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FUZZY FAULT TREE ANALYSIS

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Abstract

Conventional fault tree analysis has several problems as the estimations and tolerances of the failure probability values. To overcome these problems, fuzzy concepts with natural language can be applied to conventional fault tree analysis. And, we propose the evaluation method of the imprecision of top/basic events and possibility importances of basic events.

1. Introduction

Fault tree analysis(FTA) is a technique of reliability and safety analysis and generally applicable to complex dynamic systems. FTA provides an objective basis for analyzing system design, performing trade-off studies, analyzing common mode failures, demonstrating compliance with safety requirements, and justifying system

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change and additions. A fault tree is a model that graphically and locally represents the various combinations of possible events, both faulty and normal. occurring in a system that lead to the top undesired event. Lee et al.(1985) summarized the results of FTA.

Probabilistic FTA have the following several problems.

First, for conventional FTA it is necessary to collect lots of data to estimate the probability values of the occurrences of basic events. However, in practice, it is not likeiy that enough data can be collected to estimate these values. So these values are estimated by experts based on their engineering judgement. Second, conventional fault tree and reliability analysis do not give a picture concerning the tolerances of the probability values of hazards. Third, conventional FAT is not applied to in case of describing occurrences of basic events as linguistic variables. Therefore, fuzzy set considerations can be used for FTA. We will replace probabilistic considerations by possibilistic ones and diminish the difficulties arising from the uncertain knowledge of the distribution funtions of basic events.

Onisawa(1988, 1990, 1991a, 1991b), Onisawa and nishiwaki(1988) showed applications of fuzzy concepts to modelling of reliability analysis. Cai et al. (1990, 1991a, 1991b) developed the theory of fuzzy reliability in the possibility context. Furuta and Shiraishi(1984) studied about fuzzy importance in fault tree analysis and Singer (1990) developed a fuzzy set approach to fault tree and reliability analysis. Gmytrasiewicz, Hassberger, and Lee(1990) proposed fault tree based diagnostics using fuzzy logic. Many books provide the fuzzy concepts[Dubois and Prade(1980), Kaufmann and Gupta(1988), and Zimmermann(1991), etc.].

2. Logical Operator

Supposing that the events are mutually exclusive, the reliability function of the system can be obtained by replacing all independent events $A_i(1, 2, \cdots, n)$ by probabilities(relative frequencies of thier occurences) and substituting the logical AND and OR operators by the algebraic multiplication and addition.

The probability function by the n-ary AND operator is

$$P_{AND}(T) = AND(p_1, P_2, \cdots, P_n) = \prod_{i=1}^{n} Pi \quad \cdots\cdots (1)$$

Where the pi's denote the probabilities of the input events Ai and $p_{AND}(T)$ the probability of the output event. The probability function by the n-ary OR operator is

$$P_{OR}(T) = OR(p_1, P_2, \cdots, P_n) = 1 - \prod_{i=1}^{n} (1-Pi) \quad \cdots\cdots (2)$$

Where the pi's denote the probabilities of the input events Ai and $P_{OR}(T)$ the probability of the output event.

3. Fuzzy Number in [0, 1]

A fuzzy number is a fuzzy subset in R Which is both 'normal' and 'convex'.

There is an infinite set of fuzzy numbers. Singer[1990] used L-R fuzzy numbers for fuzzy fault tree. Also flat fuzzy numbers[Dubois and Prade(1979)] can be used。 But、 here we will define a special class of fuzzy numbers called triangular fuzzy numbers(T。 F。 N。) and trapezoidal fuzzy numbers(Tr。 F。 N。) for easy operation。 A T。 F。 N。 Can be define by a triplet(a、 b、 c) Where b is the recommended value of the occurrence of a basic event、 a is its lower bound、 and c is its upper bound。 For the T。 F。 N。、 the membership function is defined as

$$\mu_A(X) = \begin{cases} 0 & , x < a \\ \dfrac{x-a}{b-a} & , a \le x \le b \\ \dfrac{c-x}{c-b} & , b \le x \le c \\ 0 & , x > c. \end{cases}$$

Algebraic operations on T. F. N. are as following.

