Pusan Kyŏngnam Math. J 9(1993), No 2, pp. 283–294

ON SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR OF A SASAKIAN SPACE FORM

Jong Joo Kim

0. Introduction

One of typical submanifolds of a Sasakian manifold is the so-called contact CR submanifold which are defined as follows: Let M be a submanifold of a Sasakian manifold $\tilde{M}$ tangent to the structure vector field V with almost contact metric structure (ϕ, G, V). If there exists a differentiable distribution such that it is invariant under ϕ and the complementary orthogonal distribution is totally real with respect to ϕ [9], [10], [12]. Many subjects for such submanifolds of a Sasakian space form have been studied in [1], [3], [4], [5], [6], [7] and so on, one of which done by Ki and Kon asserts that the following:

Theorem A([5]). *Let M be an $(n+1)$-dimensional contact CR submanifold of a Sasakian space form $\tilde{M}^{2m+1}(c)$ with nonvanishing parallel mean curvature vector. If the f-structure Q in the normal bundle is parallel, and if the second fundamental forms A^x and the f-structure f on M commute, then each eigenvalue of the operator A^* in the direction of the mean curvature vector is constant.*

The purpose of this paper is to improve Theorem A.

1. Preliminaries

In this section, the basic properties of submanifolds of a Sasakian manifold are recalled [2], [10], [11].

Let $\tilde{M}$ be a Sasakian manifold of dimension $2m+1$ with almost contact metric structure (ϕ, G, V). Then for any vector fields X and Y on $\tilde{M}$, we have

$$\phi^2 X = -X + v(X)V, \ G(\phi Y, \phi X) = G(Y, X) - v(Y)v(X),$$

Received October 11, 1993
Research supported by Dong-A University, 1992 .

$$v(\phi X) = 0, \quad \phi V = 0, \quad v(V) = 1, \quad G(X, V) = v(X).$$

We denote by $\tilde{\nabla}$ the operator of covariant differentiation with respect to the metric G on $\tilde{M}$. We then have

$$\tilde{\nabla}_X V = \phi X, \quad (\tilde{\nabla}_X \phi)Y = -G(X, Y)V + v(Y)X. \tag{1.1}$$

Let M be an $(n+1)$-dimensional Riemannian manifold covered by a system of local coordinate neighborhoods $\{U; x^h\}$ and immersed isometrically in $\tilde{M}$ by the immersion $i : M \longrightarrow \tilde{M}$. When the argument is local, we may identify M with $i(M)$. We represent the immersion i locally by

$$y^A = y^A(x^1, \cdots, x^{n+1}), \quad (A = 1, \cdots, n+1, \cdots, 2m+1)$$

and put $B_j{}^A = \partial_j y^A$, $(\partial_j = \partial x^j)$ then $B_j = (B_j{}^A)$ are $(n+1)$-linearly independent local tangent vector fields of M. We choose $2m - n$ mutually orthogonal unit normals $C_x = (C_x{}^A)$ to M. Throughout this paper, the indices $h, i, j, \cdots$ run over the range $\{1, \cdots, n+1\}$ and $u, v, w, \cdots$ the range $\{n+2, \cdots, 2m+1\}$ and the summation convention will be used with respect to those indices. The immersion being isometric, the induced Riemannian metric tensor g on M and the metric tensor δ of the normal bundle are then respectively obtained :

$$g_{ji} = G(B_j, B_i), \quad \delta_{yx} = G(C_y, C_x).$$

In the sequel, we assume that *the submanifold M of $\tilde{M}$ is tangent to the structure vector field* V. Then we have

$$V = \xi^i B_i, \quad \xi^i = G(B_i, V). \tag{1.2}$$

The transforms of B_i and C_x by ϕ are respectively represented in each coordinate neighborhood as follows:

$$\phi B_j = f_j{}^i B_i + J_j{}^x C_x, \tag{1.3}$$

$$\phi C_x = -J_x{}^i B_i + Q_x{}^y C_y, \tag{1.4}$$

where we have put

$$f_{ji} = G(\phi B_j, B_i), \quad J_{jx} = (\phi B_j, C_x), \quad J_{xj} = -G(\phi C_x, B_j),$$

$$Q_{xy} = G(\phi C_x, C_y), \quad f_j{}^h = f_{ji} g^{ih}, \quad J_j{}^x = J_{jy}\delta^{yx}, \quad Q_x{}^y = Q_{xz}\delta^{zy},$$

