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A Simplified Fatigue-Damage Model for the Comparison of the Randomness in Load and Strength.

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Abstract

The only property of the random stresses to affect the life length are their damage amount and are independent of the especial realization in stress case. This paper shows the result is discussed that the randomness of the life length is caused by the randomness of the strength rather than by the randomness of the stress when the load is a random function and the strength is random as well. This special model is well-suited model for comparative calculations since it connects fatigue life for random stresses to fatigue life for periodically curve loads, which is usually measured in experiments.

1. Introduction

There exist several complicated method giving different results. In this paper two commonly used principles together imply the existence of a rather simple model which connects the life lengths when arbitrary loads are ap-

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plied, to the life lengths when curve loads are applied.

The time invariance assumption, commonly used that the damage caused by a stress depends only on the sequence of extremes of the load and not on the special form of the load function between its extreme values. This principle is satisfied when load cycles and amplitudes are recorded by means of the famous rain flow count [6].

The palmgren-Miner linear damage accumulation rule says that a unit fails when the total damage exceeds a prescribed value. The total damage is the sum of the damages caused by each load which has been applied to the unit. The sum does not depend on the order of the loads.

This paper gives that these two principles imply the existence of a simple model which determines the life length of a unit completely. The main feature of the model is the existence of an exhaustion function which is determined by the damage caused by periodically curve loads.

The function connects the damages by periodically curve loads to the damages by a load varying in an arbitrary way.

The arrangement and compose of this paper is followed as; section 2 is considered loads S which are random functions $\{S(t) : t \in R\}$ and we also discuss the case when the random strength is included in the model in section 3. Finally in section 4, we discuss the analytical example and case.

Notation

c,c(s)	: Crossing density function
F	: Fatigue distribution
f	: Fatigue density function
g	: Freguency of load S.
Ms	: Damage functional distribution
M(t)	: Damage function of time t
m	: Damage intensity
P	: Increasing and continuous time
Q	: Spectral distribution(Spectrum)
q	: Spectral density
S, Sk, S(t)	: Load distribtion

T	: Life length – time
V	: Variance
Y(gauf)	: Gaussian random function(Normal random)
γ, ϕ	: Nonnegative random variables

2. Damage and Fatigue Density Function in the Random Stress(Load)

There are various patterns of load variation with respect to time. That is, constant load, cyclic load with constant amplitude, and random load with spectrum. In general these are subjected to a complicated pattern of randomly varying load amplitudes and frequencies. For example, the load in an aircraft created by atmospheric gusts, and the loads produced in a surface vehicle's suspension unit by random irregularity of road surfaces.

Even when different units are subjected to the same fluctuating stress, they fail at different cycles because of nonhomogenuity in material proportion, variation in surface conditions, etc. To predict the average life of a unit, a number of test specimens are tested at various stress levels until failure.

2.1. Damage and Fatigue Density

Let a load function S be any piecewise continuous real function $\{S(t): t \in R\}$. When introducing the time in variance property of the damage it is convenient that the load function S(t) is defined for all real t.

The palmgren–Miner rule implies that –

$$M_{s1}+S_2=M_{s1}+M_{s2},\ \text{if } S_1(t)\cdot S_2(t)=0 \ni t \quad \cdots\cdots (Z-1)$$

Let p be a increasing and continuous time transformation which satisfies –

$$\lim_{t\to\infty} P(t)=\infty \text{ and } \lim_{t\to-\infty} P(t)=-\infty$$

Then the time invariance property states that –

$M_1 = M_{s2}$ when $S_2(t) = S_1(P(t))$, $t \in R$. ……(Z-2)

This time invariance property is stated and used in rain flow count of load cycls [1], [8].

It is further assumed that only stress changes cause to fatigue :

$M_s = 0$, if S is constant. ……(Z-3)

and that total damage caused by S can be approximated by the damage caused on a bounded time interval. This means that−

$$\lim_{t\to\infty} M_{s.1}[-n : n] = M_s \quad \text{……(Z-4)}$$

$$S \cdot 1_{[-n:n]}(t) = \begin{cases} S(t), & t \leq n \\ 0, & \text{otherwis.} \end{cases}$$

Theorem 1)

$$M_s = \sum_{K=-\infty}^{+\infty} \{F_+[S(T_k)] - F_+[S(t_k)] + F_-[S(t_{k+1})] - F_-[S(T_k)]\} \quad \text{……(1)}$$

$$R = \bigcup_{K=-\infty}^{+\infty} [t_k ; t_{k+1}]$$

S is increasing on $[t_k ; T_k]$ and decreasing on $[T_k ; t_{k+1}]$.

