Spectral Estimation of Nonstationary Signals Using RLS Algorithm with a Variable Forgetting Factor

시변 망각 인자를 갖는 RLS 알고리즘을 이용한 Nonstationary 신호의 스펙트럼 추정

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ABSTRACT

This paper presents a new adaptive method of estimating power spectra which vary spatially (i.e., are spatially nonstationary). This method estimates downstream power spectra from a single data set obtained by slowly moving a probe over the spatial region of interest. The concept of a variable forgetting factor, which compensates for the nonstationarity of a signal by forgetting "old" upstream data, is developed and incorporated into the recursive least square (RLS) algorithm to estimate power spectra of spatially nonstationary signals obtained from a moving probe. The feasibility and practicality of the moving probe approach is applied to the spectral estimation of a spatially nonstationary signal encountered in transition to turbulence studies conducted in a small wind tunnel. The resulting spectra compare very well with spectra estimated via the traditional approach, i.e.,the fixed probe approach.

요 약

본 논문은 공간적으로 변하는 스펙트럼을 추정하는 새로운 적응 방법을 제안한다. 제안한 방법에서는 오래된 upstream의 데이타를 망각함으로서 신호의 nonstationarity를 고려해주는 시변망각인자의 개념을 recursive least square (RLS) 알고리즘에 도입하였으며, 관심이 있는 공간영역에서 탐사침을 천천히 움직여 얻은 하나의 데이타 군으로부터 downstream 스펙트럼을 추정하였다. 제시한 방법의 실현 가능성은 실제 실험 (wind tunnel 이용)을 통해서 얻은 공간적으로 변하는 nonstationary 신호의 스펙트럼을 추정하는 과정에서 입증되며 또한 기존의 방법들과 비교함으로서 그 우수성을 보인다.

I. Introduction

Spectral analysis, a powerful tool in time series analysis, has proven to be very useful in the study of acoustis, speech, communications, radar, sonar, ultrasonics, biology, biomedicine, optics, etc. Nonparametric methods for nonstationary spectral estimation are well developed, i.e., the short-time Fourier transform, complex demodulation, and several transformations leading to time-frequency representations, the most important being the Wigner Distribution [1]. The short-time Fourier transform approach is certainly the most popular one. However, short-time analyses are known to suffer various drawbacks, especially with respect to the quasi-stationary assumptions. On the other hand, one may look for a better time-frequency representation : a recent series of papers has shown that the Wigner-Ville

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Distribution and the windowed Wigner Distribution, known as the psuedo Wigner Distribution, are good candidates with their well-understood mathematical properties. The Wigner Distribution procedure offers better resolution in time or frequency than the short-time Fourier transform approach. The defects of the Wigner Distribution approach for multicomponent signals include the possible generation of negative-going regions in the time-frequency plane and the creation of cross-terms. In most practical situations, it is ne-necessary to apply some sort of smoothing in the time and frequency domains in order to suppress the negative regions and cross-terms. However, such a smoothing process results in the loss of some time and frequency resolution of the Wigner Distribution procedure.

Recently developed parametric approaches are representations of nonstationary signals by time-dependent auto-regressive (AR) modelling described in [2]. The AR coefficients are allowed to change in time by a linear combination of a some set of known time functions. These parametric approaches offer the advantage of leading to the same type of identification procedures as AR models with constant parameters. However, further work is needed to develop a systematic procedure for designing the best basis functions for specific classes of nonstationary signals since a given basis set is by no means the best one for all nonstationary signals.

In contrast to existing parametric approachs, we propose another auto-regressive (AR) parametric method for the spectral estimation of nonstationary signals using recursive least square algorithm with a variable forgetting factor (RLS-VVF). The concept of variable forgetting factor was introduced in self-tuning control to avoid a "blowing-up" of the covariance matrix of the estimates and subsequent unstable control [3]. A similar scheme has been used in the techniques of adaptive filtering [4] and time-varying spectral estimation [5]. A different adaptation scheme [6] has been proposed by varying the memory length. The method described in the paper can quickly estimate the global trend in a nonstationary situation by decreasing the forgetting factor. This is automatically accomplished via an extended error criterion; i.e., when the signal exhibits stationaryity, the actual memory length is increased by increasing the forgetting factor. This results in a frequency estimation of the signal with high accuracy.