(i) Addition :

$$A(+)B = (a_1, b_1, c_1)(+)(a_2, b_2 c_2) \quad \cdots\cdots (3)$$
$$= (a_1+a_2, b_1+b_2, c_1+c_2), \text{ a T. F. N.}$$

(ii) Subtraction :

$$A(-)B = (a_1, b_1, c_1)(-)(a_2, b_2 c_2) \quad \cdots\cdots (4)$$
$$= (a_1-c_2, b_1-b_2, c_1-a_2), \text{ a T. F. N.}$$

(iii) Multiplication :

The result of fuzzy multiplication A(·)B of two T.F.N.s A and B is not T.F.N.. However, we can approximate the $C=A(\cdot)B=(a_1, b_1 c_1)(\cdot)(a_2, b_2 c_2)$ by a T. F. N. given by $D=(a_1a_2, b_1b_2, c_1c_2)$.

$$A(\cdot)B = (a_1, b_1, c_1)(\cdot)(a_2, b_2 c_2) \simeq (a_1a_2, b_1b_2, c_1c_2). \quad \cdots\cdots (5)$$

A Tr. F. N. can be defined by a quadruplet (a, b, c, d). For the Tr. F. N., the membership function is characterized as

$$\mu_A(X) = \begin{cases} 0 & , x<a \\ \frac{x-a}{b-a} & , a\le x\le b \\ 1 & , b\le x\le c \\ \frac{d-x}{d-c} & , c\le x\le d \\ 0 & , x>d. \end{cases}$$

Algebraic operations on Tr. F. N. are as following.

(i) Addition :

$$A(+)B = (a_1, b_1, c_1, d_1)(+)(a_2, b_2, c_2, d_2) \quad \cdots (6)$$
$$= (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2), \text{ a T. F. N.}$$

(ii) Subtraction :

$$A(-)B = (a_1, b_1, c_1, d_1)(-)(a_2, b_2, c_2, d_2) \quad \cdots (7)$$
$$= (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2), \text{ a T. F. N.}$$

(iii) Multiplication :

$$A(\cdot)B = (a_1, b_1, c_1, d_1)(\cdot)(a_2, b_2, c_2, d_2) \simeq (a_1a_2, b_1b_2, c_1c_2, d_1d_2). \quad \cdots (8)$$

4. Fuzzy Operator and Linguistic Variables

The possibility function of the fuzzy n-ary AND and OR operators can be obtained considering events in (1) and (2) as fuzzy events and replacing the algebraic operations by fuzzy operations given in (3) – (5) or (6) – (8).

The possibility function of the fuzzy n–ary AND operator is

$$\tilde{p}_{AND}(T) = AND(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_n) = \tilde{\prod_{i=1}^{n}} \tilde{p}_i \quad \cdots (9)$$

Where $\tilde{\Pi}$ denotes fuzzy multiplication. With T. F. N., expression (9) can be writtn as

$$\tilde{p}_T(X) = (a_1, b_1, c_1)(\cdot)(a_2, b_2, c_2)(\cdot)\cdots(\cdot)(a_n, b_n, c_n) \quad \cdots (10)$$

Expression (10) can be written in the recursive from as the following algorithm.

Algorithm 1. (the possibility function of the fuzzy n−ary And operator)

INPUT T. F. N. s (a_i, b_i, c_i), $i=1, 2, \cdots, n$

OUTPUT T. F. N. $\tilde{p}_T(X)$

For $i=1, 2, \cdots, n-1$ do

$\tilde{p}_{i+1}(X)=\tilde{p}_{i}(X)(\cdot)(a_{i-1}, b_{i+1}, c_{i+1})$

where $\tilde{p}_1(X)=(a_1, b_1, c_1)$

Set $\tilde{p}_T(X)=\tilde{p}_n(X)$

The possibility function by the fuzzy n−ary OR operator is

$$\tilde{p}_{OR}(T)=OR(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_n)=1-\tilde{\prod_{i=1}^{n}}(1-\tilde{p}_i). \qquad (11)$$

With T. F. N., expression (11) can be written as

$$\begin{aligned}\tilde{p}_T(X) = (1, 1, 1)-[((1, 1, 1)-(a_1, b_1, c_1))\\ (\cdot)(1, 1, 1)-(a_2, b_2, c_2))\\ (\cdot)\cdots(\cdot)((1, 1, 1)-(a_n, b_n, c_n))] \qquad (12)\end{aligned}$$

Expression (12) can be written in the recursive from as the the following algorithm.

