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MAGNETOSTATIC MODELS OF STARSPOTS*

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ABSTRACT

Magnetostatic models of starspots of late type main sequence stars(G5V ~K5V) have been constructed to investigate their physical characteristics by using the similarity law suggested by Schlüter and Temesvary(1958) and later employed by Deinzer(1965) and Yun(1968). The starspots are assumed to be single, circular and in horizontal magnetostatic equilibrium. In the present study we considered only those model spots whose area covers less than 12% of the entire stellar surface as suggested by observations. The computed surface field strength of our model spots ranges from 10^3 to several 10^3 gauss and their magnetic flux is found to be 10~100 times that of sunspots. The field strength is sensitive to spectral type, which increases with later spectral type. In contrast to the field strength, the area of starspots depends strongly on the total magnetic flux. Finally, it is noted that the computed field strength of model spots belonging to G0V~G5V falls below the equipartition field strength at their parent stellar surface unless the coverage is less than 2%. This suggests that the observed spot on G0V~G5V stars is likely to be a group of small starspots.

Key Words : stellar magnetic fields, star spots, equipartition field, sunspots, filling factor.

I. INTRODUCTION

A starspot is a cool and dark region on the photospheric surface of a rapid rotating star. It is known that the starspot has a strong magnetic field which occupies 1~10 percent of the whole surface. Kron(1947) initiated the study to suggest a possible presence of starspots on the surface of AR Lac. Hall(1972) carried out quantitative investigations to examine the long-term light variations of RS CVn. Eaton and Hall(1979) successfully demonstrated that the peculiarity of the light curve of RS CVn can be accounted for by starspots.

In 1980's, numerous efforts have been carried out to estimate the size and the temperature of the starspots of late type stars by analyzing the observed data by the light curve modeling(Vogt 1981; Poe and Eaton 1985; Rodono *et al.* 1986) and the Doppler Imaging technique(Vogt and Penrod 1983; Strassmeier 1990). Theoretical studies of starspots, however are very scanty. The first theoretical starspot model was constructed by Bopp and Evans(1973), based on a simple use of the similarity existing between starspots and sunspots. They inferred the magnetic field strength of BY Dra and CC Eri to be about 2000 gauss by assuming 10% of the energy suppression in spots due to the presence of the magnetic field. Mullan(1974) also considered a magnetostatic starspot model, simply assuming that 1% temperature difference between a spot and the surrounding photosphere is maintained at each depth throughout the convective envelope. He estimated the strength of magnetic field by allowing a certain Wilson depression of the starspot. In his model, the curvature force of the spot's magnetic field has not been taken into account. The surface field strength was estimated to be about 20000 gauss, which seemed too large as compared with the observations.

The purposes of the present study are to explore the possibility of the existence of giant starspots and to examine their physical characteristics by constructing a set of magnetostatic models of starspots with the use of the similarity law(Schlüter and Temesvary 1958). In constructing the starspot models, we followed the scheme that Yun(1968, 1970) had used in his calculations of sunspot models. In the present study, we have considered the effect of the

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curvature force arising from fanning out of the magnetic field.

II. CALCULATIONS

(a) Basic Assumptions and Formulation

In the present study a starspot is assumed to be single, circular and in magnetostatic equilibrium. It is further assumed that the magnetic field is untwisted and axially symmetric. Therefore, the angular component of the magnetic field vanishes and the field strength becomes only a function of r and z in a cylindrical coordinate system. The z-axis is taken to be perpendicular to the stellar surface and directed toward the center of the present star. The depth of $\tau = 2/3$ of the normal star is taken as $z = 0$.

Under these assumptions, the horizontal magnetostatic equation becomes

$$\frac{\partial P}{\partial r} = \frac{B_z}{4\pi}\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right), \tag{1}$$

where P is the gas pressure and B_r and B_z refer to the radial and vertical components of the field.

