Measurement of One-dimensional Vibration Intensity Carried by Bending Vibration

휨 진동에 의한 1차원 진동인텐시티의 측정

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ABSTRACT

In this paper, energy flow with propagation of bending vibration was measured in a beam. Measurements were realized by applying the cross-spectral method One-dimesional vibration intensity was determined based on the imaginary part of the cross-spectrum of the outputs of four acceleration pick-ups attached to the structure. Our results showed that vibration intensity could be accurately measured with the method. Errors of the measurement were investigated. The combined effect of the different factors(i.e. finite difference approximation, and differences in gain and phase among channels) causing error in the measurements with four acceleration pick-ups was also investigated.

요 약

본 논문은 1 차원에서의 휨 진동에 의한 에너지의 흐름(벡터량)을 측정하였다. 측정은 4개의 가속도 센서를 이용하여 Cross-spectrum 방법으로 하였다. 측정결과는 입력 파워와 비교한 결과, 잘 일치하였다.

센서 이득과 위상의 차, 유한 차분 근사가 포함되었을 경우, 측정오차에 관하여 조사하였다. 그 결과, 파장정수와 센서의 간격 $k\triangle$를 1.0으로 두는 것이 측정오차를 최소로 줄일수 있었다.

I. INTRODUCTION

A clear understanding of an energy flow in soild structures is useful to reduce sound generated by vibration in the structure, such as structure-borne sound. Vibration intensity measurement is based on a principle similar to that of acoustic intensity measurement[1-3] as vibration is also a vector intensity. Vibration intensity is defined as the amount of power flow per unit width of cross section perpendicicular to the direction of the flow.[4] It can be studied in various ways depending on the type of vibration being considered.[5,6] Among the different types of vibration, beding vibration has a close correlation with the radiated sound.[7] In this paper, ond-dimensional vibration intensity transmitted by bending vibration was investiaged. A measurement

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system was first constructed. Experiments were then performed by using the 4-channel (4-ch) method[8-10] which is used four acceleration pick-ups, and the 2-channel (2-ch) method[11] which is used two acceleration pick-ups. With regrads to errors accompanying the measurements, various cases were considered.[10,12] The soures of errors of the 4-ch method investigated, theoretically and experimentally, include the followings: the finite difference approximation used the implementation, the disparity in the pick-ups intervals, and the differences in gain and phase responses between channels of the measurement system. Initial results showed that these factors largely affect the meaurement of vibration intensity.[13-15] In this paper, for actual measurements, the 4-ch method, which gives less error in near field measurements, is found to be preferable to the 2-ch method, which is theoretically valid only for far (free) field.

Ⅱ. VIBRATION INTENSITY THEORY

One-dimensional vibration intensity propagating in the x-direction, with respect to the force and bending moment contributions, is expressed by the equation

$$\mathbf{Wx}(x:t)=B\left[\frac{\partial^3\eta}{\partial x^3}\frac{\partial\eta}{\partial t}-\frac{\partial^2\eta}{\partial x^2}\frac{\partial^2\eta}{\partial x\partial t}\right] \qquad (1)$$

where B is the bending stiffness of the material and η is the normal displacement. In implementing this equation, finite difference approximations were used in place of partial derivatives. In the previous paper[10], we showed that the vibration intensity in the **x**-direction is computed from accelerations measured with pick-ups placed at equal intervals (see Fig.1) using cross-spectrum with the following equation:

$$\mathbf{Wx}(x:f)=\frac{B}{\triangle\omega^3}\int_0^\infty Im\,[G_{a_2a_3}-G_{a_1a_3}-G_{a_2a_4}]df \qquad (2)$$

Here, Im denotes imaginary part of the given argument: $G_{a_pa_q}$ is cross-spectrum of $\alpha_p(t)$ and $\alpha_q(t)$, the acceleratoions at points p and q, respectively: and $\triangle$ is the distance between acceleration pick-ups. In the free field, only two measurement points are needed as indicated by

$$\mathbf{Wx}(x:f)=\frac{2\sqrt{Bm}}{\triangle\omega^2}\int_0^\infty Im\,[G_{a_2a_3}]df \qquad (3)$$

where m is the mass per unit area of the material of thickness of h. In the 2-ch method, the effect of the near field was ignored.