δ^{yz} being the contravariant components of δ_{yz} and $(g^{ji}) = (g_{ji})^{-1}$. From these definitions, we verify that $f_{ji} + f_{ij} = 0$, $J_{jx} = J_{xj}$ and $Q_{xy} + Q_{yx} = 0$.

By the properties of the Sasakian structure tensor, it follows from (1.2), (1.3) and (1.4) that we have

$$(1.5) \qquad f_j{}^t f_t{}^h = -\delta_j^h + \xi_j\xi^h + J_j{}^x J_x{}^h, Q_x{}^y Q_y{}^z = -\delta_x^z + J_x{}^t J_t{}^z,$$

$$(1.6) \qquad f_j{}^t J_t{}^x + J_j{}^y Q_y{}^x = 0,$$

$$(1.7) \qquad \xi^j J_j{}^x = 0, \quad \xi^j f_j{}^h = 0, \quad \xi_j\xi^j = 1.$$

By denoting ∇_j the operator of van der Waerden-Bortolotti covariant differentiation with respect to g and G, the equations of Gauss and Weingarten for the submanifold M are respectively given by

$$(1.8) \qquad \nabla_j B_i = A_{ji}{}^x C_x, \ \nabla_j C_x = -A_j{}^h{}_x B_h,$$

where $A_{ji}{}^x$ are the second fundamental forms in the direction of C_x and related by

$$A_j{}^h{}_x = A_{jix} g^{ih} = A_{ji}{}^y g^{ih}\delta_{yx}.$$

Differentiating (1.3) and (1.4) covariantly along M and making use of (1.1), (1.8) and these equations, we easily find

$$(1.9) \qquad \nabla_j f_i{}^h = \delta_j^h\xi_i - g_{ji}\xi^h + A_j{}^h{}_x J_i{}^x - A_{ji}{}^x J_x{}^h,$$

$$(1.10) \qquad \nabla_j J_i{}^x = A_{ji}{}^y Q_y{}^x - A_{jt}{}^x f_i{}^t,$$

$$(1.11) \qquad \nabla_j Q_y{}^x = A_{jt}{}^x J_y{}^t - A_{jty} J^{tx}.$$

We also have from (1.2)

$$\nabla_j \xi_i = f_{ji}, \tag{1.12}$$

$$A_{jt}{}^x \xi^t = J_j{}^x \tag{1.13}$$

because of (1.1), (1.3) and (1.8).

In the rest of this section we suppose that the ambient Sasakian manifold $\tilde{M}$ is of constant ϕ-holomorphic sectional curvature c, which is called a Sasakian space form, and is denoted by $\tilde{M}^{2m+1}(c)$. Then we see, using (1.2), (1.3), (1.4) and (1.8), that equations of the Gauss, Codazzi and Ricci for M are respectively given by

$$\begin{aligned} R_{kjih} =& \frac{1}{4}(c+3)(g_{kh}g_{ji} - g_{jh}g_{ki}) + A_{kh}{}^x A_{jix} - A_{jh}{}^x A_{kix} \\ &+ \frac{1}{4}(c-1)(\xi_k\xi_i g_{jh} - \xi_j\xi_i g_{kh} + \xi_j\xi_h g_{ki} - \xi_k\xi_h g_{ji} \\ &+ f_{kh}f_{ji} - f_{jh}f_{ki} - 2f_{kj}f_{ih}), \end{aligned} \tag{1.14}$$

$$\nabla_k A_{ji}{}^x - \nabla_j A_{ki}{}^x = \frac{1}{4}(c-1)(J_k{}^x f_{ji} - J_j{}^x f_{ki} - 2J_i{}^x f_{kj}), \tag{1.15}$$

