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The Designed and Simplified Markov Models for Systems Based on Kronecker Algebra

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Abstract

The Purpose of this paper contribute to design the multistate Markov process for the reliability of a system when the transition-rates of each unit depend on the current state of the system. The system transition-rate matrix has the form of the kronecker sum of transition rate matrices for the units, is analyzed and investigated. As a result, the system which has the state-dependent units is detaily analyzed and introduced by the approach of an algorithm for the system transition-rate matrix based on the kronecker algebra.

1. Introduction

Many aspects of the dynamic probabilistic behaviour of systems can be analyzed more easily by this approach especially when more than two states are allowed for each unit and the transition rates between each pair of states depend on the state of other units, on the state of the system, the numerical difficulties associated with the large transition-rate matrices involven in the Markov models of large system have been reduced

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by using a systematic ordering of the system states. An alternative approach has been developed which Kronecker algebra to obtain the state transition matrix[7].

For a continuous-time homogeneous Markov model, such description is the state transition-rate matrix $\wedge$ of the system. An efficient tool for setting $\wedge$ from the matrices $\wedge_1, \cdots\cdots, \wedge_N$ which correspond to individual units, is Kronecker algebra[5] because $\wedge$ equals the kronecker sum of $\wedge_1, \cdots\cdots, \wedge_N$ if the units are s-independent[6]. This paper discusses the generalization of the result to the case that state-transition rates of units depend upon the current states of other units, viz, on the state of the system. As a consequence, different state-transitions of the system can take place for s-dependent and s-independent units. On the other hand, such generalization is admissible also in stand-by redundent models, where the state spaces of units should be formed in a due fashion.

This paper is organized as follows :

Section II briefly reviews the foundations of Kronecker algebra, analyzes and investigates the transition rate matrix of each unit to the transition-rate matrix of the system, and simply explains the structural properties of this matrix. Section III introduces an algorithm for setting-up the state-dependence transition rate matrix is an effective tool, and Section IV has the example for numeric substitution about section III.

* Assumption and Notation

① The system is described by a continuous-time Markov model.

② The transition-rates of each unit at time t can depend on the state of other units at t, on the state of the system.

(1) $\wedge$	: state-transition-rate matrix of the system
(2) $\wedge(N)$	: $\bigoplus_{k=1}^{N} \wedge_k$
(3) $\wedge_d$	: trnsition-rate matrix of the system in the s-dependent case
(4) $\wedge_k$	: state-transition-rate matrix of Uk
(5) $\wedge_{s:k}$	: sxs transition-rate matrix obtained from $\wedge_k$
(6) N	: number of units of the system
(7) $\otimes$	: Kronecker product
(8) $\oplus$	: Kronecker sum
(9) Im	: m $\times$ m identity matrix
(10) ϕ_k	: number of states of unit
(11) U_N	: N units of the system
(12) S	: $\prod_{k=1}^{N} \phi_k$: number of states of the system
(13) T	: $\sum_{k=1}^{N} \phi_k$
(14) $\lambda_{k:ij}$	: rate of a state-transition $j \rightarrow i$ generic element of $\wedge_K$ (i and j)
(15) Z_K	: $\{1 ; \cdots, \phi_K\}$

(16) Z : $Z_1 \times Z_2 \times Z_3 \times \cdots \times Z_N$ (Cartesian Product)

(17) $Z(I)$: $(Z_1(I), \cdots Z_N(I)) \in Z$ the N-tuple of states of units corresponding to a state I of the system.

(18) W_k : $\sum_{n=1}^{k-1} \phi_n$, for $k \in \{1 ; \cdots ; N\}$

(19) d_{k-1} : $\prod_{i=1}^{k-1} \phi_i$, for $2 \leq k \leq N$, and $d_1 \phi 1$

(20) d_{k+1}' : $\prod_{i=k+1}^{N} \phi_i$, for $1 \leq k \leq n-1$ and $d_{N+1}\phi 1$

(21) In : index of states of the system

(22) i_k : index of states of unit k

2. State Transition Matrix of the System Using Kronecker Algebra

In a Markov model, the system behaviour is described by a set of linear homogenous differential equations (by a homogenous continuous-time discrete-ststе Markov process). The matrix can be obtained from the Λ_k $(k=1, 2, \cdots, N)$ means of Kornecker algebra. Let A and B be the matrices of order $m+n$ and $p \times q$, respectively. The Kronecker Product of two matrices $A(m+n)$ and $B(p+q)$ is the $mp \times nq$ matrix.

