⊙ 研究論文

Trajectory Tracking Control of the Wheeled Inverse Pendulum Type Self-Contained Mobile Robot in Two Dimensional Plane

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역진자형 자주로보트의 2차원 평면에서 궤도주행제어에 관한 연구

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Key words : Inverse Pendulum(역진자), Self Contained Mobile Robot(자주로보트), Posture Control(자세제어), Steering Control(진로제어), Tracking Control(주행제어)

Abstract

In this paper, we discuss on the control algorithm to make the wheeled inverse pendulum type mobile robot move in two dimensional plane. The robot considered in this paper has two independently driven wheels in same axel which suport and move it-self, and is assumed to have the gyro type sensor to know the inclination algle of the body and rotary encoders to know wheel's rotation angular velocity. The control algorithm is divided into three parts. The first part is for the posture and velocity control for forward-backward direction, the second is the steering control, and the last part is for the control of total system to track the given trajectory. We handle the running velocity control of the robot as a part of the posture control to keep the balance because the posture relates deeply with the velocity and can be controlled by the velocities of the wheels. The control problem is analyzed as the tracking control, and the controller is realized with the state feedback and feed-forward of the reference velocity. Constructing the control system which contained one integrator in forward path, we also realized the control system without observer for the estimation of the accumulated errors in the inclination angle of the body. To prevent the robot from being unstable state by sudden variation of the reference velocity when it starts and stops, or changes velocity, the reference velocity of which acceleration is slowly changing, is ordered to the robot. To control its steering, we give the different reference velocities for both wheels which are calculated from the desired angular velocity of the

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body. Finally, we presents the experimental results of the experimental robot Yamabico Kurara in which the proposed control algorithm had been implemented.

Ⅰ. Introduction

The wheeled vehicle which have two driving wheels and two casters, are widly used. However, if the casters get large gravity force, the trajectory of the vehicle is sometimes got disturbance by the irregular force caused by casters. So, the vehicle without casters is a counterplan of this problem. The control problem of the mobile inverse pendulum is worth considering not only for the technical interests but also for the practical applications. Some previous researches on the wheeled inverse pendulum type robot[1,2,3] has been reported. T. Kawamura and K. Yamafuji[1] proposed posture and driving control algorithm of the coaxial bicycle using classical control theory. Their algorithm needed the tactile sensor to detect to posture between ground and body. And their robot could not be moved in two dimensional plane because its both wheels are driven by one motor. O. Matsumoto, S. Kajita and K. tani[2] presented the estimation and control algorithm of the posture using the adaptie observer. The presented algorithm also did not considered the control on the two dimensional plane. E. Koyanagi *et al.*[3] proposed two dimensional trajectory control algorithm for this type robots. The algorithm needed the observer for estimation of the accumulated errors in inclination of the body and could only realized while the robot moving slowly.

In this paper, we propose the control algorithm to make the wheeld inverse pendulum type mobile robot move at desire constant velocity in two dimensional plane. The robot considered in this paper has tow independently driven wheels in same axel which suport and move itself, and is assumed to have the gyro type sensor to know the inclination angle of the body and rotary encoders to know wheel′s rotation angular velocity. The proposed algorithem includes the posture control to make its balance, the velocity control, and the steering control to move in two dimensional plane. In the construction of the posture and velocity control system, we handled the velocity deeply with each other and it can be controlled directly by the velocities of the wheels. When we use the rate gryo sensor to know the rotation angle, the accumulation error becomes the big problem. However, by constructing the control system which contained one integrator in forward path, we also realized the control system without the observer for the estimation of the accumulated errors in the inclination angle of the body. The reference velocity of which acceleration is slowly changing, to prevent the robot from being unstable state by sudden variation of the reference velocity when it starts and stops, or changes velocity, is ordered to the robot.

The robot can not be modeled mathematically as one dimensional wheeled inverse system when it makes rotation motion or steering control. However, such rotation motion does not seriouslay affect to the balancing control. So we assume the control of the robot′s rotation angular velocity are possible to the separated from the posture and velocity control. We adapt usual PWS(Power Wheeled Steering) control method, in which the reference velocity of both wheels are calculated from desired translational velocity and rotation angular velocity, And each left and right wheels is independently controlled by same algorithm based on the one dimensional model.