As done by Yun(1970), we have introduced the similarity assumption to make the system of the partial differential equations manageable. The vertical component of the field $B_z(r,z)$ is normalized to its axial value, $B_z(0,z)$ and the normalized function $B_z(r,z)/B_z(0,z)$ is assumed to be decreasing monotonically with increasing r. Here we assumed the function had an unique shape at all depths except for a z-dependent scale factor $\xi(z)$ governing the radial direction. As a result the ratio becomes

$$\frac{B_z(r,z)}{B_z(0,z)} = \frac{D(\alpha)}{D(0)} \tag{2}$$

with $\alpha = r\xi(z)$ where $D(\alpha)$ is the shape function and D(0) is the normalization factor determined from

$$\Phi = 2\pi\int_0^\infty D(\alpha)\alpha d\alpha, \tag{3}$$

where Φ is the total magnetic flux of a starspot.

By virtue of the similarity assumption and $\nabla \cdot \vec{B} = 0$, the components of the magnetic field can be written as

$$B_z(r,z) = D(\alpha)\xi^2 \tag{4}$$

$$B_r(r,z) = -\alpha D(\alpha)\frac{d\xi}{dz}. \tag{5}$$

With the use of equations (4) and (5), equation (1) can be transformed into an ordinary differential equation

$$fyy'' - y^4 + 8\pi\,\Delta\,P = 0, \tag{6}$$

where $y = \sqrt{B_z(0,z)}$ and $\Delta P = P(\infty,z) - P(0,z)$. For our choice of the shape function $D(\alpha) = D(0)\exp(-\alpha^2)$ the quantities f and $D(0)$ become $f = \frac{\Phi}{2\pi}$, $D(0) = \frac{\Phi}{\pi}$ and $y(z)$ is given by

$$y(z) = \sqrt{2f}\cdot\xi(z) = \sqrt{D(0)}\cdot\xi(z), \tag{7}$$

On the axis of the spot we have the ordinary hydrostatic equation

$$\frac{dP}{dz} = \rho g, \tag{8}$$

The temperature and pressure distribution are calculated by using the diffusion approximation in the radiation region and the mixing length theory in the convection region. The extent of the suppression of convection due to presence of the magnetic field in the starspot is determined by the value of the ratio of the mixing length to the pressure scale height ℓ/H. For the normal stellar photosphere the value of the ratio is taken to be unity and for the starspots it is determined as an eigenvalue by requiring the solution of equation (6) subject to the certain boundary conditions.

In order to solve the equations of energy transport, it is necessary to specify the specific heat C_p per unit mass at constant pressure, the adiabatic gradient ∇_{ad} and Rosseland mean opacity κ as a function of the local conditions. In the present calculation C_p and ∇_{ad} are computed according to the procedure suggested by Vardya(1965), and κ has been taken from the opacity table of Cox and Tabor(1976) and Alexander *et al.*(1983).

(b) Boundary Condition

By virtue of the similarity assumption the fanning out of the magnetic field at the surface z_D is specified by the angle of inclination ψ_p at the outer edge of the penumbra of the starspot which is determined by α_p, the value of α passing through at the outer edge of the starspot. The value was estimated to be 1.63 for sunspots(Yun 1970), which had been adopted in the present study. The quantity ψ_p is regarded as a free parameter, connecting y and its derivative which must be met at the surface z_D, given by Yun(1970) as

$$y'(z_D) = \frac{y^2(z_D)}{\sqrt{2f}} \frac{\tan\psi_p}{\alpha_p}. \tag{9}$$

At the lower boundary we assumed that the magnetic field becomes uniform below a certain depth z_B. It would seem that the depth z_B could be taken at any depth. However, the freedom in selecting z_B is restricted by requiring the condition that the field strength at the upper boundary should not be changed appreciably with the choice of z_B. A reasonable set of values in our study is found to be $1.5 \times 10^9 cm$ for G5V and K0V and $1.7 \times 10^9 cm$ for K5V starspots. The condition of the uniform field below z_B requires

$$y'(z_B) = 0 \;\; and \;\; y''(z_B) = 0 \tag{10}$$

so that equation (6) leads to

$$\Delta P(z_B) = \frac{y^4(z_B)}{8\pi}. \tag{11}$$

The derivative of $\Delta P(z)$ with respect to z at z_B becomes zero,

$$\frac{d(\Delta P)}{dz}|_{z_B} = (\rho - \rho^*)g|_{z_B} = 0. \tag{12}$$

Thus, the density of the starspot ρ^* becomes equal to that of its parent star, $\rho(z_B)$ at z_B.