$$A \otimes B \equiv \begin{matrix} A_{11}B & A_{12}B \cdots\cdots A_{1n}B \\ A_{21}B & A_{22}B \cdots\cdots A_{2n}B \\ \vdots & \vdots \quad\quad \vdots \\ A_{m1}B & A_{m2}B \cdots\cdots A_{mn}B \end{matrix}$$

The Kronecker sum of $A(m \times m)$ and $B(q \times q)$ is definited by :

$$A \oplus B \equiv A \otimes I_q + I_m \otimes B \quad \cdots\cdots (2)$$

The properties of Kronecker algebra are well described in [5], [6]. :

for s-independent system $\Lambda = \Lambda_1 \oplus \Lambda_2 \oplus \cdots \oplus \Lambda_N = \bigoplus_{k=1}^{N} \Lambda_k = \oplus \Lambda_k$ $\cdots\cdots$ (3)

- $A \otimes B \neq B \otimes A$
- $A \oplus B \neq B \oplus A$
- $(A \otimes B) \otimes C = A \otimes (B \otimes C)$
- $(A \oplus B) \oplus C \neq A \oplus (B \oplus C)$

[Example 1]

$\wedge_1$ and $\wedge_2$ has 2−state :

$$\wedge_1 = \begin{bmatrix} \lambda_{1:11} & \lambda_{1:12} \\ \lambda_{1:21} & \lambda_{1:22} \end{bmatrix} \qquad \wedge_2 = \begin{bmatrix} \lambda_{2:11} & \lambda_{2:12} \\ \lambda_{2:21} & \lambda_{2:22} \end{bmatrix}$$

$$\wedge(2) = \wedge_1 \otimes \wedge_2$$

$$= \begin{matrix} \lambda_{1:11}+\lambda_{2:11} & \lambda_{2:12} & \lambda_{1:12} & 0 \\ \lambda_{2:21} & \lambda_{1:11}+\lambda_{2:22} & 0 & \lambda_{1:12} \\ \lambda_{1:21} & 0 & \lambda_{1:22}+\lambda_{2:11} & \lambda_{2:12} \\ 0 & \lambda_{1:21} & \lambda_{2:21} & \lambda_{1:22}+\lambda_{2:22} \end{matrix} \quad \cdots\cdots(4)$$

Each element $\lambda_{k:ij}$ appears twice in the matrix $\wedge(2)$, for each i, j, $k \in \{1 ; 2\}$.
In the general case, the $S \times S$ matrix $\wedge(N)$ contains each element $\lambda_{k:ij}$ S_k−times, for $k \in \{1 ; \cdots ; N\}$ and i, $j \in \{1 ; \cdots ; \phi_k\}$.
Fixing a unit U_{k0} of the system.

$$S_{k0} = \prod_{\substack{k=1 \\ k \neq k0}}^{N} \phi_k$$

There are combinations of possible states of U_k, for $k \neq k_0$. The values $\lambda_{k0:ij}$ in the matrix $\wedge(N)$ are state transition rates $j \rightarrow i$ of U_{k0} corresponding to those S_{k0} possible states of the rest of the system.

[Example 2]

$$\wedge_1 = \begin{bmatrix} \lambda_{1:11} & \lambda_{1:12} \\ \lambda_{1:21} & \lambda_{1:22} \end{bmatrix}$$

$$\wedge_2 = \begin{bmatrix} \lambda_{2:11} & \lambda_{2:12} & \lambda_{2:13} \\ \lambda_{2:21} & \lambda_{2:22} & \lambda_{2:23} \\ \lambda_{2:31} & \lambda_{2:32} & \lambda_{2:33} \end{bmatrix}$$

$$\wedge_3 = \begin{bmatrix} \lambda_{3:11} & \lambda_{3:12} \\ \lambda_{3:21} & \lambda_{3:22} \end{bmatrix}$$