To verify utility of the algorithm and assumptions proposed in this paper, we implemented the algorithm on our experimental self-contained autonomous robot Yamabico Kurara. The several experimental results in which the autonomous mobile inverse pendulum moves on the real floor, are presented finally in this paper.

Ⅱ. The Scheme of Control Systems

The system which is discussed in this paper, is shown in Fig. 1. This system is a self-cotained mobile robot which is including the controller, sensor, battery and motors etc. in its own body. Its wheels are independently driven through the reduction gears by individual DC motors. The relative angular velocity between the body and the driving wheels can be measured by rotary encoder sensors which are attached on driving motrs′ s axles. Also, the inclination angular velocity of the body can be detected by vibration type rate gyro sensor which is mounted on the body.

In this paper, we deal with what make the robot move on given x-y plane, at the desired speed and with keeping the balance of the posture by itself. Its motion on x-y plane is represented by the position, the direction angle θ, the locomotion velocity υ and the direction angular velocity ω.

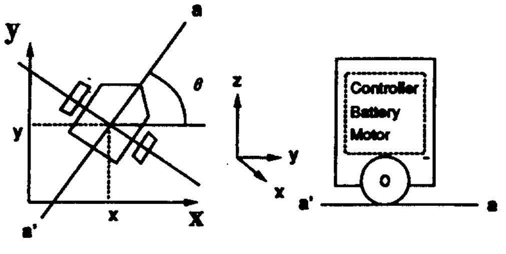

Fig. 1 The wheeled inverse pendulum on x y plane

Considering the dynamical property of this system, we can assume that the posture of the body is deeply related to its velocity, but the spin motion will not much affect the posture control unless the direction angular velocity ω is a large value. So, we propose the basic schemes of the locomotion control for such robots in two dimensional plane based on two ideas as following :

1. To deal the velocity control of the robot with its posture control
2. To keep small the direction angular velocity, and to treat the posture and velocity control independently with steering control

Ⅲ. Posture and Velocity Control

A. *Model of the inversion and locomotion*

When considering only the motion of the posture(its inclination angle)and forward and backward position, the system is modelled on wheel′ s axles and its vertical axies. And body′ s motion is determined by the inclination and forward and backward motion. Figure of the robot when body has a little inclination angle, is shown in Fig. 2. Where, θ and ϕ are the wheel′ s rotation angle and the inclination angle of the body, respectively, and β be the wheel′ s relative rotation angle($\theta - \phi$) to body. Lagrange′ s motion equation of this model is given as equation(1).

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\beta}}\right)-\frac{\partial T}{\partial \beta}+\frac{\partial U}{\partial \beta}+\frac{\partial D}{\partial \dot{\beta}}=Q_{\beta}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial U}{\partial \theta}+\frac{\partial D}{\partial \dot{\theta}}=Q_{\theta} \qquad (1)$$

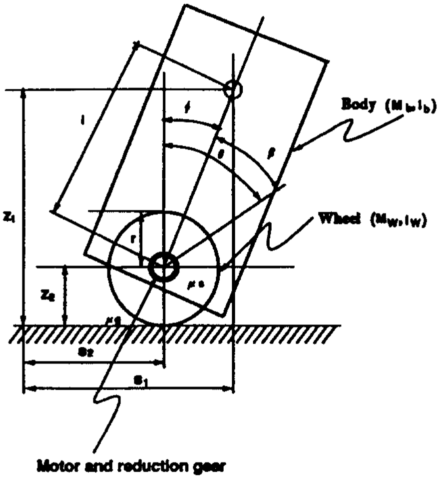

Fig. 2 **The model of the wheeled inverse pendulum**

Where,
T : Kinetic energy
U : Potential energy
D : Dissipation energy function
Q_β : Extenal force to β axis
Q_θ : Extenal force to θ axis

and

$$T = \frac{1}{2}M_w(\dot{s}_2^2 + \dot{z}_2^2) + \frac{1}{2}M_b(\dot{s}_1^2 + \dot{z}_1^2)$$
$$+ \frac{1}{2}I_w\dot{\theta}^2 + \frac{1}{2}I_b(\dot{\theta} + \dot{\beta})^2 + \frac{1}{2}I_M\eta^2\dot{\beta}^2$$
$$U = M_w gr + M_b gl\cos(\theta - \beta)$$
$$D = \frac{1}{2}(\mu_s\dot{\beta}^2 + \mu_g\dot{\theta}^2)$$
$$Q_\beta = \eta\tau_t u, \quad Q_\theta = 0$$

