Some Statistical Properties of MUSIC Null-Spectrum

MUSIC영 스펙트럼의 몇가지 통계적 성질

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ABSTRACT

A statistical performance analysis of the multiple signal classification (MUSIC) method is addressed in this paper. Some statistical properties of the MUSIC null-spectrum are obtained. From this we obtain a more exact expression of the resolution threshold which is then used to evaluate the resolution capability for two closely located signal sources.

요 약

다중 신호 분해 방법을 통계적 성능 분석하여 이 방법의 통계적인 성질을 몇가지 얻었다. 이 결과로부터 이제까지 알려진 것보다 더 정확한 분해 문턱값을 얻었고 아주 써서 가까이 있는 두 신호에 대한 다중 신호 분해방법의 분해능력을 살펴 보았다.

I. Introduction

Various techniques for finding directions of multiple signal sources have been developed in the last two decades. In [1], for instance, the multiple signal classification (MUSIC) was proposed : variations of the MUSIC method have also been studied [e.g., 2]. The common feature of these methods is the null-spectrum, which is a nonnegative function of direction of arrival (DOA) with values ideally zero at the DOA's of signal sources. The DOA's of multiple signals can be found by taking the directions at which the value of the null-spectrum is zero. In practice all that is available is the sample null-spectrum, from which the DOA's of multiple signals should be estimated.

The performance of the sample null-spectra, e.g., the MUSIC and Min-Norm sample null-spectra, has been investigated in [1] by simulation. The statistical properties of the sample MUSIC null-spectrum have also been analyzed. For example, the first and second order statistics of the sample MUSIC null-spectrum are obtained in [3], and a more exact expression of the second order statistic is obtained in [4]. In addition the statistical properties of the DOA estimates obtained by the MUSIC method are derived in [5].

In performance analysis the resolution capa-

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bility of the sample null-spectrum for two closely-located signal sources is of importance, as shown in [3,4,6]. The resolution capability of the sample null-spectrum can be characterized by the probability of resolution (PR) [3,4] which is the probability that the values of the sample null-spectrum at DOA's of two closely-located signal sources are smaller than that at the middle of the two DOA's. Although the PR is a reasonable measure for the resolution capability of the sample null-spectrum it reguires heavy computation load. To evaluate the resolution capability the resolution threshold (RT) can alternatively be used, which is defined in [3,6]. An important advantage of the RT when compared with the PR is its computation facility.

In this paper we focus on the statistical properties and resolution capability of the sample MUSIC null-spectrum: we obtain a more exact expression for the RT which can also be used when the two DOA's are not quite close.

II. Preliminaries and Assumptions

Let us consider an array of L sensors of unity gain. The array output vector is denoted by $y(t) \in C^{L\times 1}$, where $C^{L\times 1}$ is the space of $L\times 1$ complex valued column vectors. For narrow-band sources, we assume the standard model of observation:

$$y(t) = Ax(t)+n(t),\ t=1,2,\ldots,N, \tag{1}$$

where the column vector $x(t)$ is an $M\times 1$ zero mean complex random vector of source time series as observed at the array phase center, and $n(t)$ is the additive noise vector.

Assumptions on the signal source $x(t)$ and the noise $n(t)$ are as follows:

A1. The $x(t)$, $t=1, 2, \ldots$, are independent zero mean circular normal random vectors with positive definite covariance matrix $E[x(t)x^H(t)]=R_x$.

A2. The $n(t)$, $t=1,2,\ldots$, are independent circular normal random vectors with zero mean and covariance matrix σI.

A3. The two vectors $x(t)$ and $n(s)$ are independent for any $t,s=1,2,\ldots$

In (1) the matrix A is an $L\times M$ $(L>M)$ complex matrix having the particular structure

$$A=[a(\theta_1),a(\theta_2),\ldots,a(\theta_M)],$$

where θ_i is the DOA of the i-th signal. Here $a(\theta_i) \in C^{L\times 1}$ is called the steering or transfer vector.

The covariance matrix of $y(t)$ is

$$R_y=AR_xA^H+\sigma I, \tag{2}$$

where H denotes the Hermitian. Assumption A1 implies that the covariance matrix R_y is a full rank matrix. Let $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L$ denote the ordered eigenvalues of R_y. Since $rank(AR_xA^H) = M$, it follows that

$$\lambda_i > \sigma \quad for \quad 1 \leq i \leq M$$

and

$$\lambda_i = \sigma \quad for \quad M+1 \leq i \leq L$$

Let the normalized eigenvector corresponding to λ_i be denoted by e_i, $i=1,2,\ldots,L$, with which we define two matrices

$$S=[e_1,e_2,\ldots,e_M]$$

and

$$G=[e_{M+1},e_{M+2},\ldots,e_L]$$

of size $L\times M$ and $L\times(L-M)$, respectively. The ranges of the matrices S and G are called the signal and noise subspaces, respectively.