Finally, we need an additional boundary condition to be assigned to our model envelopes both for the starspot and its parent star, which is the relation between effective temperature T_{eff} and surface gas pressure P_{eff} at $\tau = 2/3$. In the present study we adopted a grid of the model atmospheres computed by Mould(1976) and VandenBerg(1983) to obtain the relation between P_{eff} and T_{eff}, which is summarized in Table 1. The physical parameters of our selected late type stars are listed in Table 2, which are taken from Allen(1976).

Table 1. log P_{eff} of Late Type Stars

T_{eff}	log P_{eff}			
	G2V	G5V	K0V	K5V
3000	5.59	5.65	5.65	5.65
3500	5.33	5.39	5.39	5.39
4000	5.30	5.36	5.36	5.36
4500	5.22	5.28	5.28	5.28
5000	5.10	5.16	5.16	5.16
5500	5.06	5.06	5.06	5.06

Table 2. Physical Parameters of Late Type Stars

Sp.type	T_{eff}(K)	$R/R_\odot$	g(cm/sec^2)
G2V	5800	1	27400
G5V	5520	0.93	31620
K0V	4700	0.85	31620
K5V	4130	0.74	31620

III. RESULTS AND DISCUSSIONS

(a) Computed Model Starspots

A set of magnetostatic model starspots of late type stars ranging from G5V to K5V has been constructed that uses as parameters the ratio of effective temperature of starspots to that of its parent star T^*_{eff}/T_{eff}, the total magnetic flux Φ and the angle of inclination ψ_p at the outer edge of the penumbra. The selected values taken in the present calculation are listed in Table 3. The characteristics of the computed model spots are summarized in Table 4, where we presented the cases for $T^*_{eff}/T_{eff} = 0.76$ with $\psi_p = 67°$and 75°. According to sunspot observations ψ_p turns out to be larger than 75°. The choice of $T^*_{eff} = 0.76T_{eff}$ is based on the recent starspot observations summarized in Table 5.

Table 3. Free Parameters of Late Type Stars

z_B	1.5×10^9cm(G5V,K0V), 1.7×10^9cm(K5V)
T^*_{eff}/T_{eff}	0.70, 0.76, 0.86
ψ_p	67^0, 75^0
Φ	5×10^{23}Mx, 1×10^{24}Mx, 2×10^{24}Mx, 3×10^{24}Mx

The area that the model spot occupies in the table is percentile. The quantity B is the field strength at the surface on the axis of symmetry of the computed starspots.

(b) Characteristics of Computed Model Starspots

The surface field strength B of the computed starspots characterized by $T^*_{eff} = 0.76T_{eff}$ and ψ_p=75°is plotted with respect to the total magnetic flux in Figure 1, where 3 different spectral types are considered. As seen from the figure the surface field strength increases with the decrease of the total magnetic flux and with later spectral types. The percentile area that the computed spots occupy is plotted as a function of the total magnetic flux in Figure 2. It is noted that the size of the starspots increases with earlier spectral types, while for a given spectral type, it increases rather sensitively with the total magnetic flux passing through a starspot.

The dependence of the surface field strength and the area of starspots on the parameter ψ_p is shown in Figure 3 and Figure 4, where we considered the starspots in a K0V star with the surface temperature $T^*_{eff} = 0.76T_{eff}$. As is expected, the surface field strength decreases with ψ_p by less than 1000 gauss in this case, while the area increases with ψ_p within less than a few percent.