In this paper, the RLS-VFF method is applied to the spectral estimation of a spatially nonstationary signal associated with the transition to turbulence in mixing layers [7]. In constrast to the classical method, which requires one to estimate the spectrum for each location by repeating the same experiment, the proposed method enables one to estimate downstream power spectra over the section of interest with a single data set obtained by slowly moving a probe downstream. Then, the downstream power spectra of a spatially nonstationary signal obtained from the moving probe are estimated efficiently via the variable forgetting factor scheme which "forgets" old data (or weights recent data) to a degree depending on an automatically-calculated measure of nonstationarity (the extended prediction error criterion). The performance of the RLS-VFF is compared to that of the RLS algorithm with a fixed forgetting factor (RLS-FFF). Also, the downstream spectra calculated using the movable probe and the RLS-VFF method are compared to classical spectra calculated from fixed probes.

II. Adaptive Power Spectral Estimation with a Variable Forgetting Factor

Since the proposed approach to estimate power spectra of nonstationary signals is based on the AR method with the recursive least squares (RLS) algorithm which is a sequential algorithm for adaptive AR parameter estimation, we will briefly summarize the RLS algorithm [8] in the following. Consider a class of nonstationary signals $y(n)$ defined by the following recursive equation :

$$y(n) = -\sum_{i=1}^{p} a_i(n)y(n-i) + e(n) \quad (1)$$

where $e(n)$ is a zero-mean white noise with variance σ_e^2, and $\{a_i(n), i=1,\cdots,p\}$ are time-varying parameters. The power spectral density $S_y(w;n)$ at time n is given by

$$S_y(w;n) = \frac{\sigma_e^2}{|1+\sum_{i=1}^{p} a_i(n)e^{-jwiT}|^2} \quad (2)$$

where T denotes the sampling interval. The sum of the prediction errors is defined by

$$\varepsilon(n) = \sum_{k=1}^{n} \lambda^{n-k}|e(k)|^2 \quad (3)$$

where

$$e(k) = y(k) - Y_p^T(k-1)\Theta_p(n)$$
$$Y_p(k) = [y(k), y(k-1), \cdots, y(k-p+1)]^T$$
$$\Theta_p(n) = [a_1(n), a_2(n), \cdots, a_p(n)]^T$$

In order to update the set of AR parameters that minimize the exponentially weighted squared error, an RLS algorithm with a forgetting factor λ $(0<\lambda\leq 1)$, which permits tracking of slowly varying signal parameters, is used.

$$\Theta_p(n) = \Theta_p(n-1) + K_p(n)e(n) \quad (4)$$

where

$$K_p(n) = \frac{P(n-1)Y_p(n)}{\lambda + Y_{p(n)}^T P(n-1)Y_p(n)} \quad (5)$$

$$P(n) = \frac{1}{\lambda}[P(n-1) - K_p(n)Y_p^T(n)\, P(n-1)] \quad (6)$$

Also, the weighted sum of the squares of the residual error can be expressed recursively as

$$\varepsilon(n) = \lambda\varepsilon(n-1) + [1 - Y_p^T(n)K_p(n)]e^2(n) \quad (7)$$

Now we introduce the variable forgetting factor concept to estimate time-varying spectra of nonstationary signals. Traditionally, the recursive least squares method with fixed forgetting factor is used to estimate time-varying spectra by assigning $\lambda<1$. Progressively smaller values of λ results in the AR parameters being computed with effectively smaller windows of data which are beneficial in nonstationary situations. However, if a signal is composed of subsignals with different degrees of nonstationarity, it would not be optimal to estimate the AR parameters with the same fixed value of $\lambda<1$. In order to determine the value of the variable forgetting factor to be used in calculating the next AR parameters, error sources are examined. The first error arises from finite data, called estimation error. This error has zero bias and a variance which decreases with data length. There also is error caused by nonstationarity. The variance of this error increases with data length and can be minimized by rapidly discounting the past and basing estimates predominantly on the most recent data. The other source for the error is noise, e.g., measurement noise. Then, the extended prediction error, which estimates the degree of the nonstationarity of the signal, is defined by

