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The Storage Space Versus Expected Travel Time of Storage Assignment Rules in Automated Warehousing System

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Abstract

To compute the expected travel time of storage and retrieval (S/R) machine in automated warehousing systems most of the previous studies assumed that equal number of rack openings are required regardless of the nature of storage assignment rules. It is known that randomized storage assignment rule usually needs less storage to space than needed for full turnover-based assignment rule. The objective of this paper is compute the expected travel time of each assignment rule more euqitably by taking into account the storage space required for each rule. First, the rack storage space is determined which satisfies a given service level. Then based on the standard Economic Ordering Quantity model, trade-off analysis is carried out which relates the storage space to the expected travel time of the S/R machine. Finally, example problems are solved to compare the performance of each assignment rule under varying conditions of demand pattern and service level.

1. Introduction

A scheduling problem in computer-directed warehousing systems using storage/retrieval (S/R) machine and palletized loads is how to assigning incoming pallet loads to storage locations. Three kinds of storage assignment rules are frequently appearing in the literature. They are randomized storage assignment (RAN), full turnover-based assignment (FULL) and class-based assignment (CLASS). Under RAN, the storage location is chosen randomly from all open rack locations. This rule has been used to approximate the performance of the closest-open-location rule which is widely used in practice. FULL has an opposite concept compared to RAN. For this rule, the highest-turnover item is assigned to the location closest to the input/output (I/O) point where the turnover of an item is the reciprocal of the item's length of stay in the storage rack. CLASS is a compromise between RAN and FULL. Under

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CLASS, the items and the rack locations are ranked according to turnover and distance (in travel time) from the I/O point, respectively. These ranked lists are then partitioned into two or three classes such that the class of items with the highest turnover is assigned randomly within the class of locations closed to the I/O point, etc.

Assuming the rack is square in time, Graves, Hausman and Schwarz [2, 3, 5] studied each assignment rule and calculated the percentage improvement is S/R machine travel time from FULL over the other rules. Later, Bozer and White [1] presented the travel time expression relaxing the assumption. However, all theses studies are based on the identical rack sizes regardless of storage rules, whereas RAN usually requires less storage space than required for FULL. It means the previous results tend to underestimate (overestimate) the S/R machine travel time in case of FULL (RAN).

Hence, in this paper, we determine the storage space required to satisfy a given service level under each assignment rule and then present a trade-off model of storage space and travel time. With the model developed, the storage rules are compared on the more equitable basis. Also, the effects of demand pattern and service level are studied on the performance of each rule.

Throughout this paper, the following assumptions are introduced:

Assumptions:

1. Each pallet can hold only one item.
2. All rack openings are of the same size, as are the pallets themselves, Therefore, all storage locations are candidates for storing a pallet load.
3. The I/O point is located at the lower left-hand corner.
4. The S/R machine operates either on a single or dual command.
5. The S/R machine speed in the horizontal and vertical directions, are uniform and known.
6. The S/R machine can travel simultaneously in the horizontal and vertical directions.

2. Determination of storage space

Suppose that n items are stored in the automated warehouse whose ordering policy follows the economic ordering quantity (EOQ) rule. It is assumed that the turnover rate of each item is known and the item number is indexed such that the turnover rate i is greater than or equal to that of item i+1. Let D_i be the demand rate (pallets per unit time) of item i and Q_i the ordering quantity. Then,

$Q_i = (2CD_i)^{1/2}$ where C is the ratio of ordering cost to holding cost which is assumed constant for all items.

Let X_1, X_2, ⋯, X_i, ⋯, X_n be the random variables which denote the inventory levels of each item in pallets. Then X_i follows the Uniform distribution, i.e., $X_i \sim U(0, Q_i)$, $i=1, \cdots, n$, from the assumption of EOQ.

