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Evaluation of Storage Policies with Production Lot-Sizing Consideration in an AS/RS+

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Abstract

The performance of Storage assignment policies is traditionally evaluated with the storage capacity of an AS/RS taken as given. However, the storage capacity is closely related to the inventory model used in real situations. This paper presents a model of evaluating the performance of three storage policies (random storage, class-based storage, and full turnover-based storage) considering production lot-sizing simultaneously with storage assignment of inventory items. The objective of the model is to achieve a balance of warehouse throughput and space requirements such that a total of material handling cost, production ordering cost, and inventory holding cost is minimized. The effects of the parameters involved in the model are investigated on the performance of each storage policy through example problems.

1. Introduction

The amount of material handling costs incurred in an automated storage/retrieval system(AS/RS) largely depends on the storage assignment policy used. As such, wise selection of the storage policy should improve the operating efficiency of the warehouse, which could reduce the costs considerably.

Assuming that the storage capacity of an automated warehouse is given, Hausman, Schwartz, and Graves(HSG)[2, 3] described the behavior of three kinds of storage policy (random, full turnover-based, class-based) in terms of the expected travel time of the storage /retrieval(S/R) machine. Recently, Hwang, Ko, and Jang[5] extended HSG's work[3] for a nonsquare-in-time rack.

However, the storage capacity required to accommodate the inventory items obviously varies according to both production lot sizing and storage policy decisions. Therefore, performance comparison of storage policies should be made considering the effect of production lot sizing.

Wilson[7] first adressed the joint problem of

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storage-assignment/log-sizing under full turnover-based storage in a traditional discrete storage rack and formulated it as a nonlinear program. Later, Hodgson and Lowe[4] considered the problem similar to Wilson's except that storage assignments are made on continuous AS/RS-rack face in time. In these studies, however, no analysis was done on the expected behavior of lot sizes and the different storage policies.

It is generally known that random storage requires less space than full turnover-based storage, but yields lower throughput, instead [6]. On the other hand, class-based storage can yield both the space benefit of random storage and the throughput benefit of full turnover-based storage. However, these assertions may not be valid when storage capacity of a warehouse is not given in advance, but have to be determined by jointly considering lot sizing and storage assignment. In this case, evaluation of storage policies should be made in terms of space requirements, expected throughput, and related costs including those of porduction ordering, inventory holding, and material handling.

Regarding this, we extend the work by HSG [3] for a nonsquare-in-time rack incorporating lot sizing of inventory items into storage assignment decisions. In this paper, the inventory items are treated as a continuous variable. The main advantages of this approach appear to be in the relative simplicity of problem formulation and the ability to obtain and analyze computational results while the shape of item demand distribution changes.

In the sections that follow, first the general formulation of the storage-assignment/lot-sizing model is proposed, and then results for each storage policy are given. Finally, some observations are described, which are made from the sensitivity analysis for examining the effects of changes in parameter values.

2. Development of Storage-Assignment/Lot-Sizing Models

2.1 General Model Formulation

Assumptions

A single-aisle continuous rectangular storage rack with the input/output(I/O) point being located at the lower left corner;

Uniform sized storage location and pallet each containing a single type of item;

Tchebyshev travel of the S/R machine with known vertical and horizontal speeds;

No interlieving operation;

Linear material-handling(S/R machine travel) cost;

Known demand rate of inventory items;

Economic-order-quantity(EOQ) lot-sizing model.

Rack Normalization

Consider a single-aisle storage rack whose dimensions are L(rack length)×H(rack height). Following Bozer and White[1] the rack is normalized as a 1×b continuous rectangle in time:

C =storage capacity=L×H;

v_x =horizontal speed of the S/R machine;

v_y = vertical speed of the S/R machine;

t_x = time to reach the end of the rack = L/v_x;

t_x = time to reach the top of the rack = H/v_x;

T = denormalizing factor = t_x assuming $t_x \geq t_y$

b = rack shape factor = t_y/t_x, $0 < b \leq 1$.

Note that for specific values of C and b, T can be expressed from the above relationships as

$$T = (C/bv_xv_y)^{1/2}. \quad (1)$$

General Model

Since both Wilson's and Hodgson & Lowe's approaches are based on discrete lot-size variables, it is not easy to perform sensitivity analysis such as for investigating the effect of change in the item demand pattern. In order to overcome this limitation and to provide the flexibility in the analysis, production lot sizes are here treated to be continuous over $0 < i \leq 1$ where i represents i-th percentile item being stored in the warehouse. Hereafter, we will use the term, "item i" instead of "i-th percentile item", for convenience.

Let the lot size of item i be $q(i)(\geq 0$ for $0 < i \leq 1)$ and the cummulative one, Q(i), then the following holds:

$$Q(i) = \int_0^i q(j)dj, \quad 0 < i \leq 1$$

Now the total lot size, R, becomes $R = Q(1)$, and thus the required storage capacity of the warehouse will be

$$C = wR \quad (2)$$

where w =

$$\begin{cases} 1/2 & \text{for random storage assignment,} \\ 1 & \text{for full turnover-based assignment.} \end{cases}$$

The problem concerned can be stated as:

$$\text{(PO) Min } TC(q(i)) = \int_0^1 (O(q(i)) + H(q(i))\, di + \int_0^1 (M(Q(i))di \quad (3)$$

sujbject to

$$0 < R = \int_0^1 q(i)di \leq U \quad (4)$$

$q(i) \geq 0, \quad 0 < i \leq 1$

where O(q(i)) = yearly porduction-ordering cost of item i;

H(q(i)) = yearly holding cost of item i;

M(Q(i)) = yearly material-handling cost of item i; and

U = upper bound of total lot size.