If performing the Kronecker Sum in the sequence 1. 2. 3 is obtained ;

$$\wedge(3) = \wedge_1 \oplus \wedge_2 \oplus \wedge_3$$
$$= (\wedge_1 \otimes I_3 + I_2 \otimes \wedge_2) \oplus \wedge_3$$

$$= (\Lambda_1 \otimes I_3 + I_2 \otimes \Lambda_2) \otimes I_2 + I_6 \otimes \Lambda_3$$
$$= \Lambda_1 \otimes I_3 \otimes I_2 + I_2 \otimes \Lambda_2 \otimes I_2 + I_6 \otimes \Lambda_3$$
$$= \Lambda_1 \otimes I_6 + I_2 \otimes \Lambda_2 \otimes I_2 + I_6 \otimes \Lambda_3$$

and if sequence 2. 1. 3. is required ;

$$\Lambda(3) = \Lambda_2 \oplus \Lambda_1 \oplus \Lambda_3$$
$$= (\Lambda_2 \otimes I_2 + I_3 \otimes \Lambda_1) \oplus \Lambda_3$$
$$= (\Lambda_2 \otimes I_2 + I_3 \otimes \Lambda_1) \otimes I_2 + I_6 \oplus \Lambda_3$$
$$= \Lambda_2 \otimes I_4 + I_3 \otimes \Lambda_1 \otimes I_2 + I_6 \otimes \Lambda_3$$

Equation (3) can be rearranged as the usual algebraic matrix sum [7] :

$$\Lambda = \Sigma_k (I_{dk-1} \otimes \Lambda_k \otimes I_{dk+1}) = \Sigma_k \Lambda_{s:k} \qquad \cdots (5)$$

The generic term $\Lambda_{s:k}$, associated with the single component k, has order :

$$\Pi_k \phi_k = d_{k-1} \otimes \phi_k \otimes d_{k+1} = S \qquad \cdots (6)$$

The sum(5) can be performed in two steps :

$$\Lambda'_k = I_{dk-1} \otimes \Lambda_k \qquad \cdots (7)$$

Which, be definition of Koronecker product, is a square matrix of order $\phi_k d_{k-1}$ formed by d_{k-1} diagonal blocks, each equal to Λ_k.

Theorem 1 : The diagonal elements of Λ_k are projected into diagonal elements of Λ. The diagonal elements of Λ are obtained only from diagonal elements of each Λ_k[3]

Theorem 2 : The mapping $M : \Lambda_k \rightarrow \Lambda$ generates overlapping only on the main diagonal of Λ. We define $|Z-t|$ as the distance of the element of entrices Z, t from the main diagonal of Λ. Let $Z=[i+(\delta-1)\phi_{k-1}]d_{k+1}+W$, $t=[j(\delta-1)\phi_{k-1}]d_{k+1}+W$, with $\delta \in \{1, \cdots, d_{k-1}\}$ and $W \in \{1, \cdots, d_{k+1}\}$.

$$S_0, \quad |Z-t| = d_{k+1}|i-j| \qquad \cdots (8)$$

Where $|i-j|$ is the distance of element $\lambda_{k:ij}$ from the main diagonal of Λ_k.

3. Setting-up the State-Dependence Transition Rate Matrix

The state-transition rate matrix Λ_d of the system composed of s-dependent units must differ from the Krinecker sum $\Lambda(N)$ at least in such a way that multiple repetition of a paticular $\lambda_{k:ij}$, which has been observed in $\Lambda(N)$, can disapppear in case of Λ. In other word, When the transition rates of each unit at time t depend on the state of the other units the resulting state-transition rate matrix A_d is different from that corresponding to the s-independence case only in the values of some of its non-null elements, but the same structure as Λ. This assertion is obviously equivalent to the relation.

Let $i_k \in \{i; \cdots; \phi_k\}$, for $k=1, \cdots, n$.

The state of the system determined by the N-tuple$(i_1; \cdots; i_N) \in Z$ of states of its units corresponds to column I and row I of $\Lambda(N)$:

$$I = 1 + \sum_{k=1}^{N} (ik-1) \prod_{m=k+1}^{N} \phi_m \quad \cdots\cdots (9)$$

This formulation is invertible but the explicit inverse expression is not given.

Let $I \in \{1; \cdots; S\}$ Denote $Z(I) \equiv (Z_1(I), \cdots, Z_N(I))$

the element of Z such that the equality

$$I = 1 + \sum_{k=1}^{n} (Z_k(I)-1) \prod_{m=k+1}^{N} \phi_m \quad \cdots\cdots (10)$$

is fulfilled. Let's take arbitrary I, $J \in \{1; \cdots; S\}$, $I \neq J$. Element(I, J) of the matrix M is positive if and only if there exists a unique $K_0 \in \{1; \cdots; N\}$such that :

$Z_{k0}(I) \neq Z_{k0}(J)$,
$Z_k(I) = Z_k(J)$, for each $k \in \{1; \cdots; N\}$, $k \neq k_0$

In other words, each non-zero, off-diagonal element of M corresponds to a state-transition between states of the system determined by such N-tuples of states of the units that differ exactly in a single element.