In what follows, for each storage rule we are going to determine the required storage space which is large enough to accommodate the incoming pallet loads at least $(1-\alpha)\times 100$

percent of time where $0<\alpha<1$. $(1-\alpha)$ is a desired level of protection against shortage of empty rack openings which can be interpreted as service level used in the inventory related literatures.

2.1 Storage space under RAN

We first consider the required storage space s(RAN, α) under randomized rule and a given value of α.

If n is sufficiently large,

$$Z=\frac{\sum_{i=1}^{n} X_i-\sum_{i=1}^{n} Q_i/2}{(\sum_{i=1}^{n} Q_i^2/12)^{1/2}}$$ follows approximately

the standard Normal distribution by Lindberg-the Central Limit Theorem [4].

Thus, S(RAN, α) can be represented as

$$S(RAN,\alpha)=\sum_{i=1}^{n} Q_i/2+Z_\alpha(\sum_{i=1}^{n} Q_i^2/12)^{1/2} \quad (1)$$

where $\alpha=Pr(\sum_{i=1}^{n} X_i>S(RAN,\alpha))$ and Z_α is determined by $Pr(Z>Z_\alpha)=\alpha$.

2.2 Storage space under FULL

Since X_i follows $U(0, Q_i)$, let $\beta=Pr(X_i>(1-\beta)Q_i)$ where β is the probability that an item i cannot be stored in the previously designated rack openings. Following the operating characteris-tics of full turnover-based storage assignment rule, α can be expressed as

$$\alpha=1-\prod_{i=1}^{n} Pr(X_i\le(1-\beta)Q_i)$$
$$=1-(1-\beta)^n \quad (2)$$

Then, the storage space at α under full turnover-based rule becomes

$$S(FULL,\alpha)=\sum_{i=1}^{n}(1-\beta)Q_i$$
$$=(1-\alpha)^{1/n}\sum_{i=1}^{n} Q_i. \quad (3)$$

2.3 Storage space under C2

Suppose that item 1, 2, ⋯, k are stored in class Ⅰ and item k+1, k+2, ⋯ n in class Ⅱ. Let S(C2, α) denote the storage space of total rack at α under 2-class-based rule and S(C2, γ_I and S(C3, γ_{II}) be the spaces of class Ⅰ at γ_I, $0<\gamma_I<1$, and class Ⅱ at γ_{II}, $0<\gamma_{II}<1$, repectively where γ_I and γ_{II} are defined as follows:

$$\gamma_I=Pr(\sum_{i=1}^{k} X_i>S(C2,\gamma_I)) \quad (4)$$

and

$$\gamma_{II}=Pr(\sum_{i=k+1}^{n} X_i>S(C2,\gamma_{II})). \quad (5)$$

Applying the procedure of finding S(RAN, α), we can obtain

$$S(C2,\gamma_I)=\sum_{i=1}^{k} Q_i/2+Z_{\gamma_I}(\sum_{i=1}^{k} Q_i^2/12)^{1/2} \quad (6)$$

and

$$S(C2,\gamma_{II})=\sum_{i=1}^{n} Q_i/2+Z_{\gamma_{II}}(\sum_{i=k+1}^{n} Q_i^2/12)^{1/2} \quad (7)$$

where Z_{γ_I} and $Z_{\gamma_{II}}$ are determined by

$P_r(Z>Z_{\gamma_I})=\gamma_I$ and $P_r(Z>Z_{\gamma_{II}})=\gamma_{II}$

assuming k and (n−k) are sufficiently large. Hence, S(C2, α) is obtained by

$$S(C2,\alpha)=S(C2,\gamma_I)+S(C2\ \gamma_{II}) \quad (8)$$

$$\text{where } \alpha=1-(1-\gamma_I)(1-\gamma_{II}) \quad (9)$$

3. The continuous reprsentation

Treating item indices i as continuous on the interval [0, 1] and represention the discrete rack as a square, Hausman et al. [3] showed that demand rate D(i) of item i becomes $D(i)=D_T s i^{s-1}$ where D_T is the total pallet demand per unit time and s is parameter of the "ABC" curve, $0<s\leq 1$. It follows that