Given the cummulative lot-size function, Q(i), M represents the material-handling cost which resultes from an optimal storage assignment. In (3), the production ordering and holding costs are expressed as separable by item. However, the material-handling cost cannot be separately represented, since the cost for each item is a function of the travel time from the I/O point to the storage location, which depends on q(i) for all i as well as the storage policy used.

M can be xpressed as

$$M(Q(i)) = 4c \cdot a(i) \cdot t(i) \quad (5)$$

where a(i) = yearly demand rate(measured in full pallet loads) of item i derived by the "ABC" phenomenon for inventories

$$= Asi^{s-1}, \quad 0 < i \leq 1, 0 < s \leq 1, \quad (6)$$

where s denotes the demand-curve shape factor;

t(i) = travel time to the location of item i

$$= (wR/bv_xv_y)^{1/2}z(i) \quad (7)$$

where z(i) is the normalized time; and

c = cost of S/R machine travel per

unit time.

Note that since $\int_0^1 a(i)di = A$, A will be the total number of pallets demanded annually.

Expression(5) reflects that costs are incurred for both item storage and its ultimate retrieval, and the S/R machine makes a round trip. Expression (7) is derived by using (1) and (2).

Substituting (6) and (7) into (5) yields

$$M(Q(i)) = 4cAV_1(wR)^{1/2} si^{s-1}z(i) \quad (8)$$

where $V_1 = (bv_x v_y)^{-1/2}$, and for convenience let $V_2 = (2bv_x v_y)^{-1/2}$. Also, assuming the basic EOQ model the ordering and holding costs are

$$O(q(i)) + H(q(i)) = ka(i)/q(i) + hq(i)/2 = KAsi^{s-1}/q(i) + hq(i)/2, \quad 0 < i \leq 1 \quad (9)$$

where k and h are respectively ordering and holding costs per unit per year which, for simplicity, are assumed constant for all items.

Finally, letting U=A without loss, of qenerality and substituting (8) and (9) into (3), (PO) can be rewritten as:

$$\text{(P1) Min } TC(q(i)) = \int_0^1 (KAsi^{s-1})/q(i) + hq(i)/2 di + 4cAV_1(wR)^{1/2} \int_0^1 si^{s-1}z(i)di \quad (10)$$

subject to

$$0 < R = \int_0^1 q(i)di \leq A$$

$$q(i) \geq 0, \quad 0 < i \leq 1.$$

2.2 Random Storage Assignment(RAN)

Under random storage assignment, any pallet is equally likely to be stored in any of storage location. Thus, in this case z(i) is independent of i such that

$$z(i) = b^2/6 + 1/2, \quad 0 < i \leq 1. \quad (11)$$

Since w=1/2 for RAN, rearranging (10) with (11) yields the total cost under RAN, TC_{RAN}, given by

$$TC_{RAN}(q(i)) = \int_0^1 (KAsi^{s-1}/q(i) + hq(i)/2di + 4cAV_2(b^2/6+1/2)(\int_0^1 q(i)di)^{1/2}. \quad (12)$$

The first integral term in (12) is a convex function of q(i), but the last term is not always convex. Due to this, it might be extremely difficult to find an optimal lot-size function directly from the above formulation. Instead, in this paper we use the following form:

$$q(i) = R a i^{a-1}, \ R > 0, \ 0 < a < 1+s \quad (13)$$

where the upper bound of a is derived from the condition that the production ordering cost should be positive. Them, the lot-size function has some desirable properties such as:

1) The total lot size is R, i.e., $\int_0^1 q(i)di = \int_0^1 R a i^{a-1} di = R$;
2) The cummulative lot size, $Q(i) = \begin{cases} Ri^a & \text{for } 0 < i \leq \\ 0, & \text{otherwise}; \end{cases}$
3) Let $q(i) = R \cdot f(i)$ where $f(i) = a i^{a-1}$, then f(i) can be regarded as a probility density function with the associated cummulative probability distribution, F(i), being i^a for $0 < i \leq 1$;
4) q(i) is concave for $0 < a \leq 1$ and convex for $1 < a < 1+s$;
5) The optimal lot size obtained from the basic EOQ model is a special case of q(i) defined as (13), where $R = (8kAs/h)^{1/2}/(1+s)$ and $a = (1+s)/2$.

Substituting (13) into (12) gives

$$TC_{RAN}(R, a) = L(R, a) + e_1 R^{1/2} \quad (14)$$

where $L(R, a) = kAs/a(1-s+a) + hR/2$ and $e_1 = 4cAV_2(b^2/6+1/2)$.

PROPERTY 1. There exist a single local minimum, (R^*_{RAN}, a^*_{RAN}) of $TC_{RAN}(R, a)$ such that

$R^*_{RAN}=\{R \mid e_1R^{1/2}/2+e_2R-e_3/R=0, R>0\}$, and

$a^*_{RAN})=(1+s)/2$

where $e_2=h/2$ and $e_3=4kAs/(1+s)^2$.

Proof: See Appendix.