* Example Matrix for Algorithm

$N=2$, $\phi_1=2$, $\phi_2=3$ and state-transition rate matrices $\Lambda^{k(N)}$ are :

$$\overset{(1)}{\Lambda}_1 = \begin{matrix} \lambda_{1:11}^{(1)} & \lambda_{1:12}^{(1)} \\ \lambda_{1:21}^{(1)} & \lambda_{1:22}^{(1)} \end{matrix}$$

$$\overset{(2)}{\Lambda}_1 = \begin{matrix} \lambda_{1:11}^{(2)} & \lambda_{1:12}^{(2)} \\ \lambda_{1:21}^{(2)} & \lambda_{1:22}^{(2)} \end{matrix}$$

$$\overset{(3)}{\Lambda}_1 = \begin{matrix} \lambda_{1:11}^{(3)} & \lambda_{1:12}^{(3)} \\ \lambda_{1:21}^{(3)} & \lambda_{1:22}^{(3)} \end{matrix}$$

$$\overset{(1)}{\Lambda}_2 = \begin{matrix} \lambda_{2:11}^{(1)} & \lambda_{2:12}^{(1)} & \lambda_{2:13}^{(1)} \\ \lambda_{2:21}^{(1)} & \lambda_{2:22}^{(1)} & \lambda_{2:23}^{(1)} \\ \lambda_{2:31}^{(1)} & \lambda_{2:32}^{(1)} & \lambda_{2:33}^{(1)} \end{matrix}$$

$$\overset{(2)}{\Lambda}_2 = \begin{matrix} \lambda_{2:11}^{(2)} & \lambda_{2:12}^{(2)} & \lambda_{2:13}^{(2)} \\ \lambda_{2:21}^{(2)} & \lambda_{2:22}^{(2)} & \lambda_{2:23}^{(2)} \\ \lambda_{2:31}^{(2)} & \lambda_{2:32}^{(2)} & \lambda_{2:33}^{(2)} \end{matrix}$$

Thus $S = \phi_1 \cdot \phi_2 = 2 \times 3 = 6$

$T = \phi_1 + \phi_2 = 2 + 3 = 5$

$W_1 = 0$, $W_2 = 2$

In step 2, $A_{(1)}$ and $A_{(2)}$ have the form ;

$$A^{(1)} = \begin{matrix} a_{11}^{(1)} & 1+W_1 \\ 2+W_1 & a_{22}^{(2)} \end{matrix}$$

$$A^{(2)} = \begin{matrix} a_{11}^{(2)} & 1+W_2 & 1+W_2 \\ 2+W_2 & a_{22}^{(2)} & 2+W_2 \\ 3+W_2 & 3+W_2 & a_{33}^{(2)} \end{matrix}$$

Hence, $A^{(k)}=(a_{ij}^{(k)})$ is of order $\phi_k\times\phi_k$, for each $k\in\{1;\cdots;N\}$, the elements $a_{ij}^{(k)}$ of Which are :

$a_{ij}^{(k)}=i+W_k$, for $i, j\in\{1;\cdots;\phi_k\}$, $i\neq j$,
if $i=j$, $a_{ij}^{(k)}=i-(\phi_k-1)W_k-1/2\phi_k(\phi_k+1)$.

Let $A\equiv(a_{ij})$ be the Kronecker sum $\overset{N}{\underset{k=1}{\oplus}} A^{(K)}$.

$$A=A^{(1)}\oplus A^{(2)}=\begin{array}{cccccc} \overset{(1)}{a_{11}}+\overset{(2)}{a_{11}} & 1+W_2 & 1+W_2 & 1+W_1 & 0 & 0 \\ 2+W_2 & \overset{(1)}{a_{11}}+\overset{(2)}{a_{22}} & 2+W_2 & 0 & 1+W_1 & 0 \\ 3+W_2 & 3+W_2 & \overset{(1)}{a_{11}}+\overset{(2)}{a_{33}} & 0 & 0 & 1+W_1 \\ 2+W_1 & 0 & 0 & \overset{(1)}{a_{11}}+\overset{(2)}{a_{22}} & 1-W_2 & 2+W_2 \\ 0 & 2+W_1 & 0 & 2+W_2 & \overset{(1)}{a_{11}}+\overset{(2)}{a_{33}} & 2+W_2 \\ 0 & 0 & 2+W_1 & 3+W_2 & 3+W_2 & \overset{(1)}{a_{11}}+\overset{(2)}{a_{33}} \end{array}$$