Linearizing locally near the up-right state ($\phi=0$, $\dot{\phi}=0$), equation(1) yeileds equation(2)

$$(M_b l^2 + I_b + \eta^2 I_M)\ddot{\phi} + (M_b rl - \eta^2 I_M)$$
$$\ddot{\theta} + \mu_s\dot{\phi} - \mu_s\dot{\theta} - M_b gl\phi = -\eta\tau_t u$$
$$(M_b rl + M_b l^2 + I_b)\ddot{\phi} + [(M_b + M_w)r^2$$
$$+ M_b rl + I_w]\ddot{\theta} + \mu_g\dot{\theta} - M_b gl\phi = 0 \qquad (2)$$

The parameters and variables in equation(2) are defined in table 1, and the values which are shown in table 1 are authors′ experimental robot of Yamabico Kurara.

Table 1. Parameters and variables

symbol	*prameter and variable name*	unit and value
M_b	Mass of the body	[K_g]9.01
M_w	Mass of the wheel	[K_g]0.51
I_b	Rotational inertia of the body	[$K_g \cdot m^2$]0.228
I_w	Rotational inertia of the wheel	[$K_g \cdot m^2$]5.1E-4
I_M	Rotational inertia of the motor axis	[$K_g \cdot m^2$]3.2E-6
r	Radius of the wheel	[m]0.062
l	Length between the axle of wheel and gravitational center of the robot body	[m]0.138
μ_s	Viscous between the wheel axle including motor and gear	[N.m/(rad/sec)]0.00576
μ_g	Viscous between the wheel and the ground	[N · m/(rad/sec)]0.00425
τ_t	Torque constant of the motor	[N · m/A]0.0235
η	Reduction ratio of gear	39.5
g	Gravitational acceletation	[m/sec^2]9.8
φ	Inclination angle of the body	[rad]
θ	Wheel′ s Rotation angle	[rad]
u	Motor′ s input courrent	[A]

Where,

$$A = \begin{pmatrix} 0 & 1 & 0 \\ a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \end{pmatrix}, B = \begin{pmatrix} 0 \\ b_1 \\ b_2 \end{pmatrix}, x = [\phi\ \dot{\phi}\ \dot{\theta}]^T \qquad (3)$$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{pmatrix} = \begin{pmatrix} \frac{(a_{22}-a_{12})M_b gl}{\Delta} & \frac{(a_{11}-a_{21})M_b gl}{\Delta} \\ \frac{-\mu_s a_{22}}{\Delta} & \frac{-\mu_s a_{21}}{\Delta} \\ \frac{(\mu_s a_{22}+\mu_g a_{12})}{\Delta} & \frac{-(\mu_s a_{21}+\mu_g a_{11})}{\Delta} \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \frac{-a_{22}\eta\tau_t}{\Delta} \\ \frac{a_{21}\eta\tau_t}{\Delta} \end{pmatrix}, \Delta = a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} M_b l^2 + I_b + \eta^2 I_M & M_b rl - \eta^2 I_M \\ M_b rl + M_b l^2 + I_b & (M_b+M_w)r^2 + M_b rl + I_w \end{pmatrix}$$

B. Postare and locomotion velocity control

1) *Design of the cotroller*

The purpose of this section is to derive the control law, which the inclination angle keep small and the wheel' s rotation angular velocity tracks the given reference value. When constant angular velocity θ_{ref} is given to the system as a reference input, steady state vector x_s and steady control input u_s are derived as equation(4) from equation(3).

$$\begin{pmatrix} x_s \\ \cdots \\ U_s \end{pmatrix} = \begin{pmatrix} f(\dot{\theta}_{ref}) \\ \cdots\cdots \\ g(\dot{\theta}_{ref}) \end{pmatrix} = \begin{pmatrix} \frac{-(a_5 b_2 - a_6 b_1)}{(a_1 b_2 - a_2 b_1)}\dot{\theta}_{ref} \\ 0 \\ \dot{\theta}_{ref} \\ \cdots\cdots\cdots\cdots \\ \frac{-(a_1 a_6 - a_2 a_5)}{(a_1 b_2 - a_2 b_1)}\dot{\theta}_{ref} \end{pmatrix} \qquad (4)$$

Defining Δx and Δu as

$$\begin{pmatrix} \Delta x \\ \Delta u \end{pmatrix} = \begin{pmatrix} x - x_s \\ u - u_s \end{pmatrix} \qquad (5)$$

the problem to make the velocity of the robot track reference velocity is resoluted by designing the regulator to the error system which is expressed by Δx, Δu.