Considering the upperbound constraint, (4), R^*_{RAN} is modified as

$R^*_{RAN}=\{R \mid e_1R^{1/2}/2+e_2R-e_3R=0 \quad 0<R\leq A\}$.

2.3 Full Turnover-Based Stroage Assingnment(FULL)

Under turnover-based dedicated storage, items are assigned to storage locations in increasing order of the ratio of their activity to the space requirements. In a unit-load AS/RS concerned in this study, the ratio becomes "trunover frequency"as given below(see wilson [7] and HSG[3)

$\beta(i)=2a(i)/q(i),\ 0<i\leq 1.$

The turnover frequency represents the number of times unit pallet load of a given item requires storage and retrieval per year.

Using the lot-size function in (13) again for comparison, the yearly material-handling cost is derived in the following.

In this case, w=1, in (7), and we get

$$\int_0^1 M(Q(i)\ di=4cAV_1R^{1/2}\int_0^1 si^{s-1}z(i)di. \quad (15)$$

Due to Tchebyshev tralvel of the S/R machine, the travel time distribution becomes

$$z(i)=\begin{cases}(bF(i))^{1/2}=b^{1/2}i^{a/2} & \text{for } 0<F(i)\leq b,\\ F(i) \qquad =i^a & \text{for } b<F(i)\leq 1.\end{cases}$$

Hence, the integral term in (15) will be

$$\int_0^1 si^{s-1}z(i)di=\int_0^b sb^{1/2}\ i^{s+a/2-1}di+\int_b^1 si^{s+a-1}di$$

$$=s(b^{s+(1+a)/2}/(s+a/2)$$
$$+(1-b^{s+a})/(s+a)). \quad (16)$$

Substituting (16) into (15) yields

$\int_0^1 M(Q(i))di=4cAsV_1[b^{s+(1+a)/2}/(s+a/2)+(1-b^{s+a})/(s+a)]R^{1/2}$.

Then, the total cost for FULL, TC_{FULL}, is given by

$TC_{FULL}(R,\ a)=L(R,\ a)+4cAsV_1[b^{s+(1+a)/2}/(s+a/2)+(1-b)^{s+a}/(s+a)]R^{1/2}$.

Notice that TC_{FULL} is neither convex nor concave on all (R, a). To solve the constrained nonlinear program,

$$\text{(P2)} \qquad \min_{R,\ a} \quad TC_{FULL}(R, a)$$

Subject to

$$0<R\leq A \text{ and } 0<a<1+s$$

we might use a gradient search type of technique.

One question that may arise would be whether the optimal solution obtained satisfy or not the principle of the FULL Policy under which items with larger $\beta(\cdot)$ should be placed closer to the I/O point. The following property gives the answer.

PROPERTY 2. B(i) computed with the optimal solution of (P2), (R^*_{FULL}, a^*_{FULL}), is non-increasing for $0<i\leq 1$.

Proof: See Appendix.

Finally, during the proof of PROPERTY 3 it is found that:

$a^*_{RAN}=(1+s)/2\leq a^*_{FULL}<1+s$.

An implication of this result might be that be that applying the FULL policy leads to the degree of skewness in the obtained lot-size curve being always smaller than that under RAN.

2.4 Class-Based Storage Assignment (C2, C3, C∞)

Class based storage is a more practical version of FULL. In this policy, pallets and rack locations are jointly partitioned into a small number of classes based on item turnover distribution and travel times, respectively. Within any given class, pallets are then assigned to storage locations randomly.

We first derive total cost expressions for the two-class system(C2) and the three-class system(C3), and then extend to the limiting case, the full turnover-based storage system(C∞) having the storage capacity being equal to R/2. Note that the cost expressions for production ordering and inventory holding costs are the same as those of RAN and FULL.

C2 System

Let B be the partitioning point which divides the rack into two classes such that class I region is used for the higher-turnover pallets and class II region for the lower-turnover pallets. The associated item partitioning point, I, with B will be

$$I=\begin{cases}(B^2/b)^{1/a} & \text{for } 0\le B\le b,\\ B^{1/a} & \text{for } b<B\le 1.\end{cases} \quad (17)$$

The material handling cost, M_2, for the two-class system then becomes

$$M_2=\int_0^1 M(Q(i))\,di$$

$$=4c\left(\int_0^I a(i)\bar{t}_I\,di+\int_I^1 a(i)\bar{t}_{II}\,di\right) \quad (18)$$

where $\bar{t}_I$ and $\bar{t}_{II}$ are the expected one-way travel times for class I and class II, respectively. Denoting M_2^1 the material handling cost for $0\le B\le b$, and M_2^2 for $b<B\le 1$, the following results are obtained:

Case 1: $0\le B\le b$

In this case, $\bar{t}_I$ and $\bar{t}_{II}$ are respectively expressed as

$$\bar{t}_I=(2/3)V_2 I^{a/2}(bR)^{1/2} \quad (19)$$

and

$$\bar{t}_{II}=V_2(b^2-4b^{1/2}I^{3a/2}+3)R^{1/2}/6(1-I^a). \quad (20)$$

Thus, substituting (19) and (20) into (18), one can derive

$M_2^1(R,\ a,\ I)=4cAV_2[(2/3)b^{1/2}I^{a+a/2}+(1-I^a)(b^2-4b^{1/2}I^{3a/2}+3)/6(1-I^a)]R^{1/2}$.

