20.000	68.000	91.552	20.000	68.000	163.252	-0.900
20.000	52.000	91.547	20.000	52.000	163.245	5.320
20.000	36.000	91.545	20.000	36.000	163.245	4.540
20.000	20.000	91.548	20.000	20.000	163.248	0.760

PARALLELISM TOLERANCE=15.7UM

squares and the minimum zone techniques. The roundness tolerance were evaluated as 41.9um and 33.2um respectively, confirming the minmum zone roundness is less than the least squares.

Table 8 shows the cylindricity analysis data, which are from practical measurement for a machined deep hole with a CMM. The first 3 columns give the (X, Y, Z) data of the measured feature, and the fourth column gives the cylindricity departure from the cylindrical reference. The center coordinate of the base circle and the direction vector of the central axis were found, and the cylindricity tolerance was eventually evaluated as 70.4um.

For the profile tolerance measurement, a sculptured surface has been machined, and measured. For the sculptured surface whose coefficients are given on the table 2, and the profile tolerance analysis are analysed as shown in table 9.

The first 3 columns shows the (X, Y, Z) data, the next 3 columns gives the searched

Table 6 Squareness tolerance analysis data

〈SQUARENESS ERROR ANALYSIS〉

Reference plane : 0.0000 X+ -1.0000 Y+ -0.0000 Z+d=0

Ref. X	Ref. Y	Ref. Z	X	Y	Z	Deviation
10.000	10.000	10.000	84.000	84.000	91.554	0.000
10.000	10.000	32.000	84.000	68.000	91.548	6.000
10.000	10.000	72.000	84.000	52.000	91.560	-6.000
10.000	10.000	72.000	84.000	36.000	91.553	1.000
10.000	10.000	90.000	84.000	20.000	91.547	7.000
30.000	10.000	10.000	68.000	20.000	91.548	5.220
30.000	10.000	30.000	68.000	36.000	91.546	7.220
30.000	10.000	32.000	68.000	52.000	91.552	1.220
30.000	10.000	50.000	68.000	68.000	91.551	2.220
30.000	10.000	70.000	68.000	84.000	91.552	1.220
50.000	10.000	90.000	52.000	84.000	91.553	-0.560
50.000	10.000	12.000	52.000	68.000	91.546	6.440
50.000	10.000	30.000	52.000	52.000	91.548	4.440
50.000	10.000	50.000	52.000	36.000	91.546	6.440
50.000	10.000	70.000	52.000	20.000	91.549	3.440
70.000	10.000	92.000	36.000	20.000	91.551	0.660
70.000	10.000	10.000	36.000	36.000	91.552	-0.340
70.000	10.000	30.000	36.000	52.000	91.543	8.660
70.000	10.000	50.000	36.000	68.000	91.556	-4.340
70.000	10.000	72.000	36.000	84.000	91.548	3.660
90.000	10.000	90.000	20.000	84.000	91.550	0.880
90.000	10.000	10.000	20.000	68.000	91.552	-1.120
90.000	10.000	30.000	20.000	52.000	91.547	3.880
90.000	10.000	52.000	20.000	36.000	91.545	5.880
90.000	10.000	70.000	20.000	20.000	91.548	2.880

SQUARENESS TOLERANCE=14.7UM

Table 7 Circularity tolerance analysis data

X	Y	Deviation (Least square)	Deviation (Minimum zone)
299.870	205.131	1.892	1.380
258.257	281.282	3.354	2.796
172.777	296.216	-6.877	-7.209
107.787	238.712	9.284	9.280
112.257	152.051	-10.304	-10.126
182.778	101.485	9.061	9.139
266.275	125.123	-5.168	-5.397
263.074	277.606	5.254	4.689
178.654	297.687	-7.915	-8.268
110.300	244.226	10.151	10.126
109.514	157.452	-9.766	-9.593
176.844	102.710	7.765	7.859
261.619	121.246	-4.409	-4.614
299.995	199.074	-0.542	-1.040
286.873	249.548	9.779	9.206
215.423	298.794	-9.273	-9.741
132.366	273.667	5.955	5.797
100.244	193.052	-2.340	-2.218
143.235	117.676	-2.454	-2.292
228.974	104.283	6.274	6.206
292.885	162.980	-9.324	-9.719
299.936	203.599	0.956	0.447
259.497	280.379	3.531	2.971
174.255	296.622	-6.848	-7.185
108.391	240.121	9.521	8.231

Least square method
Center= (200.000, 200.000)
Raidus=100.000
Roundness tolerance=20.5UM
Minimum zone method
Maximum Radius=100.0102
Minimum Radius=99.9900
Center= (200.0002, 200.0002)
Roundness tolerance=20.3UM

nearest point data. The developed profile tolerance algorithm has been applied to calculate the profile tolerance. The departure from the profile are given on the 7th coulmn, and the profile tolerance was eventually evaluated as 0.374mm.

Table 8 Cylindricity tolerance analysis data

X	Y	Z	Deviation
378.046	310.396	126.492	-22.262
368.869	303.042	136.492	15.559
370.658	292.596	136.491	19.646
378.146	288.457	136.492	33.200
387.399	291.936	136.492	7.737
389.360	303.942	136.492	-37.209
378.049	310.396	116.492	-16.638
368.860	303.048	116.492	25.818
370.650	292.591	116.491	23.261
378.141	288.451	116.492	33.195
387.390	291.932	116.492	1.328
389.360	303.948	116.492	-30.031
378.360	310.390	96.492	-16.396
368.868	303.042	96.492	16.105
370.652	292.591	96.491	15.932
378.143	288.455	96.492	22.503
387.397	291.934	96.492	2.541
389.367	303.941	96.492	-21.769

Least square method
Cylindricity tolerance=70.4UM
(a, b, c) = (279.304, 299.417, -0.000)
(l, m, n) = (0.000, 0.000, 0.25)
Radius=11.032MM

5. Conclusions

(1) A computer aided comprehensive module has been developed for the ISO based tolerance evaluation, and the developed module has been applied to the case of practical measurement tasks such as straightness, flatness, roundness, cylindricity, and profile folerance , showing high accuracy with efficiency.