In modelling of unstable system as this system, not only the modelling errors due to disregard of the nonlinear factors such as the backlash of the reduction gear and nonlinear friction are exist, but also accurate identification of the parameters is impossible indeed. Therefore, in design of the controller, feedback of integrated error is considered to cope with the modelling errors, the parameters variations and disturbance. Difining a new state variable z as

$$z = \int_0^t (\dot{\theta} - \dot{\theta}_{ref})dt \qquad (6)$$

the augmented system[4] is given as equation(7)

$$\Delta\dot{\tilde{x}} = \tilde{A}\Delta\tilde{x} + \tilde{B}\Delta\tilde{u} \qquad (7)$$

$$\Delta y = \tilde{C}\Delta\tilde{x}$$

where,

$$\Delta\tilde{x} = \begin{pmatrix} \Delta x \\ z \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix}, \tilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} C^T \\ 0 \end{pmatrix}^T$$

The optimal controller which asymtotically stabilizes the feedback system of the augmented system and minimizes performance index

$$J = \int_0^\infty (\Delta\tilde{x}^T \tilde{Q}\Delta\tilde{x} + \Delta\tilde{u}^T \tilde{R}\Delta\tilde{u})dt \qquad (8)$$

where,

$$\tilde{Q} = \tilde{Q}^T \geq 0, \tilde{R} = \tilde{R}^T > 0$$

is goven as

$$u = u_s - \tilde{K}\Delta\tilde{x}$$

$$= u^s - \tilde{k}_1(x - x_s) - \tilde{k}_2\int_0^t (\dot{\theta} - \dot{\theta}_{ref})dt \qquad (9)$$

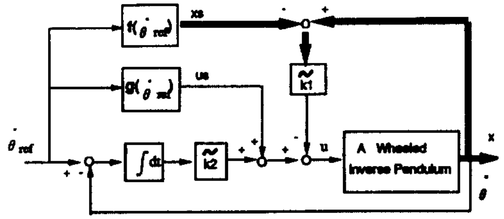

Fig 3. Block diagram of posture and velocity control system

were $k = R\ ^{1}B^{T}P$ and P is the solution of the martix Riccati equation. So the posture and velocity control system is realized by linear state feedforward and feedback controller as equation(9), which is shown in Fig. 3.

2) *Measurement of state variable*

To construct the control system as Fig. 3, all state variables, the inclination angle ϕ and angular velocity ϕ of the body, and the revolution angular velocity θ of the wheel, must be measured. The inclination anglular velocity ϕ and the revolution angular velocity θ can be measured by using gyro sensor and encoder sensor, respectively. And, ϕ is integrated to get the inclination angle ϕ. However, since measurement and integral computation errors are accumulated, the drift type error is included in the calculated inclination angle ϕ. To cope with this problem, some researches[2,3] have been done in which state observer was used. On the other hand, because the changing of the accumulated errors is very slow, we can regard it as a constant disturbance when it is fed back to control system. Therefore, the affection of the accumulated errors in the posture and velocity control is cancelled, by constructing the control system which contained integrator as Fig. 3. Hence, it is possible to control the posture and velocity of the robot without the observer for the estimation of the accumulated errors.

C. Stop and position control

When robot starts or stops, the sudden variation of the reference velocity causes body to be unstable. Hence, we have decided that the reference velocity of which acceleration is slowly changing orders to robot when making the robot starts and stops, or changes velocity. So, the reference velocity is given by equation(10) when the robot accelerates from stopped state to desired velocity.

$$\upsilon_{ref}(t+\Delta t) = \upsilon_{ref}(t) - a.\Delta t \tag{10}$$

where Δt is sampling interval and a is acceleration.