$$Q(n) = \frac{1}{M}\sum_{i=0}^{M-1} e^2(n-i) \quad (8)$$

where an appropriate averaging (M) is introduced to minimize the effect of a spurious large additive noise error, since the error coming from the noise, e.g., measurement noise, is a random process. However, M should be a small number so that the averaging does not obscure the nonstationarity of the signal. M is also used to cancel out the periodicity of the error since the prediction error, which is the difference between the signal and model output, may have periodicity if the signal under consideration is periodic.

A useful figure to determine the speed of adaptation is the asymptotic memory length N given by

$$N = \sum_{i=0}^{\infty} \lambda^i = \frac{1}{(1-\lambda)} \quad (9)$$

which implies that the information dies away with N memory length. A strategy for choosing the forgetting factor is defined by

$$\lambda(n) = 1 - \frac{Q(n)}{\sigma_e^2 N_{max}} \tag{10}$$

where σ_e^2 is the expected noise variance. The maximum asymptotic memory length (N_{max}) will govern the speed of adaptation. Note that the value of the variable forgetting factor in (10) will become close to unity ($\lambda(n) = 1 - 1/N_{max}$) for a stationary process since the extended prediction error of a stationary process will approach the noise variance (see (8) and (10)). For a signal with a high degree of nonstationarity, a small value of $\lambda(n)$ will be obtained due to the high value of extended prediction error in (8). These results are quite consistent with the basic idea behind using different values of fixed forgetting factors depending on *a priori* information of a signal in that λ is set close to unity if it is known that the signal is a stationary process, and progressively smaller values of λ are used in nonstationary environments. However, in the scheme used in this paper, the variable forgetting factor is automatically adapted to a signal by an extended prediction criterion which accounts for the nonstationarity of the signal. Since this forgetting factor adaptation scheme does not guarantee that $\lambda(n)$ does not become negative, it is best to place a reasonable lower limit on the forgetting factor (λ_{min}).

The determination of the exact value of the parameter, N_{max}, depends on *a priori* information about the nonstationary signal, which is usually not available in practice. Fortunately, the performance of the AR method with a variable forgetting factor is not sensitive to N_{max} unless N_{max} is too low. In most cases, spectral estimation of a nonstationary signal can be performed by assigning a value between 0.99 and 0.999 to λ_{max}, and, thus, assigning a value between 100 and 1000 to N_{max}. The low frequency of the signal, f_M, which characterizes the period of the prediction error, can be used to determine the parameter M in order to cancel out the periodicity of the prediction error by averaging; i.e., $M = 1/f_M$.

A similar scheme used [3] to avoid the "covariance wind-up" problem in self-tuning control is defined by

$$N(n) = \sigma_n^2 N_0 / [1 - Y_p^T(n) K_p(t)] e^2(n) \tag{11}$$

The denominator in (11) corresponds to the second term on the right-side of (7), containing new residual error information. This approach enables the parameter estimates to follow both slow and sudden setpoint changes in plant dynamics for nearly deterministic situation. However, if one is concerned with estimating time-varying spectra of nonstationary signals, the asymptotic memory length to be used in the calculation of the next value of the forgetting factor should be modified as in (8) and (10) in order to minimize the effect of a spurious large additive noise error and to smooth the periodicity of the residual.

Ⅲ. Experimental Results

The experiments were conducted in a low-turbulence wind tunnel. A sketch of the facility is shown in Fig. 1. The contraction section is separated into two parts by a splitter plate. The dimensions of the test section are 30×20×150 cm. The tunnel is instrumented with a Disa 56C/N hot-wire anemometry system. Real-time signal analysis and initial checks of the running conditions were performed using an HP3562A spectrum analyzer. The hot-wire signals are processed by a twelve-channel, twelve-bit, CAMAC system. This system is controlled by an IBM PC AT computer. Data analysis was performed on a VAX Station 3500.