⟨Proof⟩

By using the equation (Z−1) and (Z−4)

$$M_s = \sum_{K=-\infty}^{+\infty} M_{s.1}{}^{[tk;Tk]} + \sum_{K=-\infty}^{\infty} M_{s.1}{}^{[Tk;tk+1]}$$

increasing and continuous time trans p_k such that−

$P_k(W) = S^{-1}(W)$ for $S(T_k) \geq W > S(t_k)$

Then by(Z−2)/ $M_{Uk} = M_{Sk}$. Define D_k for intervals of decrease of S.

Therefore

$$M_s = \sum_{K=-\infty}^{+\infty} M_{Uk} + \sum_{K=-\infty}^{+\infty} M_{Dk}$$

Then by(Z−1)

$M_w \cdot 1_{(a,b]} = F_+(b) - F_+(a), a<b,$

$Mw \cdot 1_{(-a,-b]} = F_-(b), -F_-(a), a>b,$

Hence,

$$M_s = \sum_{K=-\infty}^{+\infty} M_{Wk} + \sum_{K=-\infty}^{+\infty} M_{Dk}$$

$$= \sum_{K=-\infty}^{+\infty} \{F_+[S(T_k)] - F_-[S(t_k)] + F_-[S(t_{k+1})] - F_-[S(T_k)]\}.$$

The functions F_- and F_+ are unique if it is assumed that

$F_-(0) = F_+(0) = 0.$

The S has local minimum at t_k and local maximum at T_k.

If the function F_+, F_- and S are absolutely continuous, (1) can be written as

$$M_s = \int_{s'(t)>0} f_+[S(t)]S'(t)dt + \int_{s'(t)<0} f_-[S(t)]S'(t)dt \quad \cdots\cdots(2)$$

When $F_+$and F_- in(1) are absolutely continuous−

$$M_s = \int_{-\infty}^{\infty} n_+(S)\, f_+(S)ds \quad \int_{-\infty}^{\infty} n_-(S)\, f_-(S)ds \quad \cdots\cdots(3)$$

where $n_+(S)$ is the number of upcrossings and $n_-(s)$ is the number of downcrossings of the level S by the function S.

Remark 2

In the symmetric case on the same demage in both

$f_1 = -f_- = f$ and

$$(S)M_s=\int_{-\infty}^{\infty} f[S(t)]\ |\ S'(t)\ |\ dt \quad \cdots\cdots (4)$$

This is that a formulation (3) is valid when F_- and F_+ are not continuous.

In the nonsymmetric case for stress function S satisfying

$$\lim_{t\to-\infty} S(t)=\lim_{t\to\infty} S(t)$$

We can appear as :

$$n(S)M_s=\int_{-\infty}^{\infty} f[S(t)]\ |\ S'(t)\ |\ dt$$

$$f(S)=[f_+(S)-f_-(S)]/2$$

Therfore, Formulation(3) reduces to

$$M_s=\int_{-\infty}^{\infty} n(S)f(S)\ ds \quad \cdots\cdots (5)$$

$$n(S)=n_+(S)+n_-(S)$$

2.2. Random load in the Stress Level

Loads S are random functions $\{S(t) : t\in R\}$ when the units are exposed to irregular loads in their natural environment. When the fatigue density f is fixed, the results are also valid as conditional results given the random fatigue density $\{f(S) : S\in R\}$ of the studied units.

$$M(t)=M_s \cdot 1_{(0,t]}=\int_0^t f[S(u)]\ |\ S'(u)\ |\ du,\ t>0$$

and the fatigue life length --

$$T=\inf\{t\geq 0 : M(t)=1\}$$

When S is an ergodic random function the damage intensity is defined as－

$$m(f)=_{t\to\infty}\{M(t)/t\}=\lim_{t\to\infty}\{t^{-1}\int_0^t f[S(u)]\,|\,S'(u)\,|\,du\}$$

The ergodicity implies that this limit exists and eguals the ensemble average with respect to the distribution of the random variable S(t).