The dependence of the field strength and area on the temperature difference between the starspot and its parent star(K0V) is shown in Figure 5 and Figure 6, where we considered the case for ψ_p=75°. As can be seen from Figure 5, for a given magnetic flux, the surface field strength can differ by 750~1200 gauss due to the surface temperature difference between the starspot and its parent star, and the area that the spot occupies, only by a few tenth to about 5%.

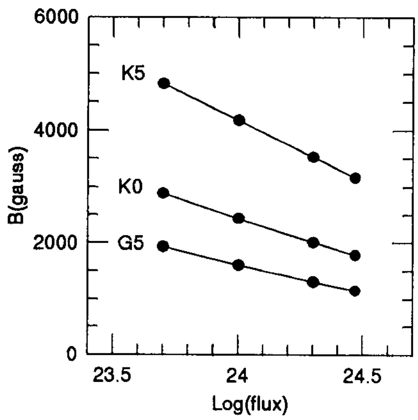
Table 4. Physical Characteristics of Computed Model Spots ($T^*_{eff}/T_{eff}=0.76$)

(a) Group A($\psi_p=75^0$)

Parent Stars	Sp.type	G5V			K0V			K5V		
	T_{eff} (K)	5520			4900			4130		
	B_{eq} (gauss)	1770			1880			2310		
Starspots	T^*_{eff} (K)	4195			3720			3139		
	$\psi_p(^0)$	75			75			75		
	Area(%)	1.5	4	11	1.5	4	11	1.5	4	11
	$\Phi(10^{24}$Mx)	0.56	1.2	2.6	0.68	1.5	3.2	0.82	1.8	4.0
	B(gauss)	1870	1530	1200	2670	2220	1760	4390	3580	2900
	dB/dz(gauss/km)	0.09	0.05	0.02	0.1	0.07	0.03	0.3	0.1	0.06

(b) Group B($\psi_p=67^0$)

Parent Stars	Sp.type	G5V			K0V			K5V		
	T_{eff} (K)	5520			4900			4130		
	B_{eq} (gauss)	1770			1880			2310		
Starspots	T^*_{eff} (K)	4195			3720			3139		
	$\psi_p(^0)$	67			67			67		
	Area(%)	1.5	4	11	1.5	4	11	1.5	4	11
	$\Phi(10^{24}$Mx)	0.7	1.5	3.3	0.84	1.8	4.0	1.0	2.3	5.1
	B(gauss)	2380	1870	1520	3420	2740	2210	5520	4590	3680
	dB/dz(gauss/km)	0.07	0.04	0.02	0.1	0.06	0.03	0.2	0.1	0.05



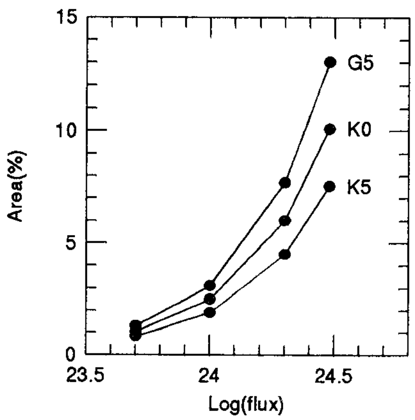
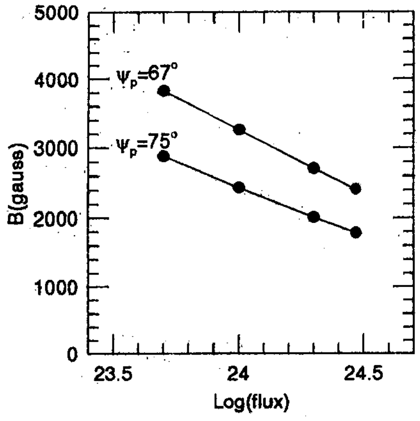


Fig. 1.(left) The surface field strength B of the computed starspots characterized by $T^*_{eff}=0.76\ T_{eff}$ and $\psi_p=75^0$

Fig. 2.(right) The percentile area of the computed spots characterized by $T^*_{eff}=0.76T_{eff}$ and $\psi_p=75^0$