$$B=\begin{array}{cccccc} \overset{(1)}{\lambda}_{1:11} & \overset{(2)}{\lambda}_{1:11} & \overset{(3)}{\lambda}_{1:11} & \overset{(1)}{\lambda}_{1:11} & \overset{(2)}{\lambda}_{1:11} & \overset{(3)}{\lambda}_{1:11} \\ \overset{(1)}{\lambda}_{1:21} & \overset{(2)}{\lambda}_{1:21} & \overset{(3)}{\lambda}_{1:21} & \overset{(1)}{\lambda}_{1:21} & \overset{(2)}{\lambda}_{1:21} & \overset{(3)}{\lambda}_{1:21} \\ \overset{(1)}{\lambda}_{1:11} & \overset{(1)}{\lambda}_{1:12} & \overset{(1)}{\lambda}_{1:13} & \overset{(2)}{\lambda}_{1:11} & \overset{(2)}{\lambda}_{1:12} & \overset{(2)}{\lambda}_{1:13} \\ \overset{(1)}{\lambda}_{2:21} & \overset{(1)}{\lambda}_{2:22} & \overset{(1)}{\lambda}_{2:23} & \overset{(2)}{\lambda}_{2:21} & \overset{(2)}{\lambda}_{2:22} & \overset{(2)}{\lambda}_{2:23} \\ \overset{(1)}{\lambda}_{2:31} & \overset{(1)}{\lambda}_{2:32} & \overset{(1)}{\lambda}_{2:33} & \overset{(2)}{\lambda}_{2:31} & \overset{(2)}{\lambda}_{2:32} & \overset{(2)}{\lambda}_{2:33} \end{array}$$

Hence, B is the matrix $T\times S$, column by column λ as follows : Fix a $J\in\{1,\cdots;S\}$ and $Z(J)=(i_1,\cdots,i_n)$ column J. contains the values of $\lambda_1:1i_1$, $\lambda_1:2i_1$, ……, $\lambda_1:\phi_1 i_1$, ……, $\lambda_N:\phi_N i_n$. And $\lambda_{k0:ij}$ means the numerical value of element(i, j) of the state-transition-rate matrix of Uk_o under condition Uk_o is in state i_k, for each $k\in\{1;\cdots;N\}$, $k\neq k_o$. It remains to assemble the $5(T)\times 6(S)$ matrix B determined above.

$$C=\begin{array}{cccccc} \overset{(1)}{\lambda}_{1:11}+\overset{(1)}{\lambda}_{2:11} & \overset{(1)}{\lambda}_{2:12} & \overset{(1)}{\lambda}_{2:13} & \overset{(1)}{\lambda}_{2:12} & 0 & 0 \\ \overset{(1)}{\lambda}_{2:21} & \overset{(2)}{\lambda}_{1:11}+\overset{(1)}{\lambda}_{2:22} & \overset{(1)}{\lambda}_{2:23} & 0 & \overset{(2)}{\lambda}_{1:12} & 0 \\ \overset{(1)}{\lambda}_{2:31} & \overset{(1)}{\lambda}_{2:32} & \overset{(3)}{\lambda}_{1:11}+\overset{(1)}{\lambda}_{2:33} & 0 & 0 & \overset{(3)}{\lambda}_{1:12} \\ \overset{(1)}{\lambda}_{1:21} & 0 & 0 & \overset{(1)}{\lambda}_{1:22}+\overset{(2)}{\lambda}_{2:11} & \overset{(2)}{\lambda}_{2:13} & \\ 0 & \overset{(2)}{\lambda}_{1:21} & 0 & \overset{(2)}{\lambda}_{2:21} & \overset{(2)}{\lambda}_{1:22}+\overset{(2)}{\lambda}_{2:22} & \overset{(2)}{\lambda}_{2:23} \\ 0 & 0 & \overset{(3)}{\lambda}_{1:21} & \overset{(2)}{\lambda}_{2:31} & \overset{(2)}{\lambda}_{2:32} & \overset{(1)}{\lambda}_{1:11}+\overset{(1)}{\lambda}_{2:11} \end{array}$$

The diagonal elements C_{IJ} of C are calculated by the requirement that each column sum of C_1 is zero. $C\equiv(C_{IJ})$ is an $S(3)\times S(3)$ matrix such that :
If $I\neq J$ and $a_{IJ}=o$ Then $C_{ij}=0$, if $I\neq J$ and $a_{IJ}=D$, Where $D>0$ then $C_{IJ}=Y_{DJ}$. The D is an integer from the set $\{1,\cdots,T\}$: See [theorem 2].