The mixing layer is formed by the merging of the flows on either side of the splitter plate. The high speed stream, U_1, is 7.17 m/sec and the low speed stream, U_2, is 1.51 m/sec. This results in a velocity differential $\Delta U = U_1 - U_2 = 5.66$ m/sec and a velocity ratio $R = (U_1 - U_2)/(U_1 + U_2) = 0.652$.

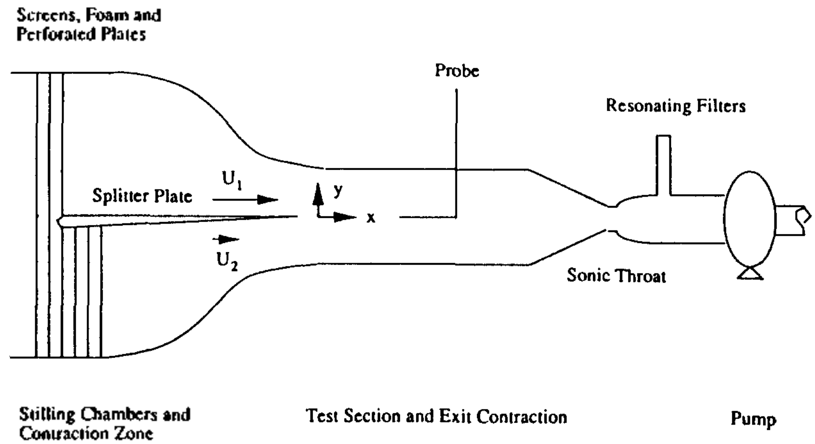
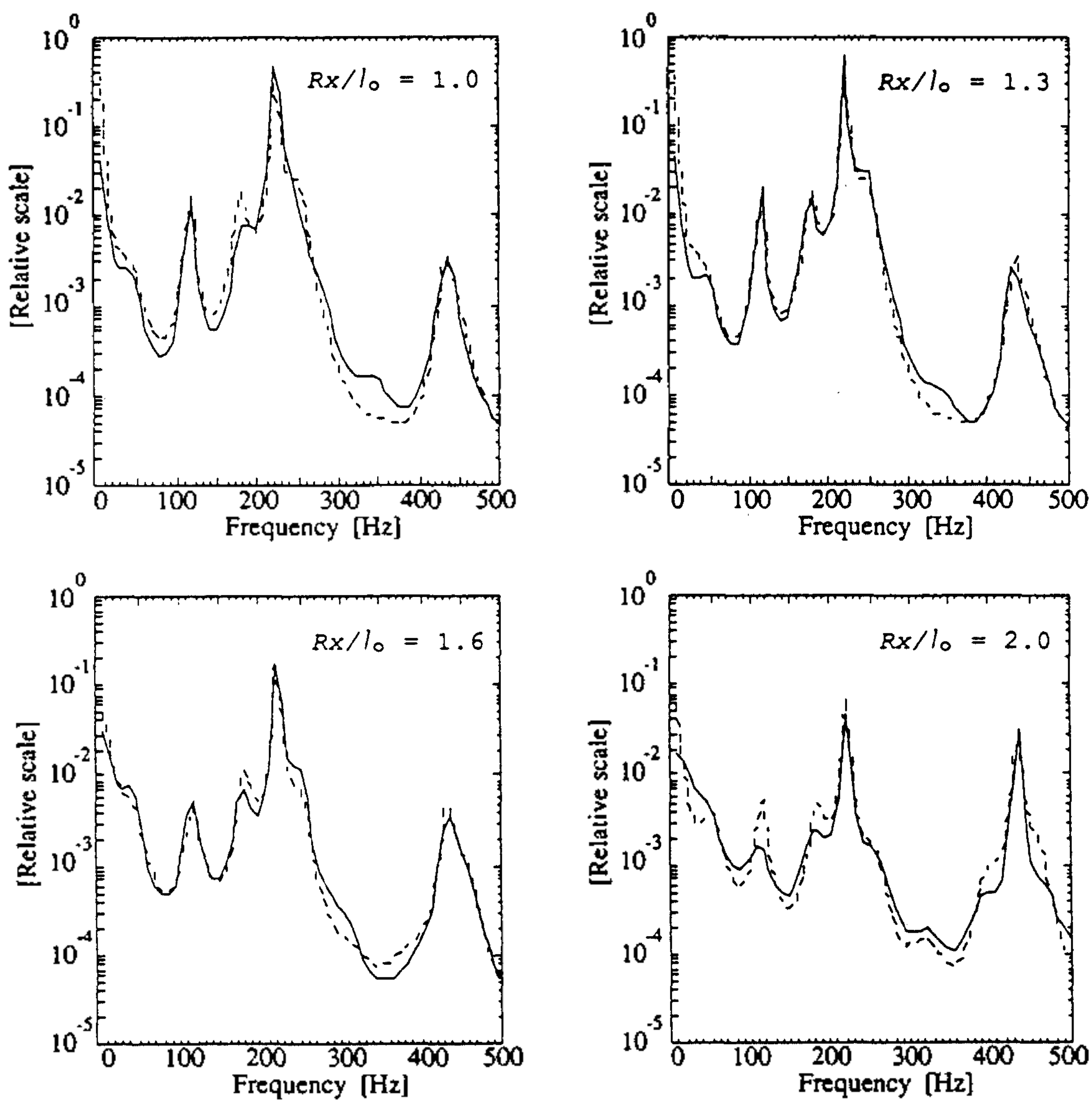