We observe that [1]

$$a^H(\theta)G=0 \quad \text{for} \quad \theta\in\Theta \tag{3}$$

where $\Theta=\{\theta_1,\theta_2,\ldots,\theta_M\}$ is the set of DOA's, because the vectors $\{a(\theta_i),\ 1\leq i\leq M\}$ are orthogonal to the noise subspace. Then the MUSIC null-spectrum $f(\theta)$ is defined by

$$f(\theta)=a^H(\theta)GG^Ha(\theta), \tag{4}$$

which is nonnegative for all θ and has zeroes only at $\theta\in\Theta$ [5].

In practice, we can not obtain R_y from a finite observation of $y(t)$. Thus let us define

$$\hat{R}_y=\frac{1}{N}\sum_{t=1}^{N}y(t)y^H(t) \tag{5}$$

to be the sample covariance matrix of $\{y(t), t=1, 2,\ldots,N\}$, which is an estimate of R_y. As we have done in the eigendecomposition of R_y, let $\{\hat{e}_1,\hat{e}_2,\ldots,\hat{e}_L\}$ denote the normalized eigenvectors of $\hat{R}_y$ with the associated eigenvalues being arranged in the order of decreasing magnitude. Note that $\hat{e}_i$, $i=1,2,\ldots,L$, are random vectors. In addition Let $\hat{S}=[\hat{e}_1,\hat{e}_2,\ldots,\hat{e}_M]$ and $\hat{G}=[\hat{e}_{M+1},\hat{e}_{M+2},\ldots,\hat{e}_L]$ be the sample signal and noise subspaces, respectively. The MUSIC sample null-spectrum $\hat{f}(\theta)$ is then defined by

$$\hat{f}(\theta)=a^H(\theta)\hat{G}\,\hat{G}^H\,a(\theta). \tag{6}$$

It is thus expected that $\hat{f}(\theta)$ has minimum points around $\theta\in\Theta$.

III. Asymptotic Statistical Properties

In this section we discuss the asymptotic (for large N, i.e.,$N\rightarrow\infty$) mean and variance of the sample null-spectrum (from now on we simply write the "sample null-spectrum" to denote the "sample MUSIC null-spectrum") with finite observations.

Let us consider the sample null-spectrum

$$\hat{f}(\theta)=a^H(\theta)\hat{G}\,\hat{G}^H\,a(\theta), \tag{7}$$

and define the projection of $a(\theta)$ on the eigenspace which is given by

$$h(\theta)=[S\vdots G]^Ha(\theta), \tag{8}$$

or

$$a(\theta)=[S\vdots G]h(\theta). \tag{9}$$

Using (9), we have from (7)

$$\begin{aligned}\hat{f}(\theta)&=h^H(\theta)\begin{bmatrix} S^H\hat{G}\hat{G}^HS & S^H\hat{G}\hat{G}^HG \\ G^H\hat{G}\hat{G}^HS & G^H\hat{G}\hat{G}^HG\end{bmatrix}h(\theta)\\ &=a^H(\theta)SS^H\hat{G}\hat{G}^HSS^Ha(\theta)+a^H(\theta)GG^H\hat{G}\hat{G}^HSS^Ha(\theta)\\ &\quad+a^H(\theta)SS^H\hat{G}\hat{G}^HGG^Ha(\theta)+a^H(\theta)GG^H\hat{G}\hat{G}^HGG^Ha(\theta).\end{aligned} \tag{10}$$

Now let us rewrite (10) as

$$\hat{f}(\theta)=\tilde{f}(\theta)+b(\theta)+v(\theta), \tag{11}$$

where

$$\tilde{f}(\theta)=a^H(\theta)GG^H\hat{G}\hat{G}^HGG^Ha(\theta), \tag{12}$$

$$b(\theta)=a^H(\theta)SS^H\hat{G}\hat{G}^HSS^Ha(\theta), \tag{13}$$

and

$$v(\theta)=2Re\{a^H(\theta)GG^H\hat{G}\hat{G}^HSS^Ha(\theta)\}, \tag{14}$$

We have for $\theta\neq\theta_i$

$$\frac{|\tilde{f}(\theta)-f(\theta)|}{\|c(\theta)\|^2}=O(1/\sqrt{N}), \tag{15}$$

$$|b(\theta)|=O(1/\sqrt{N}), \tag{16}$$

and

$$\frac{|v(\theta)|}{\|c(\theta)\|}=O(1/\sqrt{N}), \tag{17}$$

where $c(\theta)=G^Ha(\theta)$. Thus when $\theta\rightarrow\theta_i$, since $c(\theta)\rightarrow 0$, the asymptotically dominant term of $\hat{f}(\theta)-f(\theta)$ is $b(\theta)$.