Case 2: $b<B\le 1$

Similarly,

$$\bar{t}_I=V_2(b^2 I^{-a}+3I^a)R^{1/2}/6 \quad (21)$$

and

$$\bar{t}_{II}=V_2(1+I^a)R^{1/2}/2. \quad (22)$$

By substituting (21) and (22) into (18),

$M_2^2(R,\ a,\ I)=4cAV_2[(b^2 I^{-a}+3I^a)I^a/6+(1+I^a)(1-I^a)/2]R^{1/2}$.

Consequently, the total costs, TC_{c2}^j, $j=1, 2$ where j represents the case, become a function of three decision variables, R, a, and I, such that

$$TC_{c2}^j(R, a, I)=L(R, a)+M_2^j(R, a, I),\quad j=1, 2.$$

C3 System

A similar approach is applied to the three-class system. Let B_1 and B_2 denote the two partitioning points which divide the rack into three classes. Also, the corresponding item partitioning point, I_1 and I_2, can be found from (17). The material handling costs, M_3^j, j=1, 2, 3, are derived, where the superscript j represents each case defined in terms of the relative locations of B_1 and B_2.

Case 1: $0\le B_1\le b$, $0\le B_2\le b$

$$M_3^1(R, a, I_1, I_2)=4cAV_2[(2/3)b^{1/2}I_1^{s+a/2}$$
$$+2b^{1/2}(I_1^a+(I_1I_2)^{a/2}+I_2^a(I_2^s-I_1^s)$$
$$/3(I_1^{a/2}+I_2^{a/2})+(b^2-4b^{1/2}I_2^{3a/2}+3)$$
$$(1-I_2^s)/6(1-I_2^a)]R^{1/2}.$$

Case 2: $0 \le B_1 \le b,\ b < B_2 \le 1$

$$M_3^2(R, a, I_1, I_2)=4cAV_2[(2/3)b^{1/2}$$
$$I_1^{s+a/2}+(b^2+3I_2^{2a}-4b^{1/2}I_1^{3a/2})$$
$$(I_2^s-I_1^s)/6(I_2^a-I_1^a)+(1-I_2^s)(1+I_2^a)/2]R^{1/2}.$$

Case 3: $b \le B_1 \le 1,\ b \le B_2 \le 1$

$$M_3^3(R, a, I_1, I_2)=4cAV_2[I_1^s(b^2/6I_1^a+I_1^a/2)$$
$$+(I_2^s-I_1^s)(I_1^a+I_2^a)/2$$
$$+(1-I_2^s)(1+I_2^a)/2]R^{1/2}.$$

Consequently, the total cost of each case j, TC_{c3}^j, becomes a function of four decision variables, R, a, I_1, and I_2, as given by

$$TC_{c3}^j(R, a, I_1, I_2)=L(R, a)+M_3^j(R, a, I_1, I_2),\quad j=1, 2, 3.$$

As is in the case of FULL, both TC_{c2} and TC_{c3} are not proved to be either convex or concave. To solve the optimization models for the class-based storage, one can also resort to any one of existing search procedures.

C∞ System

As the number of classes gets infinitly large, the class-based system approaches to the full trunover-based storage. However, since within any class all items are to be stored under RAN, the total storage requirement of the infinite class system should be only R/2, which contradicts the definition of FULL. Due to this, the limiting case can be only used for estimating the expected limit of system throughput and the path of optimal solutions being obtained.

The total cost denoted by $TC_{c\infty}$ can be found directly by replacing R with R/2 in the material-handling cost expression for FULL, as given below

$$TC_{c\infty}(R, a)=L(R, a)+4cAsV_1[b^{s+(1+a)/2}/(s+a/2)+(1-b^{s+a})/(s+a)](R/2)^{1/2}.$$

3. Sensitivity Analysis

In this section, the effects of changes in related parameter values are investigated on the performance differences among 5 storage policies, i.e., RAN, C2, C3, C∞, and FULL. The parameters considered are s, b, k, h, and c.

A full factorial design of experiments was executed for each storage policy wich the following data set:

s=(0.065(20%/90%), 0.139(20%/80%), 0.222(20%/70%), 0.138(20%/60%), 0.431(20%/50%), 0.500(20%/45%), 1.000(20%/20%,

b=(0.4, 0.7, 1.0),

k=(4000, 42000, 80000)

h=(0.3, 300, 599.7), and

c=(10, 155, 300)

where 20%/χ% implies that 20% of the inventoried items represent χ% of the total demand, and k, h, and c are all measured in 100 won. Thus, for each storage policy, 567(=7×3⁴) problems are solved.

We believe that the data set have the sufficient generality to allow for the modeling of a wide class of systems. The other given data are:

V_x=60m/min, V_y=20m/min, and A=10000.

The computer program for the experiments was written in Quick-BASIC and implemented on an IBM-PC(386). Each problem generated was solved by using a simple grid-search technique within reasonable computer time. A sample solution is given in Table 1.

For each storage policy, we first investigated the general trend in optimal values of the decision variables, R, a, B, and (B_1, B_2), as well as some performance indices such as the expected normalized time(ENT), the expected denormalized time(EDT), and the total cost (TC). The observations made from the experimental results are listed in Table 2.