Fig 1 Schematic diagram of the wind tunnel apparatus.



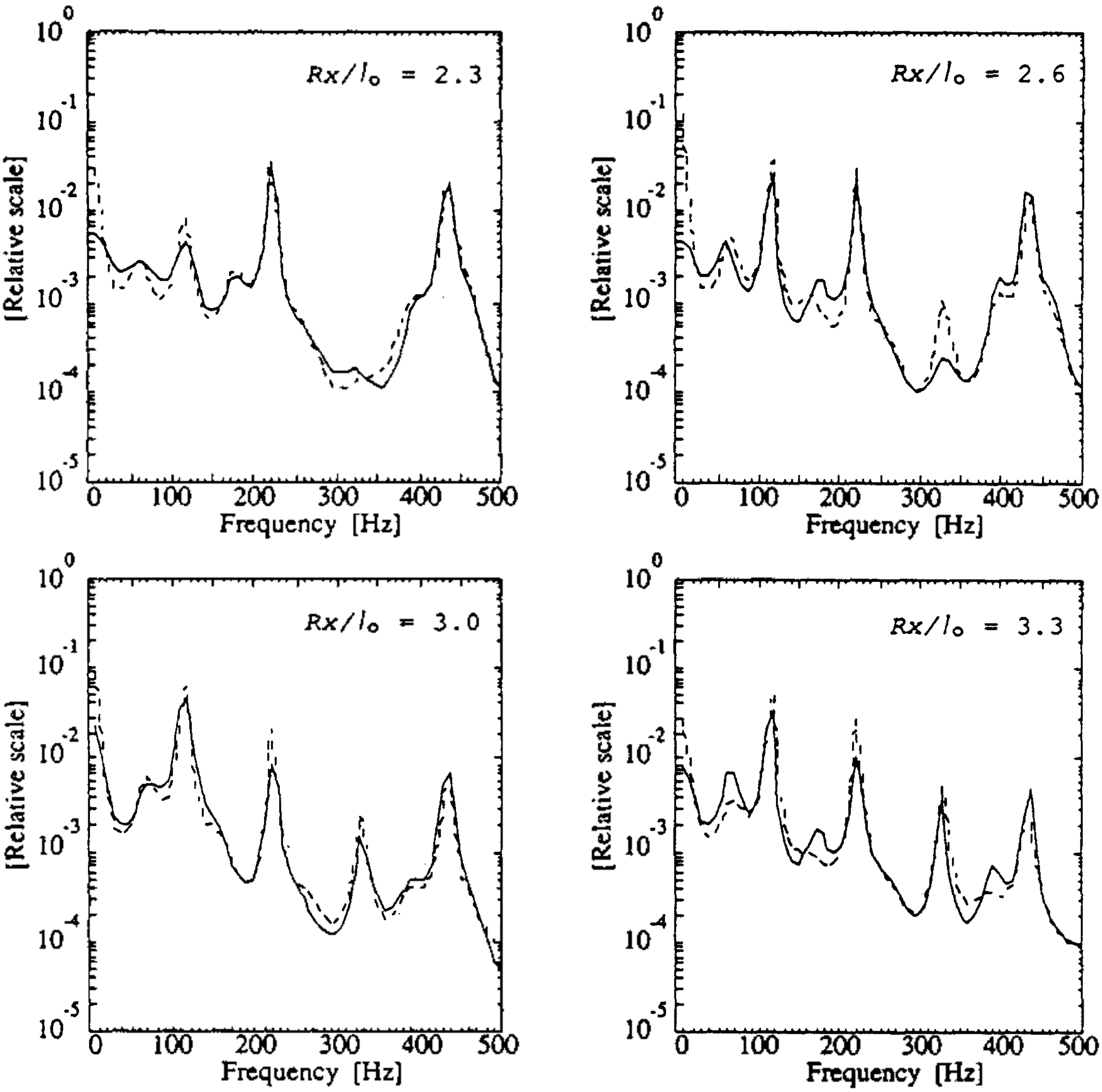


Fig 2 Power spectra of velocity-fluctuations at 8 selected downstream locations from $Rx/l_0 = 1.0$ to 3. (solid line : estimated spectrum, dotted | : reference spectrum.)

652. The measured initial wavelength of the most fundamental instability mode, l_0, is 1.98 cm. This value will be used as a reference length scale. The free-stream turbulence intensity in the vicinity of the trailing edge of the splitter plate is 0.0005 ΔU in the low speed stream. Most of this intensity is concentrated in the lower frequencies. The data obtained by moving the probe in the downstream direction from $R_x/l_0 = 0.66$(i.e.,x=2 cm) to 3.3 (i.e.,x=10 cm) at a constant velocity, $U_{probe} = 2$ cm/sec, are nonstationary since the spectrum changes as we proceed downstream. Measurements of velocity-fluctuations are digitized for 4 seconds with a sampling interval of 1 msec. The spectra of the velocity-fluctuations at 8 different but fixed locations along the x-axis and at across-stream location where u'_{rms} is a maximum were estimated beforehand using 4,096 samples each and used as a reference for comparison purposes (e.g., dotted line in Fig.2).

The RLS-VFF approach described in Section 2 has been applied to the spectral estimation of a spatially nonstationary signal in wind tunnel tests. In order to properly apply the spectral estimation method of nonstationary signals to a data set obtained by moving a probe, one needs to sel ect several parameters.

First, one needs to select the order, p, of the RLS-VFF model. Even though it is difficult to suggest the accurate choice of model order p which gives the best fit, there are a few guidelines available for model order selection. When a signal is not perturbed by noise, the theoretical model order p must be twice the number of sinusoids. When the signal is perturbed by an additive noise, the model order p must be increased to have additional poles available to represent the noise. However, too high a guess for model order introduces spurious detail into the spectrum. Since the largest for model order introdus detail into the spectrum.

Since the largest possible number of sinusoids for the signal in our experiment is 7 (see the case of $Rx/l_0=3.3$ in Fig.2) and the signal is perturbed by an additive noise, the model order for this signal is set equal to 20.

