The Output SINR of the Linearly Constrained Beamformer

선형 제한 조건을 갖는 빔형성기의 출력 SINR

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ABSTRACT

In this paper, expressions for the output signal-to-interference plus noise ratio(SINR) of the linearly constrained narrowband beamformer in noncoherent situations are derived using a vector approach.

요 약

본 논문에서는 noncoherent 환경하에서 벡터적 접근 방식을 이용하여 선형 제한 조건을 갖는 협대역 빔형성기의 출력 SINR 식을 유도하였다.

I. Introduction

Adaptive beamformers have been widely used in sonar, radar, and communication systems. The adaptive array system automatically responds to a changing signal environment and improves output signal-to-interference plus noise ratio (SINR) without prior knowledge of the interference. Frost[1] introduced a temporally adaptive procedure for minimizing output power while linearly constraining the weights to provide a prescribed steering point spectral filtering[2,3]. In [1], the optimum weights were found by the method of Lagrange multipliers. In this paper, we present how the optimum weights and an expression for the output SINR are found for the linearly constrained beamformer in a noncoherent situation. Although only a single interference is considered, the results presented could be extended to the case of multiple interferences. Futhermore, the results could be extended to the case of coherent situation such as multiple propagation paths (multipaths) or smart jamming[4]. The numerical results are included.

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II. The Output SINR of the Linearly Constrained Beamformer

Assume that the incident signals are narrowband in nature. The linearly constrained beamformer in the narrowband case is shown schematically in Fig.1, consisting of M omnidirectional equispaced elements. If we set $s(k)$ as the desired signal, $n(k)$ as the interference, and $\underline{u}$ as a zero mean uncorrelated white Gaussian noise

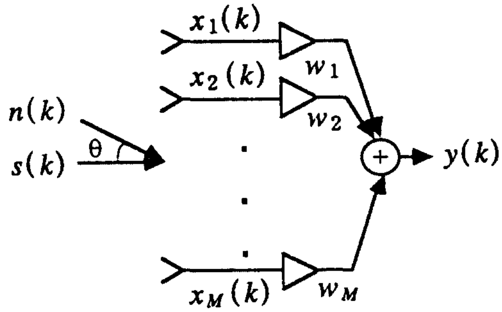

Fig. 1. Schematic diagram of a linearly constrained narrowband beamformer.

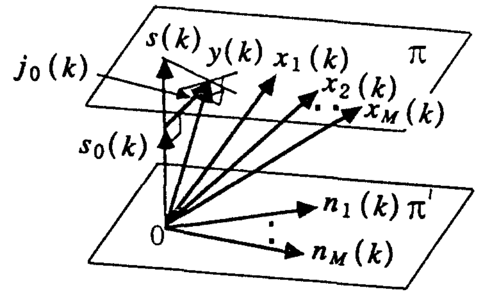

Fig. 2. The signals in vector space.

vector of length M, the output vector $\underline{x}(k)$ of the elements at the k-th time sample can be expressed as

$$\underline{x}(k)=s(k)\,\underline{1}_M+\underline{n}(k)+\underline{u} \tag{1}$$

where $\underline{x}(k)=[x_1(k)\,x_2(k)\cdots x_M(k)]^T$

$\underline{n}(k)=[n_1(k)\,n_2(k)\cdots n_M(k)]^T$

$=n(k)\,\underline{a}$

$\underline{a}=[1\,\exp(j2\pi f_0\tau)\cdots\exp(j2\pi f_0(M-1)\tau)]^T$

$\tau=\dfrac{d}{f_0\lambda}(\sin\theta_n-\sin\theta_s)$

Here, f_0 is the signal center frequency, d is the distance between neighboring elements, and λ is the signal wavelength. θ_s and θ_n are the incident angles of the desired signal and interference. $\underline{1}_M$ is an all 1's vector of length M, and τ is a presteering delay. The beamformer output $y(k)$ is given by

$$y(k)=\underline{W}^H\,\underline{x}(k) \tag{2}$$

where $\underline{W}=[w_1\,w_2\cdots w_M]^T$

The superscript H denotes Hermitian transpose. The weights are constrained to satisfy the following linear constraint equation,