$$Q(i)=(2D_T C s i^{s-1})^{1/2} \tag{10}$$

Here, we can convert approximately $\sum_{i=1}^{n} Q_i$ to $\int_0^1 Q(i)\,di$ if n is sufficiently large since $\sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} Q(i/n)\,(1/n)$. Also, $\sum_{i=1}^{n} Q_i^2$ can be approximated by $\frac{1}{n}\int_0^1 Q^2(i)\,di$ under large n since

$$\sum_{i=1}^{n} Q_i^2=\sum_{i=1}^{n} Q^2(i/n)\,(1/n)^2=\frac{1}{n}\sum_{i=1}^{n} Q^2(i/n)\,(1/n) \tag{11}$$

Thus, S(RAN, α) and S(FULL, α) can be expressed approximately as

$$\begin{aligned} S(RAN,\alpha) &= \int_0^1 Q(i)/2di + Z_\alpha\left(\frac{1}{n}\int_0^1 Q^2(i)/12di\right)^{1/2} \\ &= (2CsD_T)^{1/2}/(s+1) + Z_\alpha (CD_T/6n)^{1/2} \end{aligned} \tag{12}$$

and

$$S(FULL,\alpha)=(1-\alpha)^{1/n}\,(8CsD_T)^{1/2}/(s+1). \tag{13}$$

To facilitate comparisons it is assumed that the number of aisles, N, is identical for all storage assignment rules and each rack is square in time. Then, the storage volumes of a rack under each storage assignment rule, V (RAN, α), V(FULL, α) and V(C2, α), can be obtained by dividing S(RAN, α), S(FULL, α) and S(C2, α) by 2N. For both single and dual command operations, rack which is square in time gives a minimum expected travel time under RAN (shown by Bozer and White [1]).

Now, consider 2-class-based storage assignment rule. The optimal partitioning value, R, is already given by Hausman et al. [3]. Thus, since the item type of R^2 th pallet is $R^{4/(s-1)}$, V (C2, γ_I) and V(C2, γ_{II}) can be represented as follows:

$$\begin{aligned} V(C2\cdot\gamma_I) &= \frac{1}{2N}\left\{\int_0^{R^{4/(s+1)}} Q(i)/2\,di + Z_{\gamma_I}\left(\frac{1}{n}\int_0^{R^{4/(s-1)}} Q^2(i)/12\,di\right)^{1/2}\right\} \\ &= \frac{1}{2N}\{R^2(2CsD_T)^{1/2}/(S+1) + Z_{\gamma_I}(CD_T/6n)^{1/2}\,R^{2s/(s-1)}\} \end{aligned} \tag{14}$$

and

$$V(C2\cdot\gamma_{II})=\frac{1}{2N}\{(1-R^2)(2CsD_T)^{1/2}/(S+1) + Z_{\gamma_{II}}(CD_T/6n)^{1/2}(1-R^{4s/(s+1)})^{1/2}\}. \tag{15}$$

Thus,

$$V(C2\ \alpha)=\frac{1}{2N}[(2CsD_T)^{1/2}/(s+1) + (CD_T/6n)^{1/2}\{Z_{\gamma_I}R^{2s/(s+1)} + Z_{\gamma_{II}}(1-R^{4s/(s+1)})^{1/2}\}] \tag{16}$$

Also, since $\frac{V(C2,\gamma_I)}{V(C2,\gamma_{II})}=\frac{R^2}{1-R^2}$, from(14) and (15)

$$Z_{\gamma_I}/Z_{\gamma_{II}}=\frac{R^2(1-R^{2s/(s+1)})^{1/2}}{(1-R^2)R^{2s/(s+1)}}. \tag{17}$$

Then, we can find Z_{γ_I} and $Z_{\gamma_{II}}$ from (9) and (17) by using Zelen and Severos' approximation [6].