〈Table 1〉 Solution of the problem given that s=0.139, b=0.4, k=4000, h=0.3, and c=10.

	storage policy				
	RAN	C2	C3	C∞	FULL
R	346.8	461.7	563.9	691.5	539.5
a	0.569	0.745	0.783	0.811	0.813
B	–	0.182	–	–	–
(B_1, B_2)	–	–	(0.077, 0.330)	–	–
ENT	1.053	0.571	0.460	0.366	0.366
EDT	0.663	0.396	0.353	0.311	0.388
TC	176101.6	120324.4	106014.5	92482.8	116460.1

〈Table 2〉 Trend of optimal solutions and some performance measures as each parameter value increases.

parameter(↑)	R	a	B	(B_1, B_2)	ENT	EDT	TC
s	#	↑	↑	(↑, ↑)	↑@	↑	↑
b	↑	=@	↑	(↑, ↑)	↑	↓	↓
k	↑	=@	=	(=, =)	=@	↑	↑
h	↓	*@	=	(=, =)	=@	↓	↑
c	↓	*@	=	(=, =)	=@	↓	↑

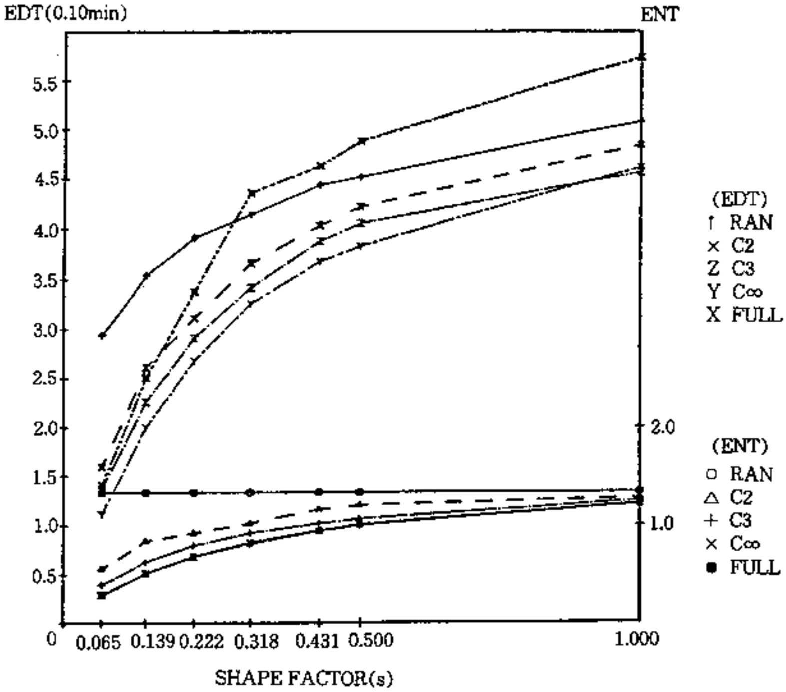
= There exists no significant difference.

@ The corresponding value of RAN is theoratically constant.

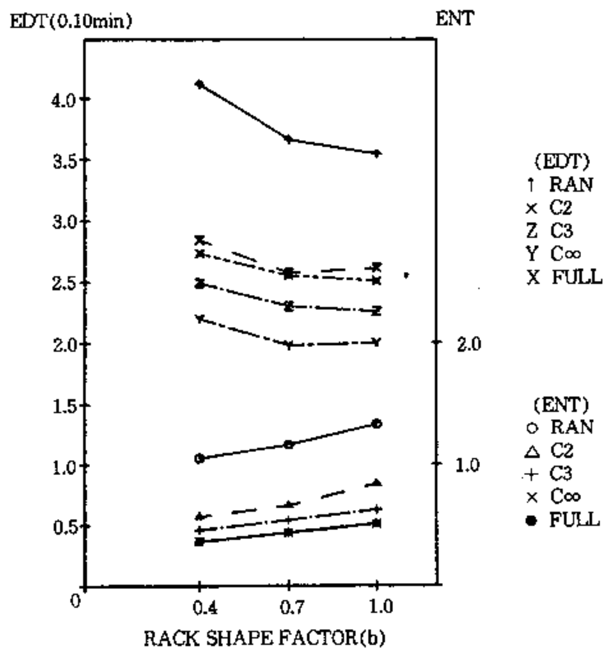
* There exist significant differences but no trend is found.

\# As a larger, R_{RAN} increases strictly, whereas R for each of the other policies does only for small value of s.