The calculation of reference velocities to make the robot stop at the goal position exactly, is not simple because starting point of the deceleration is not given as a constant time t by the affection of the response delay time or stady state error of control system. So, it should be calculated by the state feedback form as

$$\upsilon_{ref}(t+\Delta t) = \begin{cases} \upsilon_{ref} : \upsilon(t) < \sqrt{2a(x_{stop} - x(t))} \\ \upsilon_{ref}(t) - a.\Delta t : \upsilon(t) \geq \sqrt{2a(x_{stop} - x(t))} \end{cases} \tag{11}$$

where x is the robot′ s current position and x_{stop} is the position where the robot has to stop. In fact, using equation(12) is reasonable since the robot goes over the stop position x_{stop} when large acceleration is given.

$$\upsilon_{ref}(t+\Delta t) = \begin{cases} \upsilon_{ref}(t) : \upsilon(t) < \sqrt{2a[(x_{stop} - \varepsilon) - x(t)]} \\ \upsilon_{ref}(t) - a.\Delta t : \upsilon(t) \geq \sqrt{2a[(x_{stop} - \varepsilon) - x(t)]} \end{cases} \tag{12}$$

where ε is a small value which the acceleration is considered. The obtained reference velocities is ordered to control system Fig. 3. As a result, the control mode will not be changed from velocity control to position control mode until robot has a small enough velocity and comes in near enough goal position.

Ⅳ. Steering Sontrol

PWS method is applied to steering control of a wheeled inverse penulum type mobile robot. In the control of nonholonomic vehicle which has PWS method, it is good to control divide in two step as following :

(a) Determination of locomotion velocity v and direction angular velocity ω based on the deflection between the current position and direction and the refernce trajectoy.

(b) Control to makes the vehicle track the reference input v, ω which is obtained in step (a)

Authors' group had already proposed command system[5] to direct trajectoy in step(a), and made the locomotion and spin speed be determined by the command. This method is applied to trajectory control of the wheeled inverse pendulum type mobile robot. So, in this paper we discuss on step(b).

In this robot, the locomotion velocity v and direction angular velocity ω of the body relate with the right and left wheel's velocity v_r, v_l as equation(13).

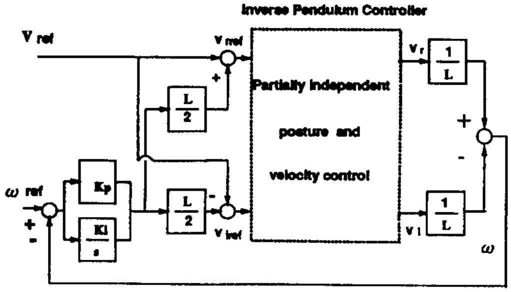

Fig. 4. Velocity and steering control scheme of the wheeled inverse pendulum

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{L} & -\frac{1}{L} \end{pmatrix} \begin{pmatrix} v_r \\ v_l \end{pmatrix} \tag{13}$$

Where, L is the distance between right and left wheel. The direction angular velocity is controlled by feed back of the difference between the right and left wheel's velocity. Regarding as an independant inverse pendulum mounted on both wheels, respectively, the reference value of the velocity control system (Fig. 3) for each wheels, is made by the reference velocity v and direction angular velocity ω of the body. Construction of the steering control system is shown Fig. 4. Where each wheel's revolution angular velocity θ are obtained from independant DC motor's encoders. And the

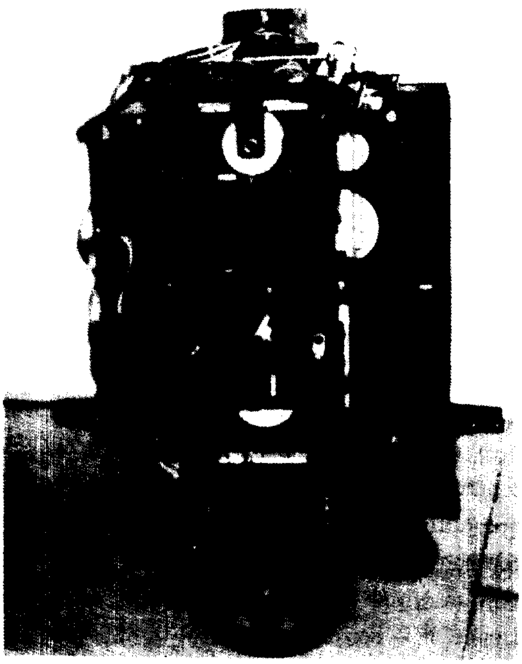

Photo. 1.Yamabico Kurara which is keeping balance by itself

inclination ϕ and angular velocity $\dot{\phi}$ are obtained from gyro, which are commonly used to the velocity control of both wheels.