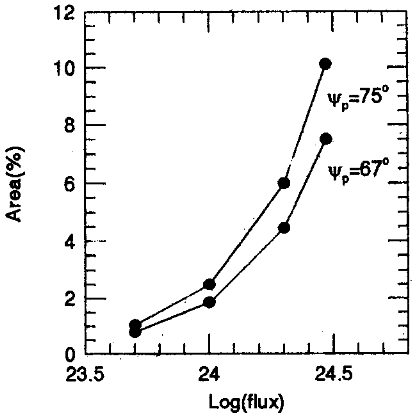
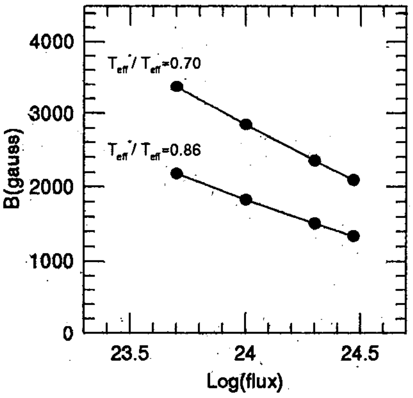


Fig. 3.(left) The dependence of the surface field strength of starspots on the parameter ψ_p

Fig. 4.(right) The dependence of the area of starspots on the parameter ψ_p



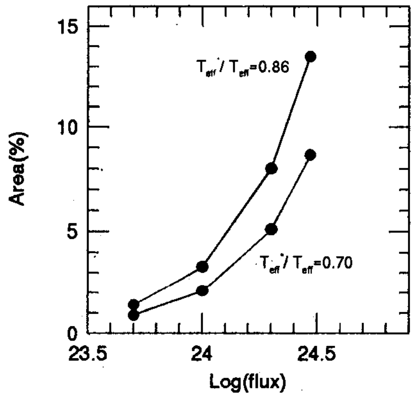


Fig. 5.(left) The dependence of the field strength on the temperature difference between the starspot and its parent star

Fig. 6.(right) The dependence of the area on the temperature difference between the starspot and its parent star

IV. THE OBSERVATIONAL CHARACTERISTICS OF STARSPOTS

Most RS CVn and BY Dra where starspots exist are composed of a close binary system and they commonly have a deep convection zone with rapid rotation. Strong magnetic fields and peculiar light curves are the main features of spotted RS CVn stars. Numerous efforts including satellite observations have been carried out to find out the characteristics of starspots. In the present study, we attempted to describe the observational characteristics of starspots of later type stars with our model spots.

(a) Temperature and Area of Starspots

The temperature and the area of a starspot have been determined by light curve modelings developed by Vogt(1981) which assume a single circular spot and use the standardized V, R band photometric data with Barns-Evans relation. This method was improved by Poe and Eaton(1985) and Rodono *et al.*(1986) who reconstructed the observed light curves by proper mixing of one or two spots. They found that the spot temperature can be quite different even for the same parent stars.

Vogt and Penrod(1983) developed another method which determines the area and the location of a starspot by means of Doppler effect of rapid rotating spotted stars. Dempsey *et al.*(1992) introduced a cross-correlation method which found a satisfactory solution with relatively low S/N about 150 by using the several strong absorption lines.

The light curve modelings and Doppler Imaging techniques yielded similar temperature and area of starspots. Tabel 5 is a collection of the temperature of starspots estimated from recent observations. As can be seen from the table, the observed surface temperature of starspots ranges from 2700K to 4100K and their occupied area, from 1.5% to 11% of the entire stellar surface. The average value of the surface temperature of starspots T^*_{eff} are found to be $T^*_{eff} = 0.76T_{eff}$, and the average coverage of starspots is about 4~5%.