Lemma 1 : The normalized error, $2b(\theta)/\sigma_\gamma^2(\theta)$, has asymptotically χ^2 distribution with degree of freedom $2(L-M)$, where

$$\sigma_Y^2(\theta) = \frac{1}{N}\left[\sum_{k=1}^{M} \frac{\lambda_k \sigma}{(\lambda_k - \sigma)^2} |a^H(\theta)e_k|^2\right]. \tag{18}$$

Thus the asymptotic mean and variance of $b(\theta)$ are given by

$$E[b(\theta)] = (L-M)\sigma_Y^2(\theta)$$

and

$$var\,[b(\theta)] = (L-M)\sigma_Y^4(\theta).$$

Proof : See[7].

Theorem 1 : The asymptotic mean of the sample null-spectrum $\hat{f}(\theta)$ for $\theta \simeq \theta_i \in \Theta$ is given by

$$E[\hat{f}(\theta)] - f(\theta) = \frac{(L-M)}{N}\left[\sum_{k=1}^{M} \frac{\lambda_k \sigma}{(\lambda_k - \sigma)^2} |a^H(\theta)e_k|^2\right]. \tag{19}$$

Theorem 2 : When θ is between θ_1 and θ_2 and the difference between θ_1 and θ_2 is sufficiently small enough to let $var[v(\theta)] \simeq 0$, we have

$$var[\hat{f}(\theta)] \simeq \frac{(L-M)}{N^2}\left[\sum_{k=1}^{M} \frac{\lambda_k \sigma}{(\lambda_k - \sigma)^2} |a^H(\theta)e_k|^2\right]^2. \tag{20}$$

Proof : See [7].

Ⅳ. Computer Simulation

The RT is the array signal to noise ratio (ASNR) at which $E[\hat{f}(\theta_1)] = E[\hat{f}(\theta_2)] = E[\hat{f}(\theta_m)]$ for two equal-power sources, where $\theta_m = (\theta_1 + \theta_2)/2$. For standard MUSIC the RT was derived in [3], and the RT for the beamspace MUSIC here was derived in [6], where the definition of the RT is extended to the case of two unequal-power sources also. Denoting the RT by $\bar{\Phi}$, we have $\min_i E[\hat{f}(\theta_m) - \hat{f}(\theta_i)] = 0$ for ASNR=$\bar{\Phi}$. If the ASNR is greater than $\bar{\Phi}$, we can discriminate two signal sources. Conversely, to discriminate two closely-located signal sources the ASNR should be greater than $\bar{\Phi}$.

In Tables 1-4 the RT's for various cases are obtained. Computer simulations show that the values of the RT are quite exact. The variance of $\hat{f}(\theta_m) - \hat{f}(\theta_i)$ obtained by computer simulation is not, however, close to the computed value. This also comes from neglecting the error $v(\theta)$ in (11). However the orders of these values are the same. In these tables it is seen that the RT is increased when the correlation of the two signal sources is increased and /or the difference of the two DOA's becomes small. In addition when the ratio of the two signal source powers becomes small, the RT is increased.

It is noteworthy as we mentioned earlier that the RT can be found when the two DOA's are not quite close as we can see from Tables 3 and 4.