Now, consider the rack structure under each storage assignment rule when the storage volumes of a rack, V(RAN, α), V(FULL, α) and V(C2, α) are given. Assuming that N_H and N_L, which is defined by numbers of loads high and long in pallets, respectively, are real values, we can calculate N_H and N_L from (18) and (19) because of the rack squared in time.

$$\frac{N_H\, d}{V_H} = \frac{L_L\, d}{V_L} \tag{18}$$

$$N_H\, N_L = V(\cdot) \tag{19}$$

Where d = height of a rack opening which is assumed to be a cube,

V_L = horizontal velocity of S/R machine,

V_H = vertical velocity of S/R machine,

$V(\cdot)$ = the storage size of a rack under any storage assignment rule.

4. Trade-off analysis of travel time and storage space

So far, we have determined the storage volume sizes when missing probability and storage assignment rule are chosen. Next, we analyze expected travel time of S/R machine when the storage volumes are given for each storage rule.

The expected travel time of S/R machine under randomized rule, t(RAN), is already known in the standard rack, the rack squared in time with 1. Also, we can intuitively identify the travel times under full turnover-based and 2-class-based rules, t(FULL) and t(C2), with the results of Hausman et al. [3] and Graves et al. [2]. Thus, the expected travel times of S/R machine can be easily computed under each storage assignment rule.

Thus, The expected travel times of S/R machine, T(RAN), T(FULL) and T(C2), under randomized, full turnover-based and 2-class-based storage assignment rules with the rack sizes of V(RAN, α), V(FULL, α) and V(C2, α), respectively, become from (18) and (19)

$$T(RAN) = \frac{N_H\, d}{V_H} t(RAN), = \left(\frac{V(RAN, \alpha)}{V_H\, V_L}\right)^{1/2} d\, t(RAN), \tag{20}$$

$$T(FULL) = \left(\frac{V(FULL, \alpha)}{V_H\, V_L}\right)^{1/2} d\, t(FULL) \tag{21}$$

and

$$T(C2) = \left(\frac{V(C2, \alpha)}{V_H\, V_L}\right)^{1/2} d\, t(C2). \tag{22}$$

Now, in order to analyze the relation between T(RAN), T(FULL) and T(C2), $\frac{T(RAN)}{T(FULL)}$ and $\frac{T(C2)}{T(FULL)}$ should be investigated. From (20)–(22), we can see

$$\frac{T(RAN)}{T(FULL)} = (1-\alpha)^{-1/2n}\{0.5 + Z_\alpha \frac{s+1}{(48ns)^{1/2}}\}^{1/2} \frac{t(RAN)}{t(FULL)} \tag{23}$$

and

$$\frac{T(C2)}{T(FULL)} = (1-\alpha)^{-1/2n}[0.5 + \frac{s+1}{(48ns)^{1/2}} \{Z_{\gamma_I} R^{2s/(s+1)} + Z_{\gamma_{II}} (1-R^{4s/(s+1)})^{1/2}\}]^{1/2} \tag{24}$$

Example : Let's consider the problem with n = 1000, α = 0.01, 0.05, 0.1, 0.2, and S = 0.065, 0.139, 0.222 and 0.318. Let t(· , S) and t(· , D) be the expected travel times of S/R machines with the normalized rack under single and dual commands, respectively. t(RAN, S), t(RAN, D), t(FULL, S), t(FULL, D), t(C2, S) and t(C2, D) are known in the previous studies [2, 3]. We can easily compute the values of

$\frac{T(RAN, S)}{T(FULL, S)}, \frac{T(C2, S)}{T(FULL, S)}, \frac{T(RAN, S)}{T(C2, S)},$

$\frac{T(RAN, D)}{T(FULL, D)}, \frac{T(C2, D)}{T(FULL, D)}$ and $\frac{T(C2, D)}{T(RAN, D)},$

where T(· ,S) and T(· ,D) are the expected travel times of S/R machine with the real storage space under single and dual commands, respectivel.