Algorithm 2. (the possibility function of the fuzzy n-ary OR operator)

INPUT T. F. N. s (a_i, b_i, c_i), $i=1, 2, \cdots, n$

OUTPUT T. F. N. $\tilde{p}_T(X)$

For $i=1, 2, \cdots, n-1$ do

$\tilde{p}_{i+1}(X)=\tilde{p}_{i}(X)(\cdot)((1, 1, 1)-(a_{i+1}, b_{i+1}, c_{i+1}))$

where $\tilde{p}_1(X)=(1, 1, 1)-(a_1, b_1, c_1)$

Set $\tilde{p}_T(X)=1-\tilde{p}_n(X)$

To supplement the binary state of a basic event, several intermediate levels which describe reliability can be used as expert's judgements. For example, the state of a basic event can be classified by one of the following terms : 'small', 'medium', 'large' like Figure 2. Then, T. F. N.s or Tr. F. N.s can be used to describe the state of a basic event assosiated with each linguistic terms.

5. Imprecision of Top Event and Possibilty Importance

The evaluation of imprecision associated with fuzzy number of top event or basic events is an important topic. As index of fuzziness, we can take energy index defined as

$$k_f(A) = \int_{a_1^{(0)}}^{a_2^{(0)}} f(\mu_A)(X))dx$$

Where $A_0 = [a_1^{(0)}, a_2^{(0)}]$ is the interval of confidence Which corresponds to $\alpha = 0$ in fuzzy number A, f(t) and f'(t) are monotonically increasing functions with reapect to t, and $k_f(A)$ satisfies the following property :

$$A_1 \subseteq A_2 \implies k_f(A_1) \leq k_f(A_2)$$

The simple from of an energy function is

$$k_2(A) = \int_{a_1^{(0)}}^{a_2^{(0)}} (\mu_A)(X))^2 dx$$

In FTA, the concept of importance is used to evaluate how far a basic event contributes to the top event. An importance analysis is useful for the design and assessment of systems. Usually the values of importance are computed by use of the probability information as the occurrence probabilities of basic events and the failure probability of the top event. However, if the state of a basic event is expressed by a fuzzy event, it may be impossible to discuss the importance of each event on the basis of only probability information. Furuta and Shiraishi[1984] suggest fuzzy importance using max/min fuzzy operator and fuzzy integral. we propose possibility importance as an alternative of structural importance or reliability importance. Let

$\tilde{p}_T(1i)$ = possibility function of top event
When the basic event A_i is occurring

$\tilde{p}_T(0i)$ = possibility function of top event
When the basic event A_i is not occurring

Then, possibility importance is defined by

$$\tilde{p}_T(i) = \tilde{p}_T(1i)(-)\tilde{p}_T(0i).$$

structural importance is based on a knowledge of the structure of the system only and reliability importance takes into account component reliabilities. But, possibility importance considers the tolerance of the probability values of the occurrences of basic events.

possibility importance is fuzzy number. Hence, possibility importance can be ordered by linear ordering with the following criteria.

(i) the removal : $r(A) = 1/2(r_l(A) + r_r(A))$

where left side removal $r_l(A)$ is defined as the area bounded by y-axis in possibility function plane and the left side of the fuzzy number A and right side removal $r_r(A)$ is defined similarly

(ii) the mode

(iii) the divergence

In the case of a T. F. N., the removal is given by

$\hat{A} = (a + 2 * b + c)/4$, where T. F. N. a=(a, b, c),

the mode is b, and the divergence is c−a, Of course, there are many methods comparing fuzzy numbers[Bortolan and Degani(1985), Chen(1985), McCahon and Lee(1990)]. But, we select criteria as we haved said above for easy operation.

6. Numerical Example

Suppose a fault tree as Figure 1. The basic events contributing to the accident to are summarized in Table 1. Which Contains linguistic Variable with respect each basic event. Figure 2 gives the membership functions for linguistic variables.