We can write－

$$m(f)=E\{f[S'(t)]\,|\,S'(t)\,|\,\,|\,f\} \quad\cdots\cdots(6)$$

If S(t) and S'(t) are statistically independent Gaussian random variables. S is a Gaussian

$$m(f)=E\{f[S(t)]\,|\,f\}\,E\{\,|\,S'(t)\,|\,\}=(2\lambda_2/\pi)^{1/2}\,E\{f[S'(t)]\,|\,f\}$$

$$\lambda_2=E\{[S'(t)]^2\}$$

$$\lambda_0=\mathrm{Var}\{S'(t)\}\ \text{and}\ \mu=E\{S'(t)\},$$

Hence,

$$E\{f[S(t)]\,|\,f\}=(2\pi\lambda_0)^{-1/2}\int_{-\infty}^{\infty}f(S)\exp[-(2\lambda_0)^{-1}\,(s-\mu)^2]ds$$

So,

$$m(g)=(\lambda_2/\lambda_0)^{1/2}\,\pi^{-1}\int_{-\infty}^{\infty}f(S)\exp[-(2\lambda)^{-1}(s-\mu)^2]ds \quad\cdots\cdots(7)$$

Here $(\lambda_2/\lambda_0)^{1/2}/(2\pi)$is the mean frequency of S, and where Q is the spectral distribution of S and q is the spectral density of S when the spectrum is continuous.

$$(\lambda_2/\lambda_0)^{1/2}/(2\pi)=[\lambda_0^{-1}\int_{-\infty}^{\infty}g^2mQ(g)]^{1/2}=[\lambda_0^{-1}\int_{-\infty}^{\infty}g^2q(g)dg]^{1/2}$$

Theorem 3

Let C(s) be the crossing intensity of the ergodic random function S, that is, S is a stationary process and

$$C(s)=\int_{-\infty}^{\infty} | X | 1(s,x)dx.$$

Then the damage intensity m(f) is obtained –

$$m(f)=\int_{-\infty}^{\infty} f(S)c(S)ds \quad \cdots\cdots (8)$$

(proof)

Using the equation(5) and (6),

$$m(g)=E\{f[S(u)] | S'(u) | \; | f\}=E\{\int_0^1 f[S'(u)] | S'(u) | du | f\}$$

Hence,

$$E\{M(1) | f\}=E\int_{-\infty}^{+\infty} n(s)f(s)\, ds | f\}=\int_{-\infty}^{\infty} c(s)f(s)ds$$

Whre S is ergodic and n(s) denotes the number of s–crossing by S in (0,1) in accordance with Equation(6).

3. Random Strength for Fatigue Life Length

The load function and the density F determine the life length uniquely. In real world life lengths vary between units of the same kind. This means that {f(s) : S∈R} has to be interpreted as a random independent function. As usual this function may not be catched directly. The only available observation is the resulting life length of the unit.

The fact that the life lengths can vary considerably shows that the random variation between units may not be omitted in the model. On the other side, the model must be simple. Let's suppose that

$f(s)=\bar{f}(S)/\gamma$, S∈R, Where $\bar{f}$ is a random fixed function and γ is a positive random variable.

Now, let T be the life length of a unit

$$T=\inf\{t\geq 0 : \int_0^t f[S(u)] | S'(u) | du=1\}=\inf\{t\geq 0 : \int_0^t \bar{f}[S(u)] | S'(u) | du=\gamma\}$$

For a deterministic period load function S with period g^{-1} the damage per time unit is

$$\bar{m}_s = g\int_0^{g^{-1}} \bar{f}[S(t)]\,|S'(t)|\,dt$$

When the deterministic fatigue density $\bar{f}$ is used.

The life length T of the unit with fatigue density $\bar{f}/\gamma$ satisfies $T=\gamma\bar{m}_s$.