$$\begin{aligned} R_{jiyx} =& \frac{1}{4}(c-1)(J_{jx}J_{iy} - J_{ix}J_{jy} - 2f_{ji}Q_{yx}) \\ &+ A_{jtx}A_i{}^t{}_y - A_{itx}A_j{}^t{}_y, \end{aligned} \tag{1.16}$$

where R_{kjih} and R_{jiyx} are the Riemannian curvature tensor of M and that with respect to the connection induced in the normal bundle of M respectively. We see from (1.14) that the Ricci tensor S of M can be expressed as follows :

$$\begin{aligned} S_{ji} =& \frac{1}{4}\{n(c+3) + 2(c-1)\}g_{ji} - \frac{1}{4}(c-1)(n+2)\xi_j\xi_i \\ &- \frac{3}{4}(c-1)J_j{}^z J_{iz} + h^x A_{jix} - A_j{}^{tx}A_{itx}. \end{aligned} \tag{1.17}$$

with the aid of (1.5), where $h^x = g^{ji}A_{ji}{}^x$.

Differentiating (1.13) covariantly along M and using (1.10) and (1.12), we find

$$(\nabla_k A_{jrx})\xi^r + A_{jrx}f_k{}^r = A_{kj}{}^y Q_{yx} - A_{krx}f_j{}^r,$$

which together with (1.7) and (1.15) implies that

$$\xi^r \nabla_r A_{jkx} = A_{kj}{}^y Q_{yx} - A_{krx}f_j{}^r - A_{jrx}f_k{}^r. \tag{1.18}$$

On the other hand, the Lie derivative of the shape operator A^x in the direction of C_x with respect to the structure vector field ξ is given by

$$L_\xi A_j{}^{hx} = \xi^r \nabla_r A_j{}^{hx} + A_r{}^{hx} f_j{}^r - A_j{}^{rx} f_r{}^h$$

because of (1.12). From the last two equations, it follows that we have

$$L_\xi A_j{}^{hx} = A_j{}^{hy} Q_y{}^x. \tag{1.19}$$

A submanifold of a Sasakian manifold is called a *generic* submanifold if Q vanishes identically [9]. If a submanifold of a Sasakian space form $\tilde{M}^{2m+1}(c)$ is generic, then $L_\xi A^x = 0$ is satisfied.

Let H be a mean curvature vector field of M in a Sasakian manifold. Namely, it is defined by

$$H = g^{ji}A_{ji}{}^x C_x/(n+1) = h^x C_x/(n+1),$$

which is independent of the choice of the local field of orthonormal frames $\{C_x\}$.

In the following we suppose that the mean curvature vector field H of M is nonzero and is parallel in the normal bundle. Then we may choose a local field $\{e_x\}$ in such a way that $H = \sigma C_{n+2} = \sigma C_*$, where $\sigma = |H|$ is nonzero constant. Because of the choice of the local field, the parallelism of H yields

$$\begin{cases} h^x = 0, & x \geq n+3 \\ h^* = (n+1)\sigma. \end{cases} \tag{1.20}$$

In the sequel the index $n+2$ will be denoted by the symbol $*$.

2. Lemmas

Let M be a submanifold satisfying $L_\xi A^x = 0$ of a Sasakian space form $\tilde{M}^{2m+1}(c)$. Then by (1.19) we have

$$A_{ji}{}^x Q_{xy} = 0. \tag{2.1}$$

This together with (1.6) and (1.13) implies that

$$f_j{}^t J_t{}^x = 0, \quad J_j{}^y Q_y{}^x = 0. \tag{2.2}$$

Hence, by (1.5) we see that $f^3 + f = 0$ and $Q^3 + Q = 0$, namely f and Q define the f-structure in M and that in the normal bundle of M respectively [8]. In such a case M is called a *contact CR submanifold* of a Sasakian manifold [5], [12].