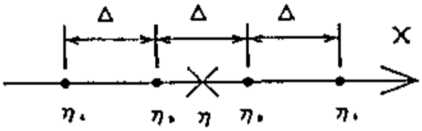


Fig 1. Configuration of pick-ups for one dimensional vibration intensity measurement

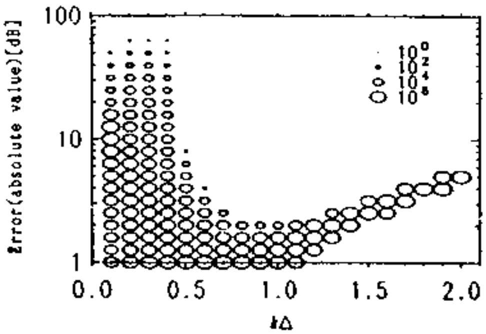
Ⅲ. ERROR DUE TO COMBINATION OF FACTORS

Error is introduced in the measurement of vibration intensity due to the use of finite difference approximation as shown in Eq.(2). Another possible cause of error is that of having differences in the gain and phase responses among the channels. Denoting the errors of gain and phase of each channel as α_i and ϕ_i, respectively, where i is the channel number, the vibration intensity in the **x**-direction is given by

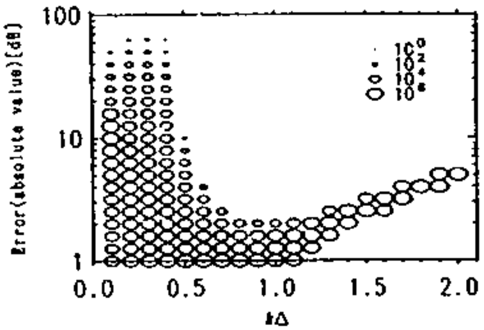
$$\begin{aligned}\mathbf{Wx}\langle x:f\rangle_t=\frac{B\omega}{2\triangle^3}\\
&G_{\alpha_2\alpha_3}\{4A_+^2\sin(k\triangle-\phi_2+\phi_3)-4A_-^2\sin(k\triangle+\phi_2-\phi_3)\}\\
&-\alpha_2\alpha_4\{A_+^2\sin(2k\triangle-\phi_1+\phi_3)-A_-^2\sin(2k\triangle+\phi_1-\phi_3)\}\\
&-\alpha_1\alpha_3\{A_+^2\sin(2k\triangle-\phi_2+\phi_4)-A_-^2\sin(2k\triangle+\phi_2-\phi_4)\}\\
&+2A_+A_-\{_4\alpha_2\alpha_3\cos(skx)\sin(-2\phi_2+\phi_3)\}\\
&-\alpha_2\alpha_4\cos(skx+k\triangle)\sin(-\phi_1+\phi_3)\\
&-\alpha_1\alpha_3\cos(skx-k\triangle)\sin(-\phi_2+\phi_4)\}],\end{aligned} \qquad (4)$$

where x is the distance from the source. k is the wave number given by $2\pi f/\sqrt{1.8C_{L}hf}$, where

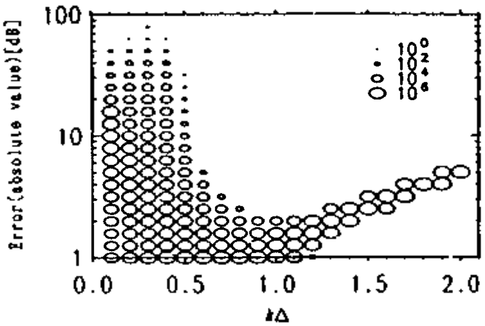
h is the beam thickness and C_{LI} is the longitudinal wave velocity in the material ; A_+ and A_- are the amplitudes of the progressive and retrogressive waves, respectively ; and $\langle \rangle_t$ denotes the time-average of the given argument. Based on Eq.(4), we numerically estimated the combined error due to the finite difference approximation and the differences in gain and phase among channels. As for the finite difference approximation, we assumed $k\triangle$, which was the product of the wave number and the pick-up interval, to be form 0.1 to 2.0 at every 0.1. As for gain differences, the gain for pick-ups 1, 2, and 4(α_1, α_2 and α_4) was assumed to lie within 0.95 and 1.05 relative to that of pick-up 3 ; i.e. error of ±5% was implied. As to phase difference, the phase for pick-ups 1, 2, and 4(ϕ_1, ϕ_2 and ϕ_4) was assumed to be within −0.5 and +0.5 degrees around that of pick-up 3. We further assumed that the standing wave ratio ($SWR = (A_+ + A_-)/(A_+ - A_-)$)(6) of the bending wave was 1.5, 2.0, and 3.0 while the kx used was $2/\pi$, π, $3/2\pi$, and 2π. Changing these parameters independently, errors (i.e. 10log(Eq.(4))/(Eq.(2)))) for about 1.4×10^8 cases were computed for each SWR condition. Figure 2 shows the frequency of occurrence of the computed error as a function of $k\triangle$. The size of ellipses in the figure is in proportion to the frequency of occurrence that the computed error take the specific value. Panels (a), (b), and (c) of Fig.2 correspond to SWR's of 1.5, 2.0, and 3.0, respectively. We can see from Fig.2 that the intensity measurement becomes robust against the gain and phase difference among channels and the error could be min9mized f the interval of the pick-ups is set so that $k\triangle$ is around 1.0.