In Table 1, Z_{γ_I} and $Z_{\gamma_{II}}$ are shown for the various values of α and s. From Table 2 and 3, it can be seen that the relation T(RAN, ·)>T(C2, ·)>T(FULL, ·) holds when α and s are relatively small. As α and s increase, C2 tends to perform better compared to the other rules. Since FULL requires the alomost same storage space regardless of α, s seems to have more significant effects than α on FULL. On the other hand, RAN needs substantially less storage space as α increases.

5. Conclusion

For each storage assignment rule, we determine the rack storage size satisfying a given service level of protection against shortages. Then based on the required rack size the expected travel time of S/R machine is calculated. Through an example, comparisons are made among the storage rules in terms of the travel time and the effects of α and s are studied. The following observations are made:

1) In general, 2-class-based rule performs better than the other two rules.

2) Full turnover-based rule is better only when the value of s is fairly small (e.g., less than 0.1).

3) The performance of randomized rule is better than that of full turnover-based rule when the value of s is somewhat large (e.g., larger than 0.25).

Note that the previous studies reported that full turnover-based storage assignment rule gives a minimum expected travel time. Based on a more equitable comparison, we find that in general class-based storage assignment rule with small number of classes performs better.

〈Table 1〉 The values of Z_{γ_I} and $Z_{\gamma_{II}}$, Note that* and** indicate the values of Z_{γ_I} and $Z_{\gamma_{II}}$, respectively.

α / s	0.01	0.05	0.1	0.2
0.065	1.248*	1.146	1.018	0.763
	51.598	47.746	42.430	31.799
0.139	2.326	1.645	1.282	0.842
	28.720	20.309	15.822	10.390
0.222	2.326	1.645	1.282	0.842
	15.205	10.751	8.736	5.501
0.318	2.326	1.645	1.282	0.842
	9.942	7.029	5.477	3.599

〈Table 2〉 Results of example under single command. Note that *, ** and *** indicate $\frac{T(RAN, S)}{T(FULL, S)}$, $\frac{T(C2, S)}{T(FULL, S)}$ and $\frac{T(RAN, S)}{T(C2, S)}$, respectively.

α / s	0.01	0.05	0.1	0.2
0.065	2.4982*	2.4681	2.4520	2.4324
	1.6671**	1.6329	1.5847	1.4834
	1.4985***	1.5115	1.5474	1.6398
0.139	1.4839	1.4706	1.4634	1.4548
	1.1607	1.0959	1.0597	1.0141
	1.2785	1.3419	1.3810	1.4346
0.222	1.1508	1.1419	1.1372	1.1315
	0.9516	0.9190	0.9011	0.8789
	1.2093	1.2426	1.2621	1.2874
0.318	0.9842	0.9773	0.9737	0.9692
	0.8591	0.8384	0.8272	0.8135
	1.1456	1.1656	1.1770	1.1915

〈Table 3〉 Results of example under dual command.
Note that,* ** and *** indicate $\frac{T(RAN, D)}{T(FULL, S)}$, $\frac{T(C2, D)}{T(FULL, D)}$ and $\frac{T(RAN, D)}{T(C2, D)}$, respectively.

α / s	0.01	0.05	0.1	0.2
0.065	2.0434*	2.0817	2.0055	1.9895
	1.5705**	1.5383	1.4928	1.3974
	1.3011***	1.3123	1.3434	1.4237
0.139	1.2618	1.2504	1.2444	1.2370
	1.0800	1.0197	0.9860	0.9436
	1.1683	1.2263	1.2620	1.3109
0.222	1.0295	1.0216	1.0173	1.0122
	0.9045	0.8734	0.8564	0.8354
	1.1382	1.1696	1.1879	1.2117
0.318	0.9119	0.9056	0.9022	0.8981
	0.8303	0.8104	0.7996	0.7863
	1.0983	1.1175	1.1283	1.1422

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