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MAGNETIC INTERACTION AND X-RAY ABSORPTION OF THE MAGNETIC COMPACT STARS

Yonggi Kim
Yonsei University Observatory,
Seoul 120-749, Korea

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ABSTRACT

Using a simple analytic model based on the MHD stability arguments we obtain the size of the magnetosphere for the magnetic compact stars. We assume the ordered, field-aligned flow in the magnetosphere and estimate the well-known Alfven radius. The dependence of the X-ray absorption in the magnetic funnel on the size of this radius is further considered. We show that such a determination of the magnetic interaction radius can be applied to the reconstruction of X-ray light curves of the magnetic binary stars.

1. INTRODUCTION

The study of compact star magnetospheres is one of fascinating physics problems, which provide a check on our understanding of plasma and magnetospheric physics under conditions that differ by many orders of magnitude from those encountered in planetary magnetospheres or terrestrial laboratories. For example the interaction of the stellar magnetic field with the accreting matter in the accretion disc can cause to channel the accretion matter in the magnetic field, and affect the intensity of X-ray produced near the surface of the magnetic compact star.

Many X-ray stars exhibiting periodic and quasi-periodic oscillations in intensity are thought to be magnetic compact stars accreting matter from a close binary companion. An accretion disc can be formed around a magnetic compact star in a close binary system if it's magnetic field is not strong enough, or if the seperation of binary system is long enough. The matter in the disc is assumed to circulate around the compact star at the Keplerian velocity. The rotational axis of the intermediate polars is not aligned with the magnetic axis. In this case the accretion flow is funneled at some radius where the ram pressure is governed by the magnetic pressure. Within this radius (Alfven radius) the plasma flows toward the magnetic poles of the compact star along the magnetic field lines and accretes onto a small arc-shaped region on the stellar surface. The intersection angle of the magnetic field with the disc and the component of the gravitational vector parallel to the field lines change around the disc. Therefore, the accretion rate toward the upper and lower pole varies as a function of the azimuthal angle of the disc. It is expected that the largest fraction of matter accretes from the side of the disc closest to the magnetic pole. Near the stellar surface the infalling matter produces a shock which gives rise to X-ray emission modulated with the rotational period. A new phenomenological approach to the distribution of accretion rate as a function of the rotational phase is presented by Kim (1992). In his paper photoabsorption by the material in the accretion funnel is considered as be mainly responsible for the X-ray modulation with the rotational phase and it is also shown that the observed column density, N_{HI}, does not come from the hard X-ray emitting region, but from the matter in the accretion funnel.

Because the structure of this accretion funnel is highly influenced by the magnetic interaction radius, it is worth to study the dependence of this radius on some physical parameter (e.g. magnetic field strength, accretion rate, size of the interaction zone etc). We present in this paper how the interaction radius in the magnetic compact star containing accretion disc can be analytically estimated and how this radius can be applied to the reconstruction of X-ray light curves of the magnetic binary stars.

2. PHYSICS OF MAGNETIC INTERACTION

The magnetosphere is in this paper defined as the volume around the compact star in which the flow of mass, energy, and angular momentum be strongly affected by its magnetic field. The observed properties of this accreting compact stars can be profoundly influenced by the scale of the magnetosphere. To get the better understanding the interaction between accretion disc and magnetosphere many theoritical and observational studies have been undertaken until now (for reviews see Frank, King and Raine 1992). Observations of the magnetic compact stars show that the matter in the accretion disc can be accreted in the magnetosphere, but the mechanism of enterance of the matter from the accretion disc into magnetosphere is only poor understood. A fundamental consideration originates from Gosh and Lamb (1978, 1979a, b) and Anzer and Börner (1980, 1982, 1983).

Because of the very complicated structure of the magnetosphere it is very difficult to model the dynamic properties of the accretion flow and the magnetic interaction. Therefore we take a simple analytic model to estimate the scale of the magnetosphere. A stable magnetospheric flow is characterized by the interaction radius, inside which ordered, field-aligned flow is possible. This radius can be estimated by the MHD stability arguments. Namely field-aligned flow is stable inside the

Alfven radius r_A, which is given implicitly by the condition that the pressure by the accretion flow be equal to the magnetich pressure.

$$\frac{B_p^2}{4\pi} = \rho v_p^2 \tag{1}$$

In the case of the accretion by the accretion disc the following assumption were taken to estimate the interaction radius as described in Frank, King, and Raine (1992). The first assumption is the poloidal dipole magnetic field.