(a)SWR=1.5



(b)SWR=2.0



(c)SWR=3.0

Fig 2. An error verse $k\triangle$ for differences in gain and phase responses among channels within ±5%, ±0.5° using channels 3 as references. The size of an ellipse is in proportion to occured the number of occurences(about 1.4×10^8).

IV. ONE-DIMENSIONAL VIBRATION INTENSITY MEASUREMENT

A. EXPERIMENTS

The set-up shown in Fig.3 was used in

measuring one-dimensional vibration intensity. One end of the beam was excited by a shaker while the other end was inserted 30[cm] into sand. The input power in the beam was measured with an impedance head on the shaker. A $1000 \times 20 \times 6$[mm] iron beam was used in the measurements. The value of the SWR with the beam inserted 30[cm] into sand was then investigated by taking measurements at 1-cm intervals from the shaker using a single acceleration pick-up.

Vibration intensity was then measured for excitations of 322, 645, and 1280[Hz]. Throughout the experiments, the acceleration pick-ups were attached to magnets and glued to a piece of paper so as to maintain their relative positions as they were moved from one measurement point to another. The transfer functions for each channel of the measurement system from the source to pick-ups were obtained priro to taking measurements. These were used to eliminate the effect of differences in gain and phase response between channels. Vibration intensity was then measured using the 4-ch method at 10-cm intervals from the source. In this case, the pick-up distance was set at 3[cm]. The input power was measured using the impedance head.

B. RESULTS AND DISCUSSION

The relative acceleration levels for each frequency tested are shown in Fig.4. From the figure, the SWR's were calculated from the measured data to be 2.25 for 322[Hz], 1.23 for 645[Hz] and 1.07 for 1280[Hz]. In this Fig.4, the effect of the near field is seen at the excited end (0 [cm]) in a beam. It can also be seen that the effect of the near field is lighter at high frequencies.

Figure 5 shows our former results[17] of the measurements for the 2-ch and the 4-ch method. The method of measurements were basically same as in the present study. Meausrements were taken at 5-cm intervals from the source with a pick-up interval of 3[cm]. The measured value at the 35-cm points, assumed to be beyond the near field, was used as reference (0[dB]). For the 2-ch method (see Fig.5(a)), the measured value increases as the measurement point moves towards the ends, presumably due to the effect of the near field. This value decreses as the excitation frequency increases since the extent of the near field becomes narrower on higher frequency. For the near field. This value decreses as the exictation frequency increases since the extent of the near field becomes

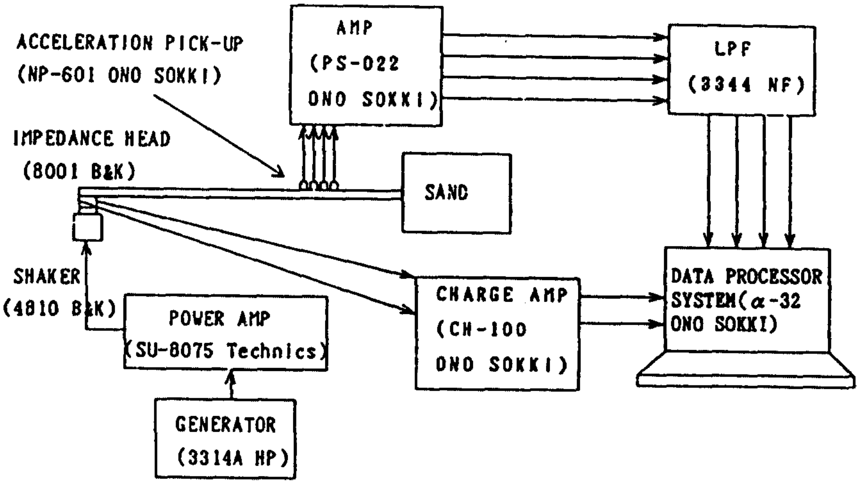


Fig 3. Block diagram of the measurement system

narrower on higher frequency. For the 4-ch method (see Fig.5(b)), there were no large errors near the end, i.e. in the near field.