$$\underline{1}_M^T\,\underline{W}=1 \tag{3}$$

The implication of equation (2) above is shown in Fig.2. In Fig.2, the additive noise component is omitted. That is, beamformer output $y(k)$ is a point on the surface π determined by the points $\{x_1(k)\,x_2(k)\cdots x_M(k)\}$. Optimum weights for the beamformer are determined so that the norm of $y(k)$ is minimized. To minimize the norm of $y(k)$, $y(k)$ must be perpendicular to the surface π. As shown in Fig.2, since $x_i(k)$ is a point on the surface π, if $y(k)$ is perpendicular to π the following equation is satisfied.

$$y(k)\perp(x_i(k)-y(k)),\qquad i=1,2,\cdots,M \tag{4}$$

where $\perp$ denotes perpendicularity. Using the definition of the inner product, the above equation becomes

$$<(\underline{x}(k)-\underline{1}_M y(k))^T,\ y(k)>=\underline{0}_M \tag{5}$$

$$<\underline{x}(k)^T,\ y(k)>\ =\ <y(k)\ \underline{1}_M^T,\ y(k)> \tag{6}$$

$\underline{0}_M$ is a zero vector of length M. The inner product $< >$ is defined as

$$<\underline{f},\underline{g}>=\underline{f}^T\ \underline{g}^* \tag{7}$$

where $\underline{f}=[f_1\,f_2\cdots f_M]^T$

$\underline{g}=[g_1\,g_2\cdots g_M]^T$

$*$ denotes complex conjugation. From equation (2) and (6),

$$<\underline{x}(k)^T,\ \underline{x}(k)^T\,\underline{W}>\ =\ <\underline{x}(k)^T\,\underline{W}\,\underline{1}_M^T,\ \underline{x}(k)^T\,\underline{W}> \tag{8}$$

$$\underline{W}=<\underline{x}(k)^T,\ \underline{x}(k)^T>^{-1}\underline{1}_M\,\underline{W}^T<\underline{x}(k)^T,\ \underline{x}(k)^T>\underline{W} \tag{9}$$

Premultiplying $\underline{1}_M^T$ to both sides of equation (9), and from equation (3)

$$1 = \underline{1}_M^T < \underline{x}(k)^T, \underline{x}(k)^T >^{-1} \underline{1}_M \; \underline{W}^T < \underline{x}(k)^T, \underline{x}(k)^T > \underline{W} \tag{10}$$

$$\underline{W}^T < \underline{x}(k)^T, \underline{x}(k)^T > \underline{W} = (\underline{1}_M^T < \underline{x}(k)^T, \underline{x}(k)^T >^{-1} \underline{1}_M)^{-1} \tag{11}$$

From equation (11) and (9)

$$\underline{W} = < \underline{x}(k)^T, x(k)^T >^{-1} \underline{1}_M (\underline{1}_M^T < \underline{x}(k), x(k)^T >^{-1} \underline{1}_M)^{-1} \tag{12}$$

$< \underline{x}(k)^T, \underline{x}(k)^T >$ in equation (12) is an autocorrelation matrix R_{xx} of the elements output, giving

$$R_{xx} = \sigma_s^2 \underline{1}_M + \sigma_n^2 \, a \, a^H + \sigma_u^2 \, I \tag{13}$$

Here, I is the identity matrix. Setting the optimum weight vector as $\underline{W}_{opt}$,

$$\underline{W}_{opt} = R_{xx}^{-1} \underline{1}_M (\underline{1}_M^T R_{xx}^{-1} \underline{1}_M)^{-1} \tag{14}$$

Equation (14) is equal to that of Frost[1]. Output $y(k)$ can be divided into the desired signal component $s_0(k)$ and interference plus additive noise signal component $j_0(k)$.