V. Experiments by Using Yamabico

A. Experimental system and determination of parameters

Yamabico Kurara robot which had removed forward and backward casters from the standard type Yamabico robot, was used to experiments. Yamabico is a series of the selfcontained autonomous mobile robot platform for the experimental research developed by authors' group. The experimental autonomous inverse pendulum Yamabico Kurara(Photo. 1) which is controlling itself to keep its own balance.

The robot has rotary encoders(resolution : 2000) and vibration type gyro sensor (TOKIMEC Co. TFG-160) to control its posture and locomotion. Both wheels are driven by DC motors(of 10 Watts). By using parameters given in Table 1, cofficient matrix A, B of this system are obtained as following.

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 38.23 & -0.085 & 0.121 \\ -51.76 & 0.347 & -0.56 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ -13.7 \\ 56.02 \end{pmatrix}$$

In experiments of locomotion control, the weighting matrics of performance index to decide the feedback gains were fixed as $Q = diag(1, 0.01, 0.01, 4)$, $R=300$. As the result, feedback gains were given as $\kappa_1 = (-9.0,\ -1.58,\ -0.123)$, $\kappa_2 = -0.115$, and poles of closedloop system were $\lambda_1, \lambda_2 = (-6.177 \pm 0.119j)$, $\lambda_3, \lambda_4 = (-1.529 \pm 0.1412j)$. And, in steering control, the parameters are chosen as $k_p = -3.05$, $k_i = -0.0003$

MC 68000 chip is used as CPU of controller, and all computation had done in fixed-point representation. The sampling interval of control program was 5[msec]. Motor's input current is controlled by feed-forward current control method[6].

B. Experiments of the posture and velocity control

At first, the step response experiment to test performance of the posture and velocity control had performed. One of the results is shown in Fig. 5, which represents that the robot can track the reference velocity while keeping its balance.

Secondly, the step response experiment to parameter variation(20.3[%] of the body's weight) had performed. In this case, the robot tracked the reference velocity with keeping balance of posture as show in Fig. 6.

When we made the robot move on the sloping (10[%]) and rough ground the results are shown in Fig. 7. In spite of the roughness of the road, the robot tracked the reference velocity with keeping balance of posture. This result shows that the proposed algorithm in this