Since the mean curvature vector assumed to be parallel in the normal bundle, it is, using (1.18), seen that $h^y Q_{yx} = 0$. Accordingly by (1.20) we obtain

$$Q_x{}^* = 0, \tag{2.3}$$

which join with the second equation of (1.5) implies

$$J_{jx} J^{j*} = \delta_x{}^*. \tag{2.4}$$

H being a normal vector field on M, the curvature tensor R_{jiyx} of the connection in the normal bundle shows that R_{ji*x} vanishes identically for any index x. Thus the Ricci equation (1.16) yields

$$A_{jtx} A_i{}^{t*} - A_{itx} A_j{}^{t*} = \frac{1}{4}(c-1)(J_{j*} J_{ix} - J_{i*} J_{jx}) \tag{2.5}$$

because of (2.3).

LEMMA 2.1. *Let M be an $(n+1)$-dimensional submanifold tangent to the structure vector field of a $(2m+1)$-dimensional Sasakian space form. Suppose that the mean curvature vector field is nonzero and parallel in the normal bundle. If $L_\xi A^x = 0$ and $\nabla_\xi A^* = 0$ on M, then we have*

$$A_{jiy} A^{ji*} = h^* P_{y**} + \frac{1}{4}(n-1)(c+3)\delta_{y*} + 2\delta_{y*}. \tag{2.6}$$

Proof. By (1.18) and (2.3), we have

$$A_{jr}{}^{*}f_{i}{}^{r} + A_{ir}{}^{*}f_{j}{}^{r} = 0 \tag{2.7}$$

because we assumed to be $\nabla_{\xi}A^{*} = 0$. Transforming (2.7) by $f_{k}{}^{i}$ and making use of (1.5) and (1.13), we get

$$(A_{jr*}J_{z}{}^{r})J_{k}{}^{z} + \xi_{k}J_{j*} - A_{jk*} + A_{sr*}f_{j}{}^{r}f_{k}{}^{s} = 0,$$

from which, taking the skew-symmetric part

$$(A_{jr*}J_{z}{}^{r})J_{k}{}^{z} - (A_{kr*}J_{z}{}^{r})J_{j}{}^{z} + \xi_{k}J_{j*} - \xi_{j}J_{k*} = 0.$$

If we transvect $J_{y}{}^{k}$ to above equation and make use of (1.5) and (2.2), we obtain

$$A_{jr}{}^{*}J_{y}{}^{r} = P_{yz}{}^{*}J_{j}{}^{z} + \delta_{y}{}^{*}\xi_{j}, \tag{2.8}$$

where we have defined

$$P_{yzx} = A_{jix}J_{y}{}^{i}J_{z}{}^{i}.$$

Thus by (2.1) and (2.2) we get

$$P_{yzx}Q_{w}^{x} = 0, \quad P_{yzx}Q_{w}^{z} = 0. \tag{2.9}$$

Transvecting $J_{z}{}^{j}J_{y}{}^{i}$ to (2.5), we find

$$P_{uzx}P_{y}{}^{u*} - P_{uyx}P_{z}{}^{u*} = \frac{1}{4}(c+3)\{\delta_{z}{}^{*}J_{y}{}^{i}J_{xi} - \delta_{y}{}^{*}J_{z}{}^{i}J_{xi}\},$$

where we have used (1.7), (2.2), (2.4), (2.8) and (2.9). Consequently we have

$$P_{yzx}P^{yz*} - P^{z}P_{zx*} = \frac{1}{4}(c+3)(p-1)\delta_{x*}, \tag{2.10}$$

$$P_{zx}{}^{*}P_{y}{}^{z*} - P_{zyx}P^{z**} = \frac{1}{4}(c+3)(J_{y}{}^{i}J_{ix} - \delta_{y}{}^{*}\delta_{x}{}^{*}), \tag{2.11}$$

where we denote $P_z{}^{zx} = P^x$ and $J_{jx}J^{jx} = p$.