$$B_p \sim \frac{\mu}{r^3}(1 + 3cos^2\theta)^{\frac{1}{2}} \tag{2}$$

where B_p is poloidal magnetic field strength, μ the magnetic moment, r the radius from the center of the star. Secondly it is assumed that the flow in the magnetosphere is field-aligned, so that

$$\frac{\rho v_p}{B_p} = const \tag{3}$$

on each flux tube. v_p and ρ is the inward poloidal velocity and the density of the accreting plasma on each flux tube respectively. Equation (3) comes from the combination of mass conservation ($\rho v_p S = \dot{M}$) and magnetic flux conservation ($S\ B_p = const$), where S the surface of the flux tube. Then the inward poloidal velocity v_p of the accreting plasma at r is assumed to be approximately equal to the free-fall velocity $v_{ff}(r)$.

$$v_p \sim v_{ff} = \left(\frac{GM}{r}\right)^{\frac{1}{2}} \tag{4}$$

The next assumption is that the accretion plasma falls on an area S at the stellar surface (accretion surface), which is a fraction area

$$f = \frac{S}{4\pi R^2} \tag{5}$$

of the whole surface.

With the above 4 assumptions we can now derive the interaction radius that satisfies the Eq. (1). The dipole geometry of the magnetic field is given in a spherical coordinate (r, θ, ϕ) at the given value of ϕ as following,

$$r = R_m sin^2\theta \tag{6}$$

where R_m is the radial distance of the magnetic field line at $\phi = 90°$. Fig. 1 shows a simple side-on view of the magnetic dipole geometry used in this paper. The magnetic axis is assumed to be inclined to the disc (offset angle β). The footpoint

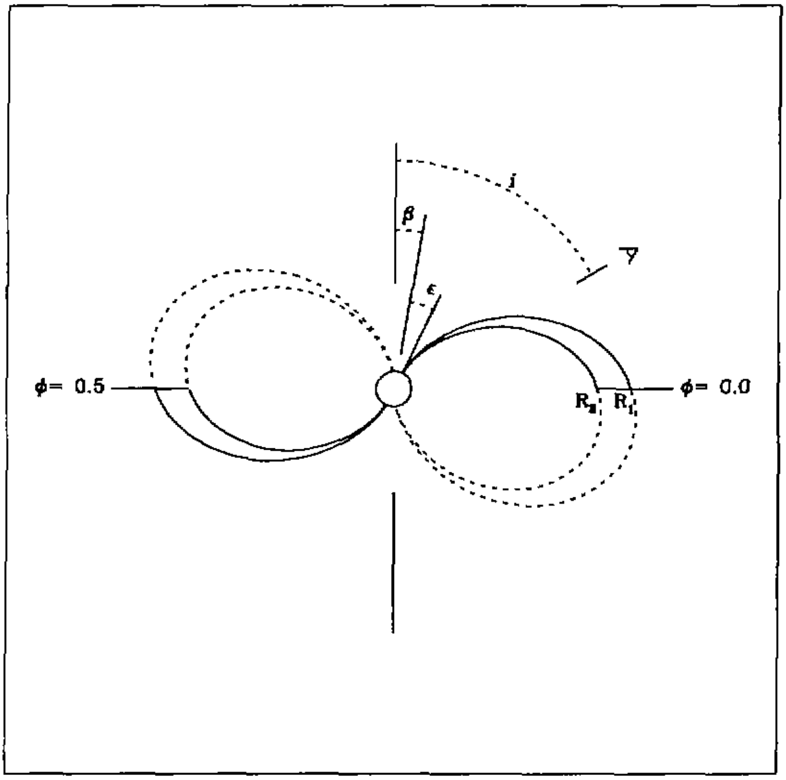

Figure 1. A simple dipole magnetic geometry for the estimate of the magnetic interaction radius. Explanation of simbols is given in the text

of the magnetic field line at the star surface has a coordinate value of $(R_{cs}, \epsilon, \gamma)$, where R_{cs} is the radius of the compact star, ϵ is the magnetic colatitude of the field line, and γ is the azimuth angle of given field line.