I many measurements of fatigue life it is observed that for a high amplitude of the load function all units will fail before the maximal testing time, however, for a lower amplitude of the load function only some units will fail, and for a very low amplitude, none of the units will fail within the maximal testing time. Generally the maximal testing time is very large and thus it is reasonable to suppose that the fatigue life is infinite for some units, when they are exposed to a load function with a low amplitude. This behaviour can easily be described by a stress scale change model. For a deterministically periodic load function S with period g－i, the damage per time unit is

$$m_s = g\int_0^{g^{-1}} \bar{f}[S(t)/\Phi]\,|S'(t)|\,dt,$$

here, $f(s)=\bar{f}(s/\Phi)$. Hence the life length

T is :

$T=1/m_s$.

Let's suppose that the function

$\bar{f}(S')=0$ for $S_1 \le S \le S_2$, where $S_1 < 0 < S_2$ satisfies that $\underset{t\ge 0}{\text{minimum}}\{S(t)\} < 0 < \underset{t\ge 0}{\text{minimum}}(S'(t)\}$.

$$M_s = g\int_0^{g^{-1}} \bar{f}[S(t)/\phi]\,|S'(t)|\,dt=0$$

if $S_1 < \phi^{-1}\min S(t)$ and $\phi^{-1}\max S(t) < S_2$.

Hence the life length T is approached infinite if $\phi > \max(S_1^{-1}\min S(t), S_2^{-2}\max S(t))$.

That is, if we want the life length with high probability, the max $|S(t)|$ should be small and also if max $|S(t)|$ is large, it has low probability.

4. Example for Analytical Solution

Let the spectral density function $q_1(g)$ be the 1 when the frequency of S_1 with mean zero is $|g| \leq 0.5$ and 0 when otherwise. Then, in the random normal load function,

$$V\{S_1(t)\} = \int_{-\infty}^{\infty} q_1(g)dg = 1$$

$$V\{S_1'(t)\} = \int_{-\infty}^{\infty} (2\pi)^2 g^2 q_1(g)dg = \pi^2/3 = 3.287$$

By using the equation (7), the damage amount and intensity is

$$m_1(f) = (\pi^2/3)^{1/2}/\pi \cdot \int_{-\infty}^{\infty} \exp(-s^2/2) \cdot f(s)ds = 1.74 \int_{-\infty}^{\infty} \exp(-s^2/2)f(S)ds$$

Meanwhile, The spectrum q_2 of S_2 is following as :

$$q_2(g) = \begin{cases} 0, & \text{otherwise} \\ 0.8, & |g| \leq 0.5 \\ 0.2, & 2 \leq |g| \leq 2.5 \end{cases}$$

$$V\{S_2'(t)\} = (6.28)^2 \cdot 1.08 = 42.6$$

$$M_2(f) = 2.08 \cdot f(s) \exp(-S^2/2)ds.$$

Comparing S_1 with S_2 in the different spectrum, q_2 of S_2 includes higher frequencies than q_1 of S_1. And T_2/T_1 is $(1/13)^{1/2}$, that is 27.92%. The mean frequency of S_2 is $(13)^{1/2} = 3.6$ times higher than the mean frequency of S_1.

This means to have decreased the life length with same condition. Hence the random variation in T is generated by the randomness of the strength rather than by the randomness of the load since the load process affects the life length only through the damage intensity m(f). Theorem 3 gives an alternative way on calculating m(f) by means of the crossing intensity C(S).

5. Conclusion

There were many tools and methods to predict life lengths. But, the relationship between the life lengths of units (components) exposed to reqularly cyclic loads and the life lengths of units(components) — exposed to irregularly cyclic loads is not obvious.

This paper has considered loads which are random functions and also the case of the random strength.

A simple model to handle fatigue — Damage life length problems was considered. As a result, the randommess in the life is generated by the random strength and not by the random load. The property of the random load in palmgren — Miner rule which affects the life is its damage intensity, which is independent of the particular realization. The dammage intensity is even independent of the distribution of random function, provided its level crossing intensity is known.

The model discussed in this paper is well — suited for comparative calculations under different load conditions since it connects fatigue — damage life for random loads to fatigue life for cyclic loads, which is usually measured in experiments.

Finally, we really hope that this suggested model in the irregular loads(by the randomness of strengths rather than loads) would by well — used and applied in the real industry field.

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