Differentiaing (2.8) covariantly and using (1.11), (1.12) and (2.9), we find

$$(\nabla_k A_{jr}{}^*)J_y{}^r - A_j{}^{r*}A_{ksy}f_r{}^s = (\nabla_k P_{yz*})J_j{}^z - P_{yz*}A_{kr}{}^z f_j{}^r + \delta_y{}^* f_{kj}$$

and hence, taking the skew-symmetric part with respect to indices k and j and taking account of (1.15), (2.2), (2.4) and (2.7)

(2.12)

$$\begin{aligned} &A_r{}^s{}_* A_{jsy} f_k{}^r - A_r{}^s{}_* A_{ksy} f_j{}^r \\ =&(\nabla_k P_{yz*})J_j{}^z - (\nabla_j P_{yz*})J_k{}^z + P_{yz*}(A_{jr}{}^z f_k{}^r - A_{kr}{}^z f_{jr}) \\ &+ \frac{1}{2}(c+3)f_{kj}. \end{aligned}$$

If we transvect f^{kj} to the last equation and make use of (1.5), (1.13), (1.20), (2.4) and (2.8), we can obtain

$$A_{ji}{}^* A^{ji}{}_y = h^* P_{y**} + P_{zwy}P^{zw*} - P^z P_{yz*} + 2\delta_y{}^* + \frac{1}{4}(c+3)(n-p)\delta_y{}^*.$$

Thus by (2.10) we arrive at (2.6).

For the shape operator A^* in the direction of the mean curvature vector field, a tensor field $(A^*)^a$ and a function $h_{(a)}$ for any integer $a \geq 2$ are introduced as follows:

$$(A_{ji}{}^*)^a = A_{ji_1}{}^* A_{i_2}{}^{i_1*} \cdots A_i{}^{i_a-1*}, \quad h_{(a)} = \sum_i (A_{ii}{}^*)^a.$$

Thus, (2.6) implies that

$$h_{(2)} = h^* P_{***} + \frac{1}{4}(n-1)(c+3) + 2. \tag{2.13}$$

When $y = *$ in (2.12), we have

$$\begin{aligned} &2A_j{}^{r*}A_{rs*}f_k{}^s \\ =&(\nabla_k P_{z**})J_j{}^z - (\nabla_j P_{z**})J_k{}^z + 2P_{z**}A_{jr}{}^z f_k{}^r + \frac{1}{2}(c+3)f_{kj} \end{aligned}$$

because of (2.7). Transforming by $A_t{}^{j*} f^{kt}$ and making use of (1.5), (1.13), (2.4), (2.7), (2.8) and (2.9), we obtain

$$h_{(3)} = P_{wz*} P^{zx*} P_x{}^{w*} - P_{xwz} P^{wx*} P^{z**} + P_{***} + P_{z**} A_{ji}{}^z A^{ji*} + \frac{1}{4}(c+3)(h^* - P^*),$$

which combine with (2.6) and (2.11) gives forth

(2.14) $$h_{(3)} = h^* \|P_{z**}\|^2 + \frac{1}{4}(c+3)(n-2)P_{***} + \frac{1}{4}(c+3)h^* + 3P_{***},$$

where $\|P_{z**}\|^2 = P_{z**}P^{z**}$.

LEMMA 2.2. *Under the same assumptions as those in Lemma 2.1, the function $h_{(2)}$ is harmonic.*

Proof. By definition we have $P_{***} = A_{ji}{}^* J_*{}^j J_*{}^i$. Differentiating this covariantly, we find

$$\nabla_k P_{***} = (\nabla_k A_{ji}{}^*) J_*{}^j J_*{}^i$$

because of (1.10), (2.1) and (2.2). Thus, the Laplacian of the function P_{***} is given by

$$\Delta P_{***} = (\Delta A_{ji}{}^*) J_*{}^j J_*{}^i + 2(\nabla_k A_{ji}{}^*) J_*{}^i \nabla^k J^{j*}.$$

This together with (1.5), (1.10), (1.13), (1.15) and (2.1) implies that

(2.15) $$\Delta P_{***} = (\Delta A_{ji}{}^*) J_*{}^j J_*{}^i - \frac{1}{2}(c-1)(h^* - P^*).$$

On the other hand, it is shown in the proof of Lemma 4.2 of [1] that

$$(\Delta A_{ji}{}^*) J_*{}^j J_*{}^i = \frac{1}{2}(c-1)(h^* - P^*).$$

Therefore (2.15) turns out to be $\Delta P_{***} = 0$. Thus, the equation (2.13) implies that $\Delta h_{(2)} = 0$ because the mean curvature vector is parallel. Hence we arrive at the conclusion.