Between the magnetic colatitude, the offset angle and the azimuth angle there is a relation which can be derived simple from the spheric rectangular triangle:

$$\begin{aligned} \sin^2 \varepsilon &= \frac{R_{cs}}{r} \sin^2 \theta \\ &= \frac{R_{cs}}{r} (1 - \sin^2 \beta \cos^2 \gamma) \end{aligned} \tag{7}$$

The accretion surface S is dependent on the position and the size of the interaction region between magnetosphere and the accretion disc (R_1, R_2, and $\Delta R = R_1 - R_2$) and the variation of the accretion surface has a strong influence on the mass accretion rate at the stellar surface and the property of the produced X-ray radiation. The accretion surface S is given by

$$\begin{aligned} S &= \int_{\varepsilon_1}^{\varepsilon_2} \int_0^{2\pi} R \sin \varepsilon R d\varepsilon d\gamma \\ &= 2\pi R^2 (\cos \varepsilon_2 - \cos \varepsilon_1). \end{aligned} \tag{8}$$

The fraction area f of Eq. (5) is then

$$f = \frac{2S}{4\pi R^2} = \cos \varepsilon_2 - \cos \varepsilon_1. \tag{9}$$

This fraction area f is also dependent on the position and the size of the interaction region in the accretion disc.

The interaction region is characterized by R_i und ΔR where R_i is defined as the interaction radius, inside which the plasma start to flow into the magnetic compact star along the magnetic field. R_i is R_1 in Fig. 1 and $\Delta R = R_1 - R_2$. We consider 2 regions in an accretion funnel, namely in a interaction region (ρ_2, v_2 and B_2) and in the stellar surface (ρ_1, v_1 and B_1). The second assumption Eq. (3) can be now rewritten:

$$\frac{\rho_1 v_1}{B_1} = \frac{\rho_2 v_2}{B_2}. \tag{10}$$

With Eq. (4) and Eq. (9), the interaction radius R_i can be solved in MHD stability condition Eq. (1):

$$\begin{aligned} R_i &= \left(\frac{f^2 \mu^4 f_b}{G M \dot{M}^2 R^2} \right)^{\frac{1}{5}} \\ &= 0.94 \times 10^8 \left(\frac{f_{-2}^2 \mu_{30}^4 f_b}{M_1 \dot{M}_{16}^2 R_9^2} \right)^{\frac{1}{5}} \end{aligned} \tag{11}$$

In this equation f_b is the θ-dependent part of the dipole magnetic field B, $f_b = (\frac{1+\cos^2\theta}{1+\cos^2\epsilon})^{\frac{1}{2}}$ and f_{-2} is the fraction area in 10^{-2}, μ_{30} the magnetic moment in 10^{30} Gcm^3, $\dot{M}_{16}$ the accretion rate in $10^{16} gs^{-1}$, R_9 the radius of the magnetic star in $10^9 cm$, and M_1 the stellar mass in the solar mass. We have now the upper limit of the magnetosphere, inside which the accreting plasma begins to flow from the accretion disc into the magnetosphere.

In the next section we present the numeric procedure to estimate the interaction radius (Eq. (11)) and results.

3. NUMERIC PROCEDURE AND RESULTS

Because the fraction area f is dependent on the position and the size of the magnetic interaction region, the magnetic interaction radius R_i can be only estimated iteratively as following: With a start radius R_{start}, the fraction area f is calculated for a given ΔR in Eq. (9) and then the magnetic interaction radius R_i is calculated in Eq. (11). If the calculated interaction radius $R_{i,c}$ is equal to R_{start}, R_{start} is regarded as the interaction radius, if not so, $R_{i,c}$ is recalculated after correction of the R_{start} until $R_{i,c}$ becomes the corrected R_{start}.

With fixed R_i and ΔR the magnetic moment can be shown as a function of the accretion rate, results of which are very useful to find the magnetic moment for a given accretion rate or vice versa. Fig. 2 demonstrates the relation between the magnetic moment and the accretion rate. For one magnetic compact star with a magnetic moment of $10^{33} Gcm^3$, an accretion rate of $3 \times 10^{16} gs^{-1}$ can be read off in this figure with $R_i = 5R_{cs}$.

In the literature, the magnetic interaction radius R_i and the size of the interaction region ΔR (or R_1 and R_2 in Fig. 1) are usually adopted as seperate model parameter for the interaction region (Mason *et al.* 1988, Rosen *et al.* 1988), but the iteration methode in this paper reduces the number of unknown parameter for the geometry of the magnetic interaction region. Only one of these two parameters needs to be taken as a model parameter. Among the parameters $(\mu, \dot{M}, R_i, \Delta R)$, if we can get a information about R_i or $\dot{M}$ from the observation, we can estimate the magnetic moment and the fraction area f with a help of the iteration methode. Such a parameter set can be used to a further analysis of the observed data as the start model parameter, which is planning to be undertaken for the magnetosphere model of the magnetic compact star with accretion disc.