In the present study, not relative but absolute error for the 4-ch method was examined. The results are shown in Fig. 6. The longitudinal axis denotes the difference in [dB] between the input power and power from the vibration intensity for the 4-ch method. For each excitation frequency, the input power measured with the impedance head was used as refrence (0 [dB]). Energy flow was in the plus direction for all measurement points, i.e. in the direction from the exciting point toward the sand. Table 1 shows the calculated error caused by the finite difference approximation with pick-up interval at 3[cm] at each frequency. In consideration of Table 1, difference between input power and measured poweer was nearly 2.2[dB] for 322[Hz], 1.8[dB] for 645[Hz] and 0.1[dB] for 1280[Hz]. This discrepancy would be attributable to the existence of modes of vibration other than bending vibration and some other factors.

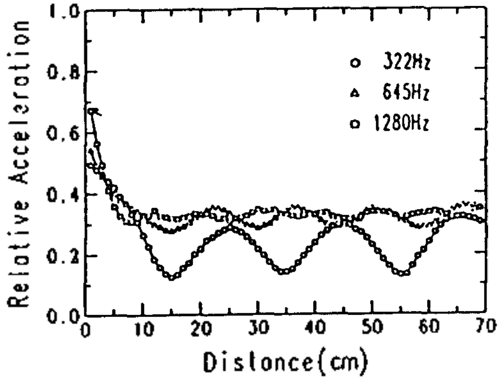


Fig 4. Relative acceleration measured at 1-cm intervals from the shaker. One end of the beam was inserted 30[cm] into sand.

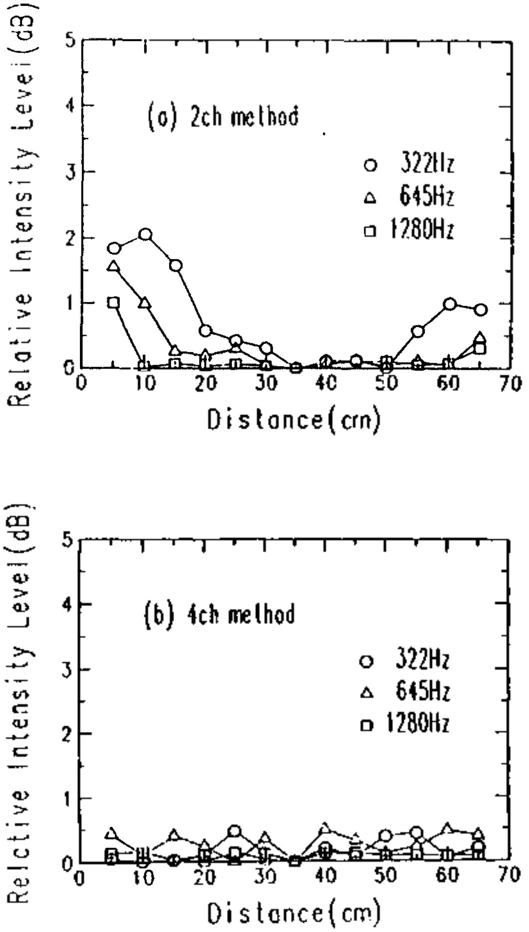


Fig 5. Results of measurements using (a)the 2-ch method and (b)the4-ch method. The measured value at the 35-cm point was used as reference(0 [dB]).

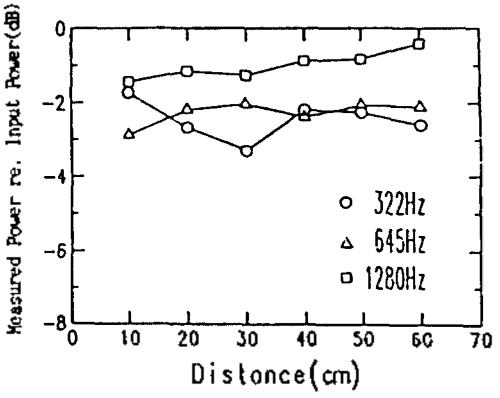


Fig 6. Difference between input power and measured vibration intensity.

Table I Errors due to finite difference approximation at the three frequencies.

Freq[Hz]	Finite Difference Approximation
322	−0.22
645	−0.45
1280	−0.90

[dB]

V. CONCLUSION

One-dimensional vibration intensity in a beam was investigated experimentally by using the 4-ch method. Moreover, the combined effect of the finite difference approximation and the differences in gain and phase responses between channels of the measurement system were investigated theoretically. Result showed that the minimum error occured at near $k\triangle=1.0$. It was also found thet the 4-ch method shows good agreement between the measured input power and vibration intensity.

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