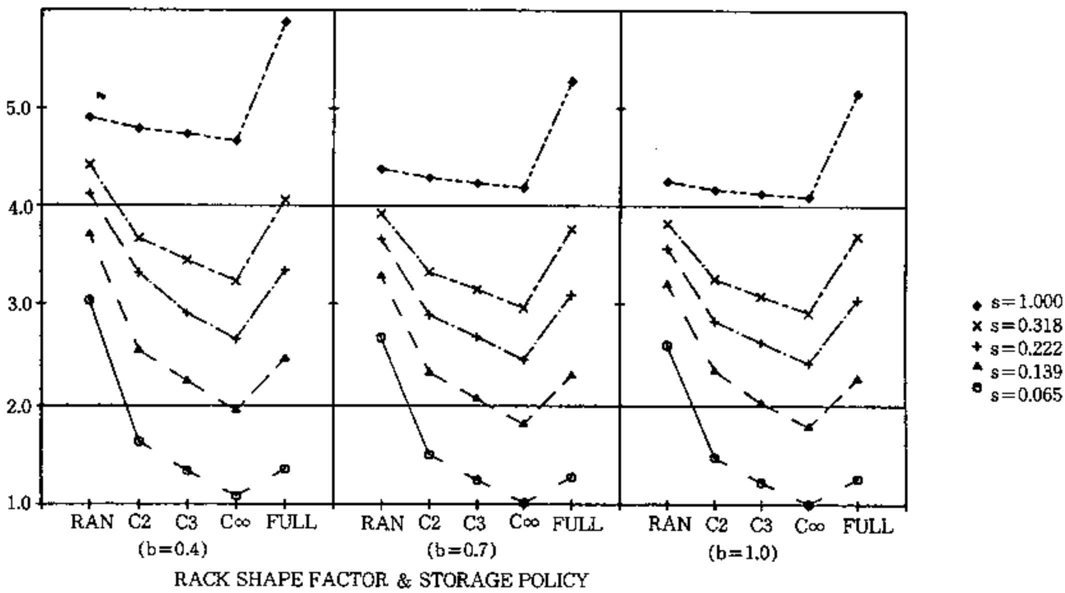
To show the typical effects of s and b on the expected travel times, Figure 1 and Figure 2 are repectively given for the case where k=42000, h=0.3, and c=300. From the figures we can find that the computational results from the models developed are almost equivalent to those derived theoretically whithout production lot-sizing consideration[5]. This is also valid for B and (B_1, B_2), that is, every B obtained was located between B_1 and B_2.



[Figure 1] The effect of s on the expected travel times when b=1.0, k=42000, h=0.3, and c=300.



[Figure 2] The effect of s on the expected travel times when s=0.139, k=42000, h=0.3, and c=300.



[Figure 3] The sensitivity of total cost to s and b when k=42000, h=0.3, and c=300.

The sensitivity of total costs according to both s and b is depicted in Figure 3.

Next, for the performance comparison among storage policies, denoting IENT, IEDT, IR, and ITC the relative measures of policy performance to RAN, which are defined as:

$IENT = (ENT_{RAN} - ENT_x)/ENT_{RAN}$,

$IEDT = (EDT_{RAN} - EDT_x)/EDT_{RAN}$,

$IR = (R_x - R)/RAN_{RAN}$, and

$ITC = (TC_{RAN} - TC_x)/TC_{RAN}$

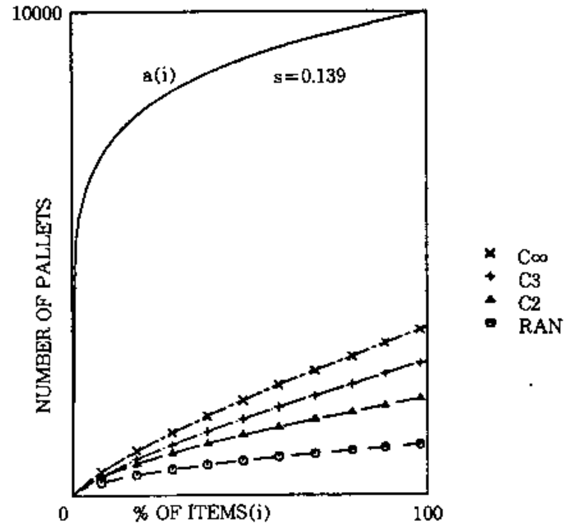
where the subscript x represents a policy, the following observations are made:

(1) With each set of given parameter values, according to the sequence of(RAN, C2, C3, C∞), R and a increase(see Figure 4), and on the contrary ENT, EDT, and TC decrease. However, those results from FULL are observed to be dependent upon the problems generated.

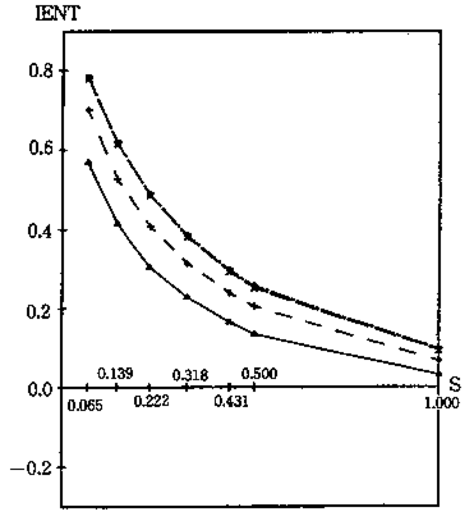
(2) As s gets smaller with other parameter values being fixed, IENT, IEDT, and ITC are all increasing. This indicates that the larger the improvement in travel times is with the most highly skewed demand distribution yielding the largest, the better the relative quality of each policy to RAN is.

As expected, $IENT_{FULL}$ is, like other policies, always founded to be greater than or equal to zero for all s. In addition, there exists no significant difference between $IENT_{FULL}$ and $IENT_{C\infty}$. However, as s increases, $IEDT_{FULL}$ becomes dramatically decreasing, and as a result especially for fairly large values of s, i.e., s≥0.431, it becomes negative valued, which yields the bad performance of FULL in terms of total cost. Figure 5 shows the experimental result reflecting these observations.