3. Theorems

THEOREM 3.1. *Let M be an $(n+1)$-dimensional submanifold of a Sasakian space form $\tilde{M}^{2m+1}(c)$ with nonvanishing parallel mean curvature vector. If $L_\xi A^x = 0$ and $\nabla_\xi A^* = 0$ on M, then we have*

$$\|\nabla A^*\|^2 = \frac{1}{8}(c-1)^2(n-p). \tag{3.1}$$

Proof. By a straightforward computation, we have

$$A_j{}^{rx} A_{irx} A^{is*} A_s{}^{j*} = A_{jr}{}^x A_{isx} A^{rs*} A^{ji*} + \{\frac{1}{4}(c-1)\}^2(p-1), \tag{3.2}$$

where we have used (1.13), (2.4), (2.5) and (2.10).

We also have

$$A^{kh*} A^{ji*} f_{kj} f_{hi} = h_{(2)} - P^z P_{z**} - \frac{1}{4}(c-1)(p-1) - (p+1) \tag{3.3}$$

because of (1.5), (1.13), (2.2), (2.7), (2.8) and (2.10).

Making use of (1.14) and (1.17), we can verify the following:

$$S_{js} A_i{}^{s*} A^{ji*} - R_{kjih} A^{kh*} A^{ji*} = \frac{1}{16}(c-1)^2(n-p), \tag{3.4}$$

where we have used (1.13), (2.6), (2.8), (2.10), (2.13), (2.14), (3.2) and (3.3).

By (1.9) we have

$$\nabla_k f_i{}^k = n\xi_i - h^* J_{i*} - A_{ir}{}^x J_x{}^r$$

because of (1.20). Hence we have

$$\begin{aligned} &A^{ji*}\nabla_k(J_{j*} f_i{}^k) \\ =&A^{ji*} A_{kr}{}^* f_j{}^r f_i{}^k + (P_{z**} J^{iz} - \xi^i)(n\xi_i - h^* J_{i*} + A_i{}^{rx} J_{xr}) \end{aligned}$$

by virtue of (1.10) and (2.8), or using (1.13), (2.4), (2.13) and (3.3) we obtain

$$A^{ji*}\nabla_k(J_{j*} f_i{}^k) = \frac{1}{4}(c-1)(n-p). \tag{3.5}$$

On the other hand, since the submanifold M has parallel mean curvature vetor field, the Laplacian $\Delta A_{ji}{}^{*}$ of A^* is given, using the Ricci formula for A^* and (1.15), by

(3.6)
$$\Delta A_{ji}{}^{*} = S_{jr}A_i{}^{r*} - R_{kjih}A^{kh*} + \frac{1}{4}(c-1)\nabla_k(J_*{}^k f_{ji} + J_{j*}f_i{}^k + 2J_{i*}f_j{}^k),$$

Transvecting A^{ji*} to (3.6) and making use of (3.4) and (3.5), we have
$$A^{ji*}\Delta A_{ji}{}^{*} = -\frac{1}{8}(c-1)^2(n-p).$$
Thus by Lemma 2.2, we obtain (3.1) because we have in general
$$\frac{1}{2}\Delta h_{(2)} = A^{ji*}\Delta A_{ji*} + \|\nabla A^*\|^2$$

By (1.5), (1.15), (2.2) and (2.4) we have
$$\|\nabla_k A_{ji}{}^{*} + \frac{1}{4}(c-1)(J_{j*}f_{ki} + J_{i*}f_{kj})\|^2 = \|\nabla_k A_{ji}{}^{*}\|^2 - \frac{1}{8}(c-1)^2(n-p).$$
This together with (3.1) yields