$$s_0(k) = \alpha \, s(k) \tag{15}$$

$$j_0(k) = y(k) - s_0(k) \tag{16}$$

Here, α is a complex scalar. $j_0(k)$ and $s(k)$ are orthogonal, giving

$$< s(k), y(k) - \alpha s(k) > = \underline{0}_M \tag{17}$$

$$\alpha = < s(k), s(k) >^{-1} < s(k), x(k)^T > \underline{W}_{opt} \tag{18}$$

$< s(k), s(k) >$ denotes the variance of $s(k)$ and $< s(k), \underline{x}(k)^T >$ is the cross correlation vector between $s(k)$ and the elements output.

$$< s(k), s(k) > = \sigma_s^2 \tag{19}$$

$$< s(k), x(k)^T > = \sigma_s^2 \underline{1}_M^T \tag{20}$$

Substituting equation (14), (19), (20) into (18) gives $\alpha = 1$. Therefore, from equation (2), (14), (15), and (16) above the output SINR is obtained as follows.

$$SINR_o = \frac{E[|s_0(k)|^2]}{E[|j_0(k)|^2]} = \frac{\sigma_s^2}{(\underline{1}_M^T R_{xx}^{-1} \underline{1}_M)^{-1} - \sigma_s^2} \tag{21}$$

From equation (13) and Woodbury's identity[5], R_{xx}^{-1} is obtained as below.

$$R_{xx}^{-1} = \underline{Q}^{-1} - \frac{\sigma_s^2 \underline{Q}^{-1} \underline{a} \, \underline{a}^H \underline{Q}^{-1}}{1 + \sigma_n^2 \underline{a}^H \underline{Q}^{-1} a} \tag{22}$$

where $\underline{Q}^{-1} = \sigma_u^{-1} I - \frac{(\sigma_s^2 / \sigma_u^2) \, \underline{1}_M \underline{1}_M^T}{\sigma_u^2 + M \sigma_s^2}$

From equation (21) and (22), the closed form of the theoretical output SINR may be obtained.

Ⅲ. Numerical Results

For the results presented, a six-element linear array of one-half wavelength spacing is used. The desired signal is assumed to be broadside along the array. The signal center frequency is 50 Hz, and the sampling frequency is 1000 Hz.

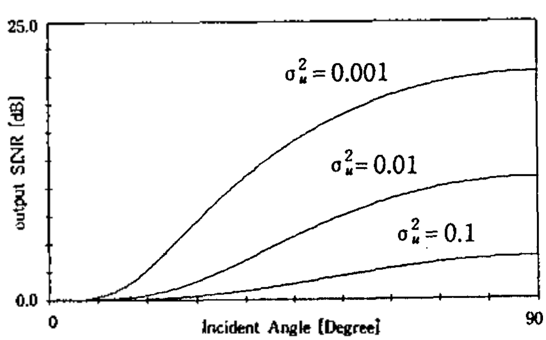

Fig. 3. Output SINR ($\sigma_s^2 = \sigma_n^2 = 1.0$).

Fig.3 shows the output SINR as a function of the incident angle for input additive white noise levels when the desired signal variance σ_s^2 and input interference variance σ_n^2 are both 1.0. The output SINR decreases nearing the look direction because the constraint allows interference, as well as desired signal, to pass in the look direction. The figure shows that the output SINR increases as the input interference-to-noise ratio (INR) increases.

Ⅳ. Conclusions

In this paper, we have derived an expression for the output SINR of the linearly constrained beamformer in a noncoherent situation using the concept of vector space. The results show that the output SINR decreases as it approaches the look direction and the output SINR increases as the input interference-to-noise ratio(INR) increases. The results could be extended to the case of coherent situations such as multiple propagation paths (multipaths) or smart jamming.

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Byung Jae Kwak was born in Korea on July 29, 1967. He received B.S. and M.S. degree in electronic engineering from Yonsei university in 1989 and 1991. He is working toward the Ph.D degree at The University of Michigan. His current research interests are in the area of radar, sonar signal processing, and spectrum estimation.

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Ki Man Kim for a photograph and biography, see pp.76 of the December 1991 issue of this journal.

▲Il Whan Cha

Il Whan Cha for a photograph and biography, see pp.52 of the December 1990 issue of this journal.

▲Dae Hee Youn

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