An *a priori* knowledge of the degree of nonstationarity of the signal beforehand is unrealistic. Hence, the maximum $\lambda_{max}=0.999$ and minimum $\lambda_{min}=0.75$ are selected and the variable forgetting factor $\lambda(n)$ is allowed to change between these limits. Thus, a maximum asymptotic memory length $N_{max}=1/\lambda max=1000$, given by (9), is used in our experiments. The largest spectral peak occuring at about 200 Hz(period = 5 msec) is used to deterimine the parameter $M=5$ in order to cancel out the periodicity of the prediction error by averaging (note that the period of the signal is 5 msec and the sampling interval is 1 msec). M is also used to average the noise error in order to prevent a large spurious noise error from misleading the next calculation of the variable forgetting factor. However, the average number $M=5$ is small compared with $N_{max}=1000$ so as not to obscure the nonstationarity nature of the signal.

In Fig. 2 spectra estimated using the RLS-VFF method on nonstationary moving probe data (solid line) and reference spectra obtained by

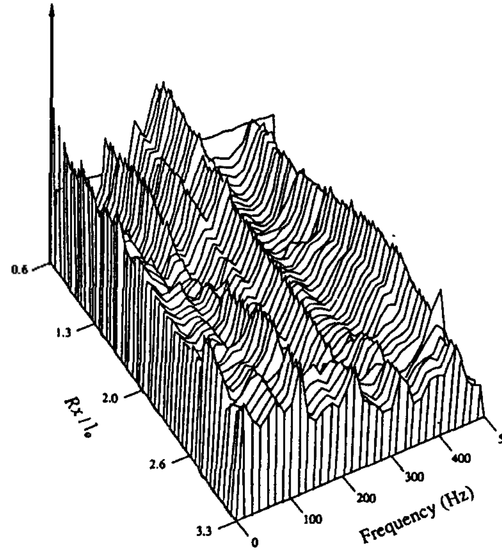


Fig 3 Downstream power spectra of velocity-fluctuations in wind tunnel estimated by recursive least squares algorithm with a variable forgetting factor. The spatial resolution is $Rx/l_0=$ 0.054 or 0.16 cm. The frequency resolution is $\Delta f=500/32=15.625$ Hz.

classical methods from measurements with fixed probes (dotted line) at selected downstream locations are shown. The estimated spectra show relatively good agreement with the reference spectra, especially at the peaks. However, the agreement is often not as good in the valleys (which are typically 20 to 30 dB's below the peaks) where signal to noise ratio (SNR) is low. The lack of agreement is due to the fact that in the moving probe approach only a single data set is used to estimate downstream power spectra over the section of interest, whereas in the fixed probe approach a large number of ensemble averages can be carried out to estimate an accurate power spectrum, even in the valleys.

The downstream evolution of power spectra estimated using the RLS-VFF moving probe approach is shown in Fig. 3. This figure is obtained by plotting every 80th power spectrum out of the 4000 total power spectra, resulting in a 3-dimensional plot with a 50 point Rx/l_0 scale. In physical coordinates, this is equivalent to spectra measured every 0.16cm, as the probe moves along. Thus, the combination of variable forgetting factor, and a moving probe can yield very closely spaced power spectra, a representation that can be extremely powerful in visualizing the changing spectral dynamics associated the downstream evolution of the flow. In order to estimate the downstream power spectra with the same spatial resolution using the fixed probe approach, one would have to repeat the same experiment 50 times, which is often a tedious and time-consuming job.

It is important to note here that the spatial evolution of the power spectra measured with a moving probe and the RLS-VFF method agree with results in [9] obtained using fixed probes and FFT techniques to estimate the power spectra in the same facility under identical experimental conditions. For example, an examination of Figs. 2 and 3 indicates that the initial region of the transition ($0.6 < Rx/l_0 < 2.0$) is characterized by a large peak at the fundamental frequency (215 Hz) as predicted by hydrodynamic linear stability analysis. Another peak at the subharmonic frequency (107.5 Hz) is also noticed. As we progress further downstream, we notice a strengthening of the peak at the subharmonic

Table 1 Normalized mean square errors between the reference spectra and estimated spectra at selected downstream locations using recursive least squares algorithm with (a) fixed forgetting factor (FFF) (b) variable forgetting factor (VFF)