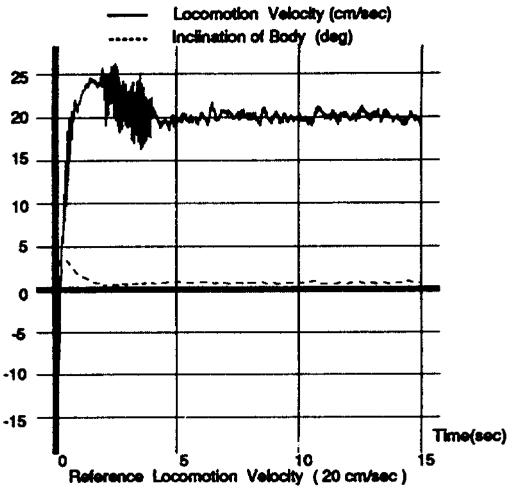

Fig. 5. **Experimental results of posture and velocity control**

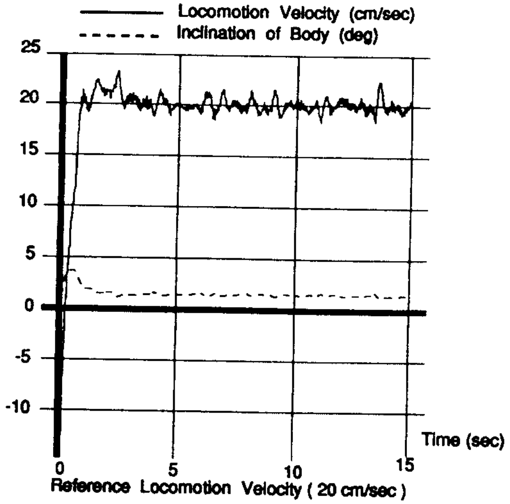

Fig. 6. Experimental results for constant parameter variation

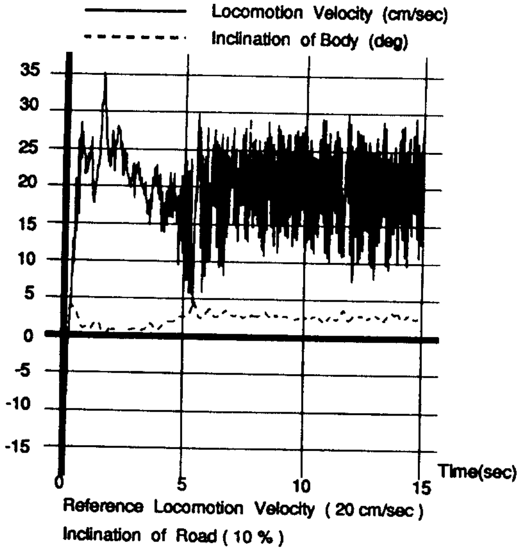

Fig. 7. Experimental results for constant disturbance

paper is robust enough for the indoor environment.

Subsequently, the experimental results when the goal point and the limited acceleration value is given to the robot, are shown in Fig. 8. This shows that the conrtol system can make the robot to move itself and stop at the given point.

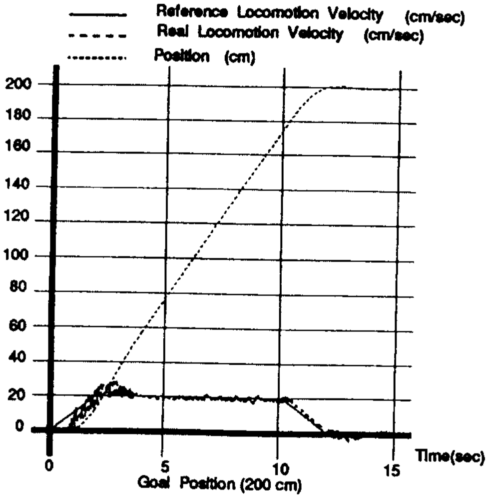

Fig. 8. Experimental results of positon control

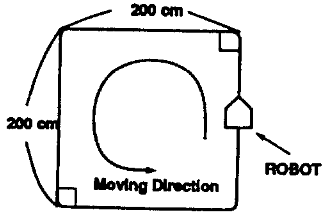

Fig. 9. locomotion map

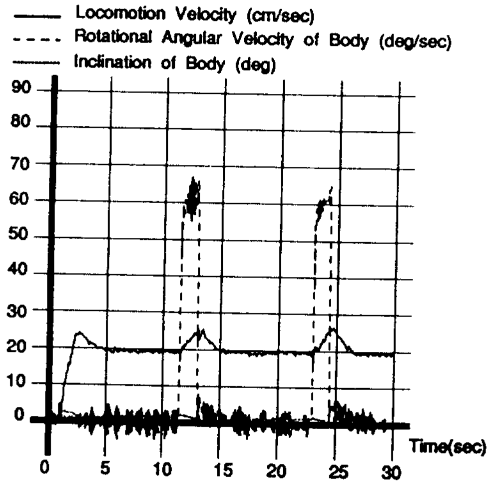

Fig. 10. Experimental results of locomotion in two dimensional plane

C. Locomotion experiments in two dimensional plane

When the trajectory as Fig. 9 is given for the robot, the resultant locomotion velocity, the inclination angle and direction angular velocity of the body are shown in Fig. 10.

The robot could track given trajectory with keeping the stability of the posture, but locomotion velocity was changed while the robot turning.

Ⅵ. Conclusion

In this paper, we proposed the control algorithm to make the inverse pendulum type mobile robot move in two dimensional plane, at the desire constant velocity, with keeping the balance of the posture by itself. The control algorithm is implemented on the experimental robot. The conclusion through experiments using the robot had obtained as following :

1. In spite of the modelling errors, the parameter variation, and the accumulated errors in the inclination angle, the robot can move robustly at the desired velocity and with keeping the balance of posture.
2. Without the locomotion velocity and direction angular velocity are too much restricted, the steering control could be seperated from the posture and velocity control.

The further relating researches as follows are being considered : (1) The steering control which the direction angular velocity is determined by given trajectory. (2) The realization of the sensor based navigation.

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