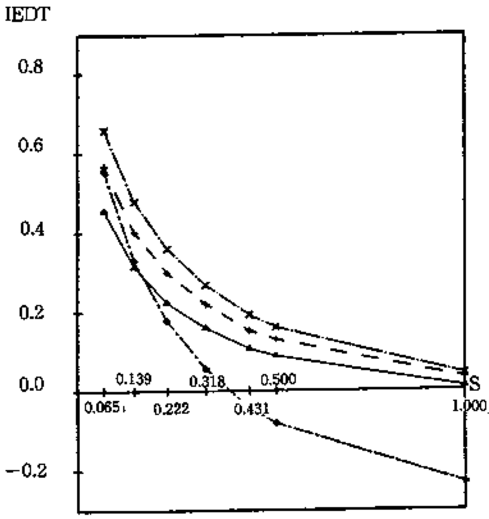
(3) For each policy, the percentage increment (IR) in total lot size over RAN is also increasing as the value of s becomes smaller. From this we know that larger reductions in



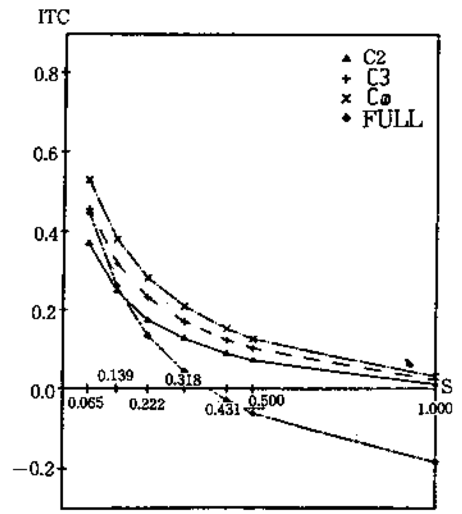
[Figure 4] The optimal lot-size function of each policy when s=0.139, b=0.7, k=42000, h=0.3, and c=10.



(a) Improvement in ENT



(b) Improvement in EDT



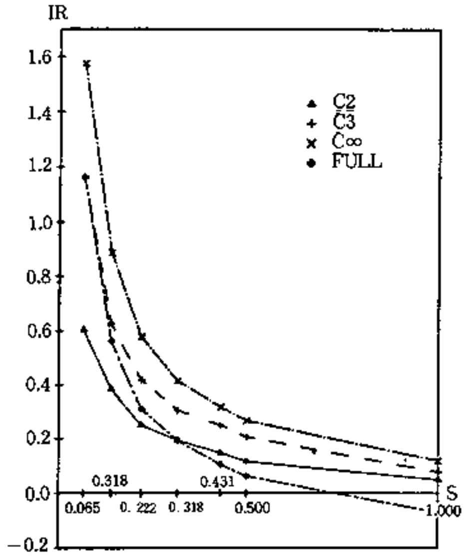
(c) Improvement in TC

[Figure 5] The effect of s on the percentage improvement in travel times and total cost for each policy, which are averaged for all b, k, h, and c.

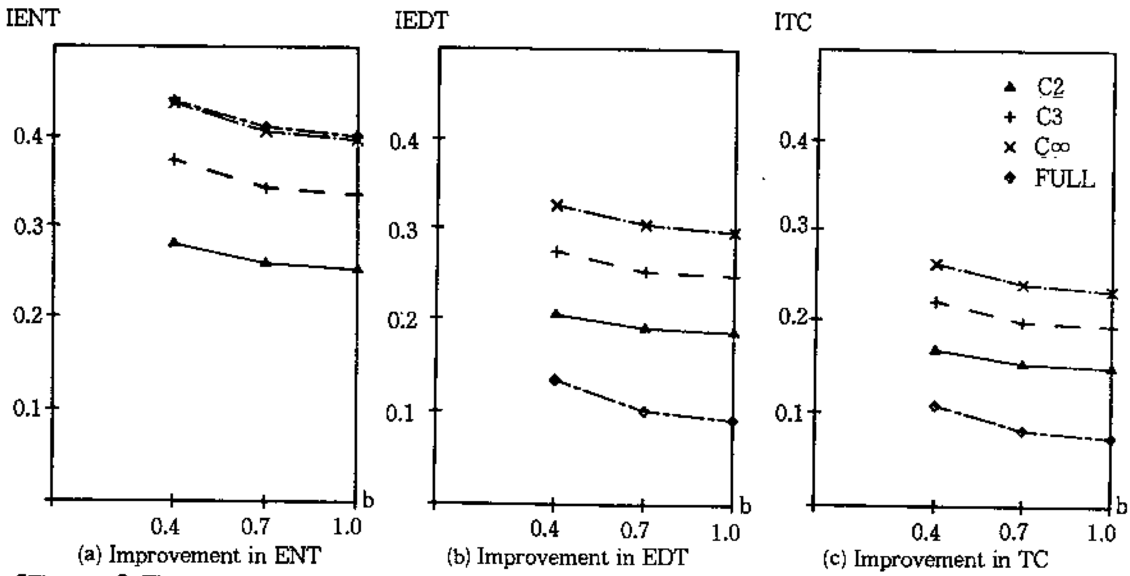
travel times result in yielding larger lot sizes, which implies larger warehouse capacity is required.

In any case of the experimental results, IR_{FULL} is observed to be less than or equal to $IR_{c\infty}$. Also, for the completely unskewed distribution (i.e., s=1.0), IR_{FULL} is always nonpositive valued. These observations are depicted in Figure 6 where the average IRs for different values of s are given.