corr. coeff.	$\rho = 0 + i0$		$\rho = 0.5 + i0.5$	
$\nu = P_2/P_1$	$\nu = 1$	$\nu = 0.5$	$\nu = 1$	$\nu = 0.5$
RT (ASNR, dB)	45.34	49.30	50.33	53.54
$E[\hat{f}(\theta_1)]$	2.34e-05 (2.37e-05)*	9.40e-06 (9.47e-06)	1.49e-05 (1.48e-05)	7.11e-06 (7.06e-06)
$var[\hat{f}(\theta_1)]$	6.87e-11 (6.86e-11)	1.10e-11 (1.09e-11)	2.78e-11 (1.96e-11)	6.31e-12 (4.42e-12)
$E[\hat{f}(\theta_2)]$	2.34e-05 (2.41e-05)	1.88e-05 (1.93e-05)	1.49e-05 (1.53e-05)	1.42e-05 (1.46e-05)
$var[\hat{f}(\theta_2)]$	6.87e-11 (6.62e-11)	4.43e-11 (4.26e-11)	2.78e-11 (2.65e-11)	2.53e-11 (2.42e-11)
$E[\hat{f}(\theta_m)]$	2.34e-05 (2.42e-05)	1.88e-05 (1.92e-05)	1.49e-05 (1.55e-05)	1.42e-05 (1.46e-05)
$var[\hat{f}(\theta_m)]$	1.72e-11 (5.13e-11)	6.24e-12 (2.59e-11)	1.27e-12 (1.13e-11)	7.56e-13 (7.90e-12)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	4.16e-13 (5.22e-07)	9.39e-06 (9.81e-06)	1.71e-14 (6.86e-07)	7.08e-06 (7.54e-06)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	5.16e-11 (8.87e-11)	1.17e-11 (3.30e-11)	2.36e-11 (2.82e-11)	5.59e-12 (1.16e-11)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	4.16e-13 (1.29e-07)	-3.33e-08 (2.53e-09)	1.71e-14 (2.39e-07)	-2.90e-08 (3.63e-08)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	5.16e-11 (9.19e-11)	2.86e-11 (5.04e-11)	2.36e-11 (3.47e-11)	2.02e-11 (2.74e-11)

Table 1.The resolution threshold when two DOA's are 15° and 17° and the mean and variance of the sample null-spectrum at the resolution threshold (* the values in parantheses are obtained by comuter simulation with 100 trials and those without parantheses are theoretical values.).

corr. coeff.	$\rho = 0 + i0$		$\rho = 0.5 + i0.5$	
$\nu = P_2/P_1$	$\nu = 1$	$\nu = 0.5$	$\nu = 1$	$\nu = 0.5$
RT (ASNR, dB)	38.18	42.11	43.25	46.42
$E[\hat{f}(\theta_1)]$	1.23e-04 (1.24e-04)	4.93e-05 (4.98e-05)	7.65e-05 (7.59e-05)	3.68e-05 (3.65e-05)
$var[\hat{f}(\theta_1)]$	1.88e-09 (1.91e-09)	3.04e-10 (3.05e-10)	7.32e-10 (5.09e-10)	1.69e-10 (1.18e-10)
$E[\hat{f}(\theta_2)]$	1.23e-04 (1.25e-04)	9.89e-05 (1.01e-04)	7.65e-05 (7.81e-05)	7.36e-05 (7.53e-05)
$var[\hat{f}(\theta_2)]$	1.88e-09 (1.75e-09)	1.22e-09 (1.15e-09)	7.32e-10 (6.69e-10)	6.78e-10 (6.25e-10)
$E[\hat{f}(\theta_m)]$	1.23e-04 (1.24e-04)	9.84e-05 (9.89e-05)	7.65e-05 (7.74e-05)	7.33e-05 (7.36e-05)
$var[\hat{f}(\theta_m)]$	4.68e-10 (1.37e-09)	1.72e-10 (6.88e-10)	2.87e-11 (2.69e-10)	1.78e-11 (1.91e-10)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	2.47e-11 (4.92e-07)	4.92e-05 (4.91e-05)	9.27e-13 (1.53e-06)	3.65e-05 (3.71e-05)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	1.41e-09 (2.36e-09)	3.18e-10 (8.71e-10)	6.35e-10 (7.33e-10)	1.52e-10 (3.00e-10)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	2.47e-11 (-1.09e-06)	-4.08e-07 (-2.34e-06)	9.27e-13 (-7.45e-07)	-3.51e-07 (-1.71e-06)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	1.41e-09 (2.46e-09)	7.98e-10 (1.37e-09)	6.35e-10 (8.89e-10)	5.55e-10 (7.24e-10)

Table 2.The resolution threshold when two DOA's are 15° and 18° and the mean and variance of the sample null-spectrum at the resolution threshold.