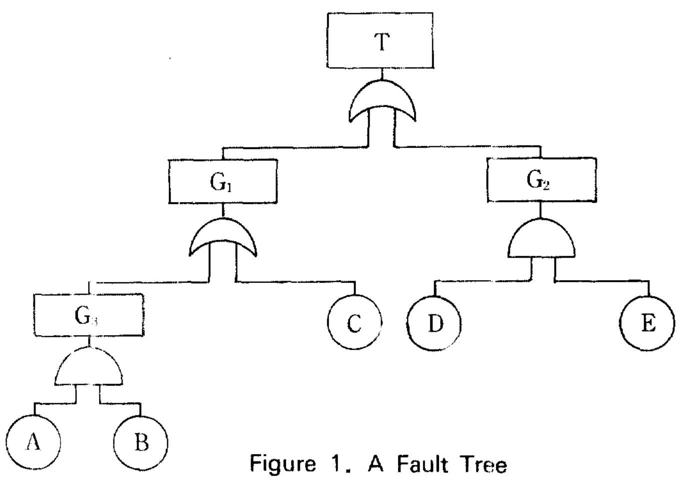

Figure 1. A Fault Tree

Table 1. Basic Events with Linguistic Variables

Basic events	Linguistic variables
A	small
B	medium
C	small
D	large
E	medium

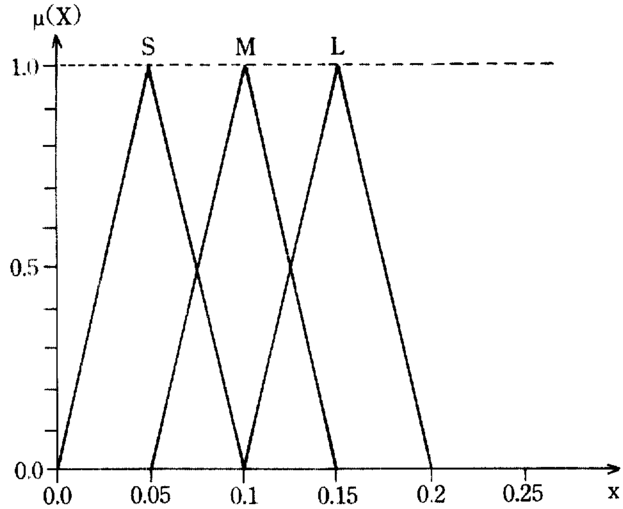

Figure 2. The Membership Functions for Linguistic Variables
(S : Small, M : Medium, L : Large)

The calculation of the possibility function of the top event is as the following :

$\tilde{p}_{G3} = AND(\tilde{p}_A, \tilde{p}_B) = (0.000, 0.005, 0.015)$
$\tilde{p}_{G1} = OR(\tilde{p}_{G3}, \tilde{p}_C) = (0.000, 0.055, 0.114)$
$\tilde{p}_{G2} = AND(\tilde{p}_D, \tilde{p}_E) = (0.005, 0.015, 0.030)$
$\tilde{p}_T = OR(\tilde{p}_{G1}, \tilde{p}_{G2}) = (0.005, 0.069, 0.140)$

Figure 3 shows the possibility function of top event. Index of fuzziness are k_2(basic event)=0.0332 and k_2(top event)=0.0450. Table 2 displays removal of each basic event.

Table 2. Removal of Basic Events

Basic event	$\tilde{P}_T(1i)$	$\tilde{P}_T(0i)$	Removal
A	(0.055, 0.158, 0.258)	(0.005, 0.064, 0.127)	0.092
B	(0.005, 0.111, 0.214)	(0.005, 0.064, 0.127)	0.090
C	(1.000, 1.000, 1.000)	(0.005, 0.020, 0.045)	0.978
D	(0.050, 0.149, 0.246)	(0.000, 0.055, 0.114)	0.093
E	(0.100, 0.197, 0.291)	(0.000, 0.055, 0.114)	0.140

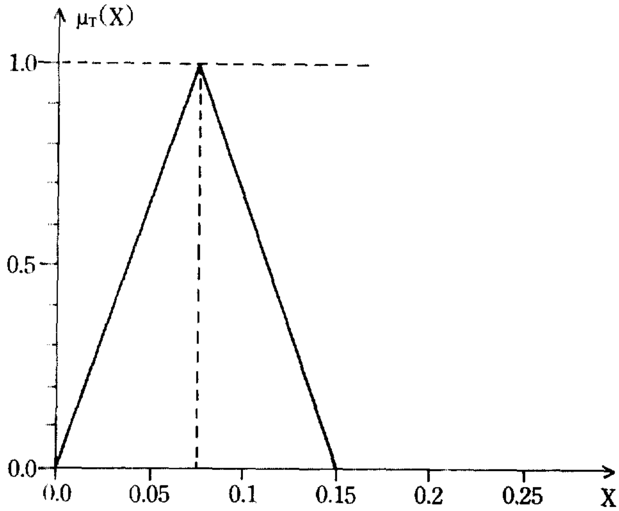

Figure 3. The Possibility Functions of Top Event

From Table 2. we can make a possibility importance ordering as C〉E〉D〉A〉B.

For reference, structural importance ordering is as following :

C(9/32) 〉A=B=C=D(3/32).

7. Concluding Remarks

Fuzzy fault tree analysis can provide possibilitic considerations under environments which many vague terms are found in the qualitative expression. The further study could be done to develop sensitivity analysis of failure and error possibilities in fuzzy tree analysis and consider dependency between events.

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