(4) As the value of b is increasing, IENT, IEDT, and ITC for each policy are all decreasing, which reflects the theoretical result that potential gains in travel times over RAN become minimum when the rack is square in time. Figure 7 represents this observation. From the figure we can find that as far as total costs are concerned, C∞ yields the best results, whereas FULL does the worst.



[Figure 6] The effect of s on the percentage increment in total lot sizes which are averaged for all b, k, h, and c.



[Figure 7] The effect of b on the percentage improvement in travel times and total costs for each policy, which are averaged for all s, k, h, and c.

(5) Finally, the effects of changes in other parameters of k, c, and h on the relative policy performance over RAN are summarized in

〈Table 3〉 Effects of changes in parameter values on the relative performance of each storage policy over RAN.

parameter	performance index			
(↑)	IR	IENT	IEDT	ITC
s	↓	↓	↓	↓
b	↓	↓	↓	↓
k	*	=	=	=
h	*	=	=	*
c	*	=	=	*

* There exist significant differences but no trend is found.

= There exists no significant differnece.

4. Conclusions

Significant reductions in inventory control and material handling costs of an AS/RS are obtainable by jointly considering storage assignment and production lot sizing. Assuming the geometric function type of lot-size distribution, this paper have formulated the joint problem and drawn some obseravtions from computaional experience. The results obtained by solving the models developed in this paper might be viewed as an achievement of a more desirable valance between throughput and storage capacity. However, the assumptions of continuous rack and continuous demend curve must make the current results be incomplete to a certain degree.

As a topic of further research, it is recommended to extend the models by including storage and retrival operation of the S/R machine.

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Appendix

Proof of PROPERTY 1 :

Let $TC_{RAN}(R, a) = g_1(R, a) + g_2(R)$ where $g_1(R, a)$ is readily shown to be convex and positive on $\{(R, a) \mid R>0, 0<a<1+s\}$.

Since g_1 can be represented by the product of two single-variable functions which are positive for all feasible values of (R, a),

$g_{11}(R) = kAs/R$ and $g_{12}(a) = (a(1+s-a))^{-1}$,

the problem can be decomposed as

$$\underset{R,a}{\text{Min}}\ TC_{RAN}(R, a) = \underset{R}{\text{Min}}(g_{11}(R) \cdot \underset{a}{\text{Min}}(g_{12}(a)) + g_2(R)) \quad \text{(A11)}$$

Since $g_{12}(a)$ itself is convex on $0<a<1+s$, the optimal value of a, a^*_{RAN}, which minimizes $g_{12}(a)$ and in turn TC_{RAN}, should satisfy the following necessary and sufficient condition

$$\frac{\partial g_{12}(a)}{\partial a} = ((1+s-a)^{-2} - a^{-2})/(1+s) = 0, \quad \text{(A12)}$$

Hence, solving (A12) for a gives the result

$a^*_{RAN} = (1+s)/2$.

Now, (A11) becomes a single-variable optimization problem by substituting a^*_{RAN} into TC_{RAN} such that

$$\underset{R}{\text{Min}}\ TC_{RAN}(R, a^*_{RAN}) = e_1R^{1/2} + e_2R + e_3/R$$

where $e_i>0$, $i=1, 2, 3$.

It follows from PROPERTY 1 given in Hodgson and Lowe[4] that there exists exactly one point, $R = R^*_{RAN} > 0$ such that

$$R\frac{\partial TC(R, a^*_{RAN})}{\partial R} = e_1R^{1/2}/2 + e_2R - e_3/R = 0.$$

Thus, (R^*_{RAN}, a^*_{RAN}) is a single point that minimizes TC_{RAN}.

Proof of PROPERTY 2 :

Let $TC_{FULL}(R, a) = g_{11}(R) \cdot g_{12}(a) + e_2R + e_4 \cdot g_{31}(R) \cdot g_{32}(a)$ where $e_4 = 4cAsV_1$; $g_{31}(R) = R^{1/2}$; and

$g_{32}(a) = b^{s+(1+a)/2}/(s+a/2) + (1-b^{s+a})/(s+a)$.

Since g_{11}, g_{12}, g_{31}, g_{32}, e_2, and e_4 are all positive on $\{(R, a) \mid 0<R\le A, 0<a<1+s\}$, and TC_{FULL} is the sum of the products of single variable functions, in order to minimize TC_{FULL} both $g_{12}(a)$ and $g_{32}(a)$ should be minimized as far as possible.

Form the fact that $a^*_{RAN} = (1+s)/2$ minimizes $g_{12}(a)$ which is convex, and $g_{32}(a)$ is strictly decreasing on $0<a<1+s$, we know that an optimal value of a, a^*_{FULL}, must be located somewhere between $(1+s)/2$ and $1+s$. With this and the relation that $s \le (1+s)/2$ for $0<s \le 1$, the following always holds :

$$a^*_{FULL} \ge s \quad \text{(A21)}$$

Substituting a^*_{FULL} into the expression for $\beta(i)$ gives

$$\beta(i) = 2a(i)/q(i) = 2si^{s-1}/Ra^*_{FULL}\, i^{a^*_{FULL}-1} = (2s/Ra^*_{FULL}\, i)^{s-a^*_{FULL}-1}, \quad 0<i\le 1.$$

It follows from (A21) that $\beta(i)$ is nonincreasing for $0<i\le 1$.