Rx/l₀ FFF	1.0	1.3	1.6	2.0	2.3	2.6	3.0	3.3
1.00	0.66	0.24	0.20	0.82	1.3	2.2	1.3	0.41
0.99	0.53	0.24	0.72	0.61	0.20	0.41	0.35	0.19
0.98	0.53	0.24	0.72	0.61	0.20	0.41	0.35	0.19
0.98	1.9	0.67	2.3	1.3	0.39	0.84	2.1	0.21
0.97	49	1.9	3.9	1.3	0.53	1.4	92	0.39
0.96	110	4.0	5.6	1.1	0.68	2.1	160	0.94
0.95	4.9	8.3	8.0	0.94	0.82	3.1	170	2.2

(b)

VFF	0.36	0.14	0.11	0.24	0.097	0.20	0.09	0.19

frequency between $Rx/l_0=2.0$ and 2.6. Further downstream, between $Rx/l_0=3.0$ and 3.3 we notice strengthening of the 3/2 harmonic at 322.5 Hz.

Normalized mean square errors between the reference (fixed-probe) spectra and the estimated (moving-probe) spectra at selected downstream locations are presented in Table 1. The performance of our approach with a variable forgetting factor (VFF) are compared to those with fixed forgetting factor (FFF) equal to 1.00, 0.99, 0.98, 0.97, 0.96, and 0.95. The normalized mean square error is defined as follows :

$$MSE=\frac{1}{64}\sum_{m=1}^{64}\frac{[S(m)-\hat{S}]^2}{S^2(m)} \qquad (12)$$

where $S(m)$ and $\hat{S}(m)$ are the reference and estimated spectrum, respectively. The normalized mean square error is averaged over 64 different frequency points (here, 64 signifies the number of frequencies up to the Nyquist frequency (500 Hz)). The upper bound of the mean square error using the variable forgetting factor method is obtained at $Rx/l_0=1.0$ and is equal to 0.36. This value is low when compared to the values obtained using fixed forgetting factors. By examing normalized mean square errors at other locations, one can see that the proposed RLS−VFF method exhibits superior performance compared with typical fixed forgetting factors ($0.95 \leq \lambda \leq 1$).

Note that when moving the probe downstream to estimate power spectra of velocity-fluctuations one encounters fewer wave fronts per unit time than when counting wave fronts at a fixed probe. This frequency shift associated with moving probe is analogous to a Doppler shift. The fractional difference between the frequency measured by moving the probe and the one by a fixed probe is given by $\Delta f/f=-U_{probe}/-U_{flow}$. Here, U_{flow} is the velocity of the flow, U_{probe} is the velocity of the moving probe. The minus sign is required for the correct determination of apparent frequency increase or decrease, depending upon whether the probe is moving with or against the flow velocity. The fractional frequency shift ($\Delta f / f$) occurring in this experiment is about (2 cm/sec)/(4 m/sec) = 0.5%, which is negligible. Thus, one needs to make sure that the probe does not move too fast (compared to the velocity of the flow so as not to create a large amount of frequency shift.

Ⅳ. Conclusion

Traditionally, the downstream evolution of power spectra of velocity-fluctuations in a wind tunnel are determined by measuring data at each poing of interest to estimate the spectrum for that point. In this paper, a new AR method of estimating power spectra of nonstationary signals is presented using *variable forgetting factors* which are adapted to the signals via the criterion of an extended prediction error. The practicality of the proposed moving probe approach using variable forgetting factors has been demonstrated via spectral estimation of a spatially nonstationary signal associated with transition to turbulance studies. The variable forgetting factor approach is shown to yield statistically meaningful, high spatial resolution plots of the power spectra as the flow evolves downstream. Our results indicate that the variable forgetting factor method exhibited better performance (in terms of mean square errors) in our experiments than typical fixed forgetting factor methods.

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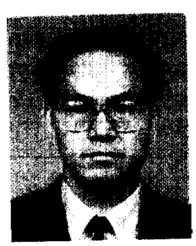

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