It is easily to be seen in Fig. 1 that the structure of the accretion funnel can be altered by the variation of the interaction region. The change of the accretion funnel geometry will cause to modify the column density for the photoabsorption of the X-rays. Therefore we expect the X-ray light curves affected by the accretion funnel geometry. The iterative method to estimate the interaction radius can be added now to the phenomenological modelling of the X-ray photoabsorption in the magnetosphere. Such a model calculation has been done in this paper. Namely, in contrast to Kim (1992), where R_1 and R_2 in Fig. 1 are taken as the two seperate model parameter, a model calculation is carried out only with ΔR as a model parameter for the geometry of the interaction region. The calculated X-ray light curves in some energy band are shown in Fig. 3. The model parameter used in this calculation are listed in table 1.

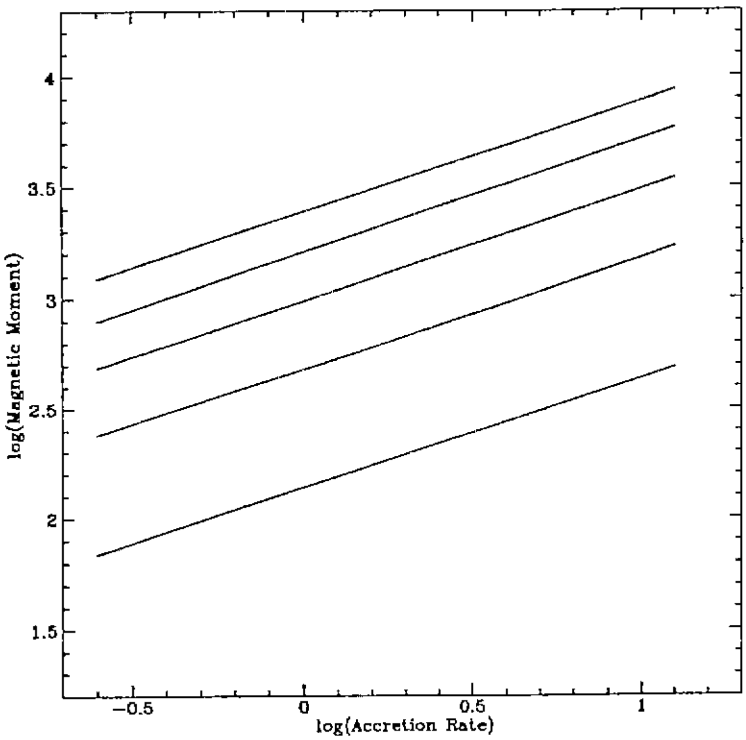

Figure 2. Magnetic moment(μ_{30}) as a function of accretion rate ($\dot{M}_{16}$) with the interaction radius R_i as the Parameter. Each curve shows the relation for a fixed R_i (25, 20, 15, 10, and 5 R_{cs} from the upperst curve on, and $\Delta R = 0.3$)

Cataclysmic variables are close binaries with a white dwarf having a strong magnetic field enough to channel the accretion flow onto restricted regions of the white dwarf surface. DQ Her type cataclysmic variables (intermediate polars, a subclass of cataclysmic variables) are thought to accrete the plasma from the accretion disc into the magnetosphere of the white dwarf. Therefore intermediate polars can be a good laboratory to apply the phenomenological model for the X-ray photoabsorption in the magnetosphere. In fact, many X-ray observations of the intermediate polars (Norton and Watson 1991, Beuermann 1987, Rosen *et al.* 1991) show a decreasing modulation depth with increasing energy, which is qualitative consistent with the results of our simple calculation in Figure 3. Because we have a phenomenological model producing a X-ray light curves qualitative same as the observed X-ray light curves, it becomes now possible that such a model can be applied to the reconstruction of the X-ray light curves for intermediate polars and to the quantitative analysis of the observed X-ray light curves.