corr. coeff.	$\rho = 0 + i0$		$\rho = 0.5 + i0.5$	
$\nu = P_2/P_1$	$\nu = 1$	$\nu = 0.5$	$\nu = 1$	$\nu = 0.5$
RT (ASNR, dB)	33.20	37.11	38.37	41.48
$E[\hat{f}(\theta_1)]$	3.87e-04 (3.91e-04)	1.57e-04 (1.58e-04)	2.38e-04 (2.35e-04)	1.15e-04 (1.14e-04)
$var[\hat{f}(\theta_1)]$	1.87e-08 (1.93e-08)	3.06e-09 (3.10e-09)	7.07e-09 (4.91e-09)	1.67e-09 (1.16e-09)
$E[\hat{f}(\theta_2)]$	3.87e-04 (3.95e-04)	3.15e-04 (3.21e-04)	2.38e-04 (2.42e-04)	2.31e-04 (2.36e-04)
$var[\hat{f}(\theta_2)]$	1.87e-08 (1.70e-08)	1.24e-08 (1.14e-08)	7.07e-09 (6.25e-09)	6.69e-09 (6.01e-09)
$E[\hat{f}(\theta_m)]$	3.87e-04 (3.92e-04)	3.12e-04 (3.13e-04)	2.38e-04 (2.40e-04)	2.29e-04 (2.30e-04)
$var[\hat{f}(\theta_m)]$	4.67e-09 (1.37e-08)	1.75e-09 (6.92e-09)	2.40e-10 (2.44e-09)	1.56e-10 (1.76e-09)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	4.16e-10 (1.13e-06)	1.56e-04 (1.55e-04)	1.45e-11 (5.06e-06)	1.14e-04 (1.15e-04)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	1.42e-08 (2.33e-08)	3.16e-09 (8.63e-09)	6.27e-09 (7.09e-09)	1.52e-09 (2.90e-09)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	4.16e-10 (-2.42e-06)	-2.32e-06 (-7.89e-06)	1.45e-11 (-1.39e-06)	-1.96e-06 (-5.82e-06)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	1.42e-08 (2.45e-08)	8.19e-09 (1.37e-08)	6.27e-09 (8.48e-09)	5.59e-09 (7.08e-09)

Table 3.Thr resolution threshold when two DOA's are 15° and 19° and the mean and variance of the sample null-spectrum at the reaolution threshold.

corr. coeff.	$\rho = 0 + i0$		$\rho = 0.5 + i0.5$	
$\nu = P_2/P_1$	$\nu = 1$	$\nu = 0.5$	$\nu = 1$	$\nu = 0.5$
RT (ASNR, dB)	29.36	33.22	34.60	37.66
$E[\hat{f}(\theta_1)]$	9.44e-04 (9.54e-04)	3.84e-04 (3.88e-04)	5.72e-04 (5.64e-04)	2.80e-04 (2.77e-04)
$var[\hat{f}(\theta_1)]$	1.11e-07 (1.16e-07)	1.84e-08 (1.87e-08)	4.09e-08 (2.84e-08)	9.82e-09 (6.83e-09)
$E[\hat{f}(\theta_2)]$	9.44e-04 (9.59e-04)	7.73e-04 (7.87e-04)	5.72e-04 (5.79e-04)	5.62e-04 (5.70e-04)
$var[\hat{f}(\theta_2)]$	1.11e-07 (9.84e-08)	7.48e-08 (6.70e-08)	4.09e-08 (3.52e-08)	3.95e-08 (3.46e-08)
$E[\hat{f}(\theta_m)]$	9.44e-04 (9.56e-04)	7.65e-04 (7.67e-04)	5.72e-04 (5.78e-04)	5.54e-04 (5.56e-04)
$var[\hat{f}(\theta_m)]$	2.77e-08 (8.21e-08)	1.06e-08 (4.18e-08)	1.21e-09 (1.33e-08)	8.22e-10 (9.86e-09)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	3.60e-09 (2.92e-06)	3.81e-04 (3.79e-04)	1.18e-10 (1.39e-05)	2.74e-04 (2.78e-04)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_1)]$	8.46e-08 (1.38e-07)	1.87e-08 (5.13e-08)	3.70e-08 (4.12e-08)	9.12e-09 (1.69e-08)
$E[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	3.60e-09 (-2.40e-06)	-8.95e-06 (-2.03e-05)	1.18e-10 (-3.84e-07)	-7.42e-06 (-1.44e-05)
$var[\hat{f}(\theta_m) - \hat{f}(\theta_2)]$	8.46e-08 (1.46e-07)	5.04e-08 (8.26e-08)	3.70e-08 (4.86e-08)	3.37e-08 (4.15e-08)

Table 4.The resolution threshold when two DOA's are 15° and 20° and the mean and variance of the sample null-sepctrum at the resolution threshold.

V. Summary

We decomposed the estimation error of the sample MUSIC null-spectrum into two errors, from which we observed that the sample MUSIC null-spectrum is a biased estimate of the MUSIC null-spectrm. The mean and variance of the sample MUSIC null-spectrm were obtained analytically.

We also derived a more exact expression of the resolution threshold which can be used to show the resolution capability : it is easier to compute than the probability of resolution. Since the expression was derived without assuming that the two DOA's are closely-located, it can also be used when the two DOA's are not quite close.

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