Table 5. Temperature and Area of Starspots

Stars			Starspots			Ref.
Name	Sp. type	T_{eff} (K)	Area(%)	T^*_{eff} (K)	T^*_{eff}/T_{eff}	
Hr 1099	K1IV	4700	1.5, 4.5	3500	0.74	(1),(3)
HK Lac	K0III	4600	4.3, 3.3	<3520	<0.77	(2)
HR 7275	K1IV	4600	6.3, 2.7	3400	0.74	(2)
IM Peg	K1III	4440	4.3, 2.4	3250	0.73	(2)
λ And	G8IV	4780	10.6, 6.7	3730	0.78	(2)
UX Ari	K0IV	4780	4.7, 3.3	3360	0.70	(2)
LX Peg	K0IV	4780	3.0, 2.4	4050	0.85	(2)
II Peg	K2IV	4500	2.0, 9.5	3300	0.73	(2),(3)
BY Dra	M0V	4100	3.5, 11.0	3500	0.85	(2),(3)
AU Mic	dM2e	3500	1.5, 11.0	2650	0.76	(3)
AR Lac	K0IV	4700	1.5, 1.5	3500	0.74	(3)
σ Gem	K1III	4440	4.0, 2.4	3400	0.77	(2),(4)
VY Ari	K3IV	4600	5.5(pol) 2.8, 2.9, 3.3	3400	0.74	(5)

Ref.: (1) Vogt and Penrod 1983; (2) Poe and Eaton 1985; (3) Rodono *et al.* 1986; (4) Dempsey *et al.* 1992; (5) Strassmeier and Bopp 1992

(pol) : polar spot

(b) Stellar Magnetic Field

Stellar magnetic field is the most important physical quantity to understand starspots. Unfortunately, classical methods measuring polarized light had made null results due to the complexity of the magnetic geometries of later type stars. It was not until 1980 that stellar magnetic fields of later type stars were reliably measured by Robinson *et al.*(1980). Robinson measured the magnetic field strength B and the surface filling factor f by comparing magnetically insensitive line profile(low Lande g lines) with sensitive line profile (high Lande g lines). This technique was improved by including the model atmosphere and the infrared band observations. As a result, Linsky and Saar(1987) found a trend that (1) the surface magnetic field strength(B) increases with later spectral types and the filling factor(f) increases with stellar rotation, and (2) the filling factor is inversely proportional to the age.

Recently, Kemp *et al.*(1987) and Donati *et al.*(1990) developed a new technique which used the stellar circular polarizations, noting that a large Doppler effect interrupted the precise measurement of the field strength by the Robinson method. Donati *et al.*(1990) found the field strength of HR 1099 to be 1000 gauss, which happens to correspond to the equipartition field of the star.

The observed average stellar field strength and the filling factor of later type stars (G5~K5) are presented in Table 6. The values in Table 6 are considered to be the lower limit of the measured magnetic fields, since the Robinson method is expected to be more sensitive to the magnetic field of active regions rather than that of starspot.

The photometric and spectroscopic observations suggest (1) that starspots cover on the average, 4~5% of their parent stellar surface, and (2) that their surface temperature T^*_{eff} is mostly given in accordance with $T^*_{eff} = 0.76T_{eff}$,

Table 6. Statistics on Observed Field Strengt and Filling Factors of Late Type Magnetic Stars

Sp.type	Magnetic field(gauss)				Filling factor (%)
	≤ 1000	1000∼2000	2000∼3000	≥3000	
G	1	8	0	0	10∼60
K	4	8	5	0	12∼80

where T_{eff} is the effective temperature of the parent stars. Accordingly, in the present study we have taken the computed starspots of Group A in Table 4 as a representative set of model starspots. Since the measurements of the field strength of starspots are not available, we can not make any direct comparison of the model spots with the observations. However, the computed field strength of starspots is found to be comparable with those of magnetically active stars as suggested by Saar(1990). Furthermore it is noted that the field strength suggested by our model spots increases with later type stars in accordance with the findings of Linsky and Saar(1987).