(3.7)
$$\nabla_k A_{ji}{}^{*} = -\frac{1}{4}(c-1)(J_{j*}f_{ki} + J_{i*}f_{kj}).$$
Thus we have

THEOREM 3.2. *Let M be an $(n+1)$-dimensional submanifold of an odd-dimensional unit sphere $S^{2m+1}(1)$ with nonvanishing parallel mean curvature vector. If $L_\xi A^x = 0$ and $A^*f = fA^*$ on M, then A^* is parallel.*

For any point q in M we can choose a local orthonormal frame field $\{E_i\}$ so that the shape operator A^* in the direction of the mean curvature vector field is diagonalizable at that point q, say $A_{ji}{}^{*} = \lambda_j\delta_{ji}$.

Transvecting (3.7) with $(A^{ji*})^{a-1}$ for any integer $a \geq 2$ and taking account of (1.13), (2.2) and (2.8), we obtain $\nabla_k h_{(a)} = 0$ and hence $h_{(a)}$ is constant. Since we have
$$h_{(a)} = \sum_i(\lambda_i)^a, \quad (a = 1, 2, \cdots),$$
it is seen that λ_i^j is constant. Thus we obtain

THEOREM 3.3. *Let M be a submanifold of a Sasakian spcae form with nonvanishing parallel mean curvature vector. If $L_\xi A^x = 0$ and $\nabla_\xi A^* = 0$, then each eigenvalue of A^* is constant.*

From (1.18) and (1.19) and Theorem 3.3, we have

COROLLARY 3.4 ([1]). *Let M be a generic submanifold of a Sasakian space form with nonvanishing parallel mean curvature vector. If $\nabla_\xi A^*$ $= 0$ or equivalently, $A^* f = fA^*$ on M, then each eigenvalue of A^* is constant.*

References

[1]. S.S Ahn and U-H. Ki, *Generic submanifolds with parallel mean curvature vector of a Sasakian space form*, J Korean Math. Soc., to appear.
[2]. U-H. Ki, N.-G. Kim, S.-B Lee and I.-Y Yoo, *Submanifolds with nonvanishing parallel mean curvature vector field of a Sasakian space form*, J. Korean Math. Soc., **30** (1993)
[3]. U-H. Ki, N -G. Kim and M Kon, *Generic submanifolds of an odd-dimensional sphere*, Nihonkai Math J., **4** (1993).
[4]. U-H. Ki and Y.H. Kim, *Generic submanifolds with parallel mean curvature vector of an odd- dimensional sphere*, Kodai Math J., **4** (1981), 353–370.
[5]. U-H. Ki and M Kon, *Contact CR-submanifolds with parallel mean curvature vector of a Sasakian space form*, Coll. Math., **64** (1993), 173–184
[6]. U-H. Ki and J.S. Pak, *On totally real submanifolds with parallel mean curvature vector of a Sasakian space form*, Bull. Korean Math Soc., **28** (1991), 55–64.
[7]. M. Kon, *On some generic minimal submanifolds of an odd-dimensional sphere*, Coll Math , **56** (1988), 311–317.
[8]. K. Yano, *On a structure defined by a tensor field of type (1,1) satisfying* $f^3 + f = 0$, Tensor N.S., **14** (1963), 99–109.
[9]. K. Yano and M. Kon, *Generic submanifolds of Sasakian manifolds*, Kodai Math J., **3** (1980), 163–196.
[10]. K. Yano and M. Kon, *CR submanifolds of Kaehlerian and Sasakian manifolds*, Birkhäuser, Boston Inc., 1983.
[11]. K. Yano and M. Kon, *Structures on manifolds*, World Scientific Pub , 1984.
[12]. K. Yano and M. Kon, *On contact CR submanifolds*, J Korean Math. Soc., **26** (1989), 231–262

Department of Mathematics
Dong-A University
Pusan 604-714, Korea