(2) The minimum zone method as well as the least squares method have been successfully implemented for most of tolerance analysis.

(3) The form error calculation algorithm has been developed for the profile tolerance evaluation, where the form error can be successfully calculated for the CAD based

Table 9 Profile tolerance analysis data

Measured Point Data			Corresponding Point Data on CAD			Deviation
140.7178	127.0760	28.5828	140.7381	127.1743	28.1163	0.4772
140.9935	135.0471	30.0983	141.0394	135.1149	29.7206	0.3865
140.8721	142.9503	31.1456	140.9196	142.9801	30.8812	0.2703
140.8833	151.0685	31.5925	140.9166	151.0710	31.4314	0.1645
140.8785	159.0240	31.1310	140.8868	159.0186	31.0829	0.0491
149.7690	158.9537	32.7080	149.7575	158.9629	32.7666	-0.0605
149.7674	151.1079	33.3686	149.7858	151.1092	33.2796	0.0908
149.6865	143.0579	32.7316	149.7285	143.0953	32.5011	0.2373
149.8028	135.1547	31.2074	149.8483	135.2492	30.8412	0.3810
149.6906	127.2035	29.0334	149.7118	127.3467	28.5648	0.4905
149.9007	119.1681	26.6672	149.8758	119.3549	26.0634	0.6325
149.8448	111.1246	24.4195	149.7614	111.3005	23.7411	0.7058
150.0178	103.0060	22.6436	149.8972	103.1058	21.9884	0.6737
149.9685	94.8366	21.8255	149.8750	94.8294	21.3791	0.4561
149.9821	87.0835	22.2592	149.9841	87.0864	22.2726	-0.0138
158.5318	86.9652	20.6041	158.5550	87.0040	20.7362	-0.1396
158.7978	94.8339	19.9665	158.7083	94.8271	19.5380	0.4378
158.8629	103.0334	21.0565	158.7373	103.1722	20.3748	0.7069
158.7297	111.1919	23.4009	158.6443	111.4318	22.6978	0.7478
158.8354	119.2398	26.3539	158.8113	119.4854	25.7498	0.6526
158.6547	127.2695	29.4765	158.6755	127.4527	29.0191	0.4932
158.6055	135.2539	32.3125	158.6473	135.3667	31.9758	0.3575
158.6305	143.1356	34.3245	158.6625	143.1723	34.1469	0.1842
158.6950	151.0506	35.1397	158.6961	151.0504	35.1341	0.0057
158.6978	159.0819	34.2379	158.6633	159.1201	34.4178	-0.1871

surface. The developed algorithm can give the form error for the case that the effect of measurement probe radius is to be considered.

(4) The developed algorithm is implemented around IBM 386/486 compatible with window environment, so that it can be applied to the most cases of metrological works with many commercial measurement equipments.

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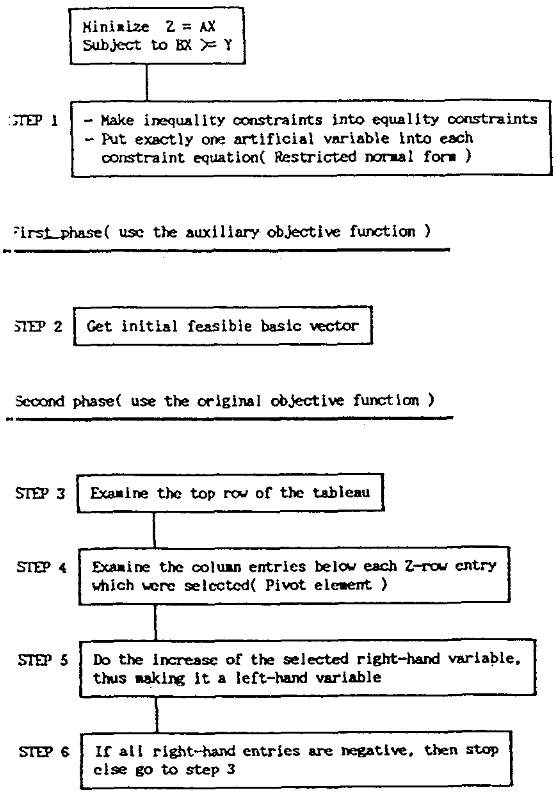
Appendix-Flow chart for the simplex search



Appendix-Flow chart for the simplex search (continued)

Maximize $Z = X1 + X2 + 3\,X3 - 1/2\,X4$

SUBJECT TO

$$X1 + 2\,X3 \le 740$$
$$2\,X2 - 7\,X4 \le 0$$
$$X2 - X3 + 2\,X4 \ge 1/2$$
$$X1 + X2 + X3 + X4 = 9$$

$$\begin{cases} X1 + 2\,X3 + Y1 = 740 \\ 2\,X2 - 7\,X4 + Y2 = 0 \\ X2 - X3 + 2\,X4 - Y3 = 1/2 \\ X1 + X2 + X3 + X4 = 9 \end{cases}$$

Restricted normal form :

$$\begin{cases} Z1 = 740 - X1 - 2\,X3 - Y1 \\ Z2 = -2\,X2 + 7\,X4 - Y2 \\ Z3 = 1/2 - X2 + X3 - 2\,X4 + Y3 \\ Z4 = 9 - X1 - X2 - X3 - X4 \\ Z' = - Z1 - Z2 - Z3 - Z4 \end{cases}$$