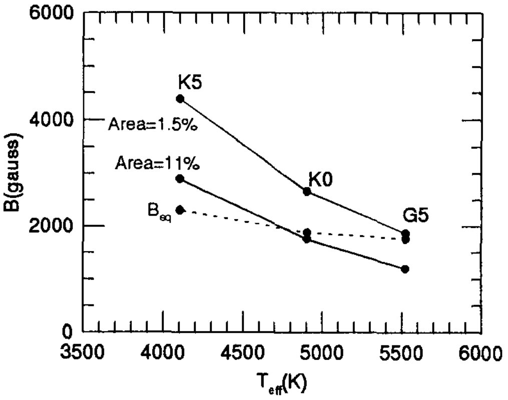


Fig. 7. The comparison of the equipartition field of each spectral type with the computed field strength of starspots for the case of Group A.

According to the results of Donati *et al.*(1990), the upper limit of the stellar magnetic fields is found close to the equipartition field strength for most magnetic stars. Hence, we may consider the equipartition field as being a lower limit of the field strength of a starspot for a given spectral type. This means that the computed field strength of a starspot should be at least, stronger than its equipartition field of its parent star. Figure 7 shows the comparison of the equipartition field of each spectral type with the computed field strength of starspots for Group A. As can be seen from the figure, most of the model starspots have the field strength stronger than their respective equipartition field. It also suggests that the starspots on the surface of G5V are likely quite small in size. In this sense the starspot of λ And(G8 type) whose coverage is found to be more than 10%(Poe and Eaton 1985), is likely to be a spot group rather than a single giant spot.

In the case of K0V and K5V stars, the computed field strength of starspots is greater than their equipartition field. This indicates that K0V∼K5V stars can accommodate single individual starspots with a reasonable size.

Finally, we compared the physical characteristics of sunspots with those of starspots, which are summarized in Table 7.

Table 7. Sunspots versus Starspots

Physical quantity	Sunspot	Starspot
Spectral type	G2	G8 ~ M0
Area of a spot(%)	~ 0.01	1 ~ 10
T^*_{eff}/T_{eff}	0.70 ~ 0.86	0.70 ~ 0.86
Total magnetic flux(Mx)	$\sim 10^{22}$	$10^{23} \sim 10^{24*}$
B(gauss)	500 ~ 3500	1000 ~ 5000*
Rotation period(days)	27	< 7
Activity period(years)	11	> 10 (10 ~ 60)

* The value of spot models

V. SUMMARY AND CONCLUSIONS

Magnetostatic models of starspots of late type main sequence stars(G5V~K5V) have been constructed to investigate their physical characteristics by using the similarity law suggested by Schlüter and Temesvary(1958) and later employed by Deinzer(1965) and Yun(1968). The starspots are assumed to be single, circular and in horizontal magnetostatic equilibrium. In the present study we considered only those model spots whose area covers less than 12% of the entire stellar surface as suggested by observations.

In order to examine the characteristics of starspots we have taken as representative, the computed model starspots of Group A(see Table 4(a)), which are characterized by $T^*_{eff} = 0.76T_{eff}$ and $\psi_p = 75°$. Some of the important findings emerged from the present study are as follows.

1. We demonstrated that the physical characteristics of starspots suggested by photometric and spectroscopic observations of starspots along with magnetic observations of magnetically active stars can be accounted fairly consistently with our computed starspot models. The field strength of the model spots are found to be 1000~5000 gauss, which do not differ greatly from the observed stellar magnetic fields ranging from 1000 to 3000 gauss suggested by Saar(1990).
2. It is found that the starspots have the total magnetic flux at least 10~100 times that of sunspots in order to maintain their sizes.
3. The field strength is sensitive to spectral type, which increases with later spectral types. In contrast to the field strength, the area of starspots depends strongly on the total magnetic flux.
4. It is noted that the computed field strength of model spots belonging to G0V~G5V falls below the equipartition field strength of their parent stellar surface unless the coverage is less than about 2%. This suggests that the observed spot on G0V~G5V stars is likely to be a group of small starspots.
5. Despite several differences in starspots and sunspots, it is found that they are qualitatively similar phenomena.
6. Finally, it is hoped that the present study provides some theoretical basis for using starspots in interpreting the peculiar behaviors of stellar light curves.

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