Table 1. Model parameter used for the calculation of X-ray light curves: another parameters for the calculation are same as in Kim (1992)

Parameter	Symbol	Value
Mass of the compact star	$M_{cs}/M_{\odot}$	1.0
Radius of the compact star	R_{cs}/R_9	1.0
Accretion rate	$\dot{M}/\dot{M}_{16}$	1
Magnetic moment	μ/μ_{30}	100
Inclination angle	i	60
Offset angle	β	10
Size of interaction zone	$\frac{\Delta R}{R_i}$	0.3

4. DISCUSSION

Light curves in Fig. 3 have the same qualitative form as the observed X-ray light curves for intermediate polars (Beuermann 1987, Norton and Watson 1991, Rosen *et al.* 1991). This result will make possible to enlarge the photoabsorption model (Kim 1992), and to fit the observed X-ray light curves. The interaction radius estimated in this paper can be also used to study reemission of the X-ray in the accretion funnel. The absorbed X-rays heat the plasma in the accretion funnel and produce optical light. Such a optical spectrum in observed for the intermediate polars by Hellier *et al.* (1987, 1991). The optical spectrum due to this reprocessing can be calculated theoritically by energy balance between input and output energy. The modelling of this reemission with the interaction radius introduced here should be undertaken in order to understand the physics of the magnetospheric structure for the intermediate polars more better.

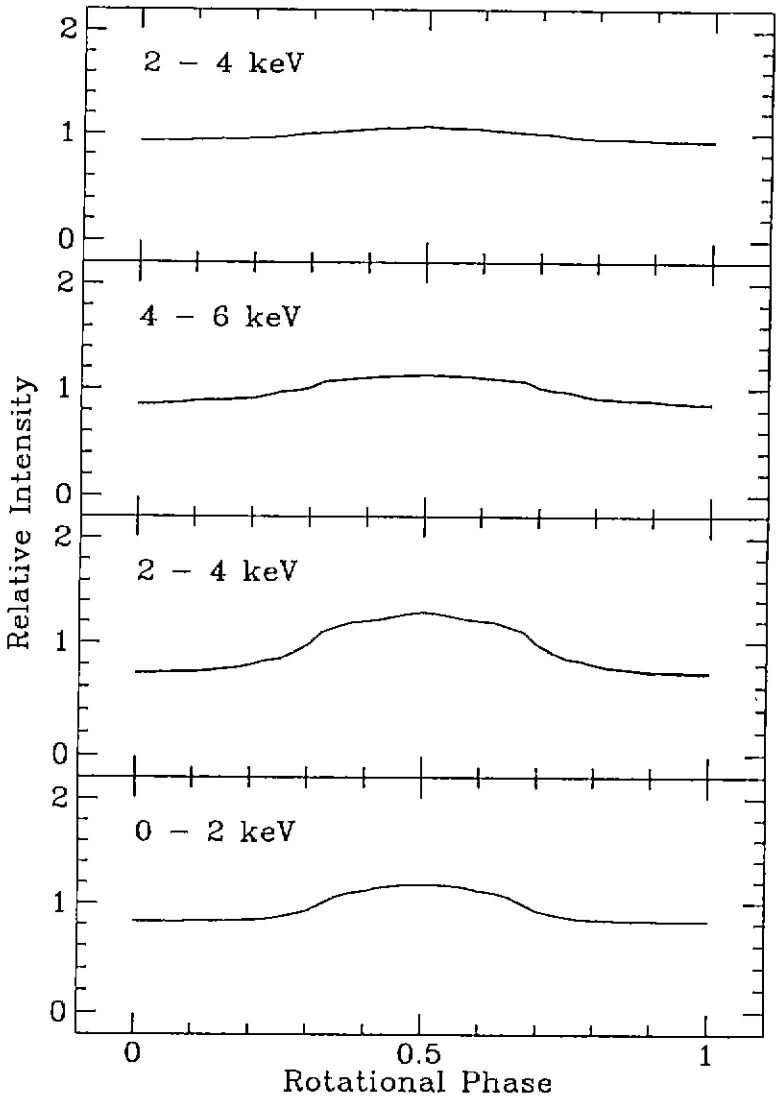

Figure 3. Rotational X-ray light curves for some energy band. The modulation depth is 30, 44, 24 und 13 % respectively for the energy band of 0 - 2, 2 - 4, 4 - 6 and 6 - 10 keV

The estimate of the interaction radius suggested in this paper is based on the assumption that the accretion occurs from the accretion disc. But it is to be mentioned that there is currently for the intermediate polars no consensus on whether the plasma accretes through a accretion disc (Rosen, Mason and Cordova 1988, Hellier, Cropper and Mason 1991) or by the direct impact of the accretion stream on the magnetosphere (Hameury, King and Lasota 1986, Norton *et al.* 1992). From the observational view on there is a considerable observational evidence for the existence of discs in many of the intermediate polars (Hellier 1991). Observational and theoretical studies are necessary for solving this problem. More detailed results of the application of the interaction radius in the phenomenological model of the intermediate polars will be reported elsewhere.

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