4. Example for Numeric Substitution

$$\Lambda_1^{(1)} = \begin{bmatrix} -10 & 21 \\ 10 & 21 \end{bmatrix} \qquad \Lambda_1^{(2)} = \begin{bmatrix} -12 & 23 \\ 12 & -23 \end{bmatrix} \qquad \Lambda_1^{(3)} = \begin{bmatrix} -14 & 25 \\ 14 & -25 \end{bmatrix}$$

$$\Lambda_2^{(2)} = \begin{bmatrix} -20 & 10 & 12 \\ 9 & -61 & 30 \\ 11 & 51 & -42 \end{bmatrix} \qquad \Lambda_2^{(2)} = \begin{bmatrix} -43 & 15 & 20 \\ 15 & -45 & 30 \\ 28 & 30 & -50 \end{bmatrix}$$

$S = \phi_1 \cdot \phi_2 = 6$, $T = \phi_1 + \phi_2 = 5$, $W_1 = 0$, $W_2 = 2$.
The $A^{(1)}$ and $A^{(2)}$ defined in step 2 have the form :

$$A^{(1)} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \qquad A^{(2)} = \begin{bmatrix} -9 & 3 & 3 \\ 4 & -8 & 4 \\ 5 & 5 & -7 \end{bmatrix}$$

so that

$$A = A^{(1)} \oplus A^{(2)} = \begin{bmatrix} -11 & 3 & 3 & 1 & 0 & 0 \\ 4 & -10 & 4 & 0 & 1 & 0 \\ 5 & 5 & -9 & 0 & 0 & 1 \\ 2 & 0 & 0 & -10 & 3 & 3 \\ 0 & 2 & 0 & -10 & 3 & 3 \\ 0 & 0 & 2 & 5 & 5 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} -10 & -12 & -14 & 21 & 23 & 25 \\ 10 & 12 & 14 & -21 & -23 & -25 \\ -20 & 10 & 12 & -43 & 15 & 20 \\ 9 & -61 & 30 & 15 & -45 & 30 \\ 11 & 51 & -42 & 28 & 30 & -50 \end{bmatrix}$$

$$C = \begin{bmatrix} -30 & 10 & 12 & 21 & 0 & 0 \\ 9 & -73 & 30 & 0 & 23 & 0 \\ 11 & 51 & -56 & 0 & 0 & 25 \\ 10 & 0 & 0 & -64 & 15 & 20 \\ 0 & 12 & 0 & 15 & -68 & 30 \\ 0 & 0 & 14 & 28 & 30 & -75 \end{bmatrix}$$

Taking an I, I $\in$ {1 ; 2 ; 3 ; 4 ; 5 ; 6}. Denote the first two elements of row I in following table by i_1 and i_2. Column I of B consists of column i_1 of $\Lambda_1^{(i2)}$ and column i_2 of $\Lambda_2^{(i2)}$. These implies that elements (I, J) of Λ, A, C equal zero whenever the N-tuples Z(I) and Z(J) differ in at least two entries.
It remains to verify that elements (I, J) of Λ and C coincide if Z(I) and Z(J) differ in exactly one entry.
In C, the diagonal elements of Λ are calculated in such a way that all column sums of Λ are also zero.

5. Conclusion

This paper dealed mainly with the case of s-dependent units U_1, $U_2 \cdots$, U_N (Assumption 2).

The state-transition rate matrix Λ_d of the system composed of s-dependent units must differ from the Kronecker sum $\Lambda_{(N)}$ at least in such a way that multiple repetition of a particular $\lambda_{k:ij}$, which has been observed in $\Lambda_{(N)}$, can disappear in case of Λ_d, that is, it may happen that any two non-zero elements of Λ d are different due to assumption 2.

An algorithm introduced in section III is an effective tool in frequent situations during a construction of a machine : component possessing different parameters are available. The basic structure of the system (numbers of unit and of their states) remains in most cases unchanged and only the state-transition rates of units change. The discribed algorithm takes advantage of this fact in the followig way : The structure of the system is described by an auxiliary Kronecker sum and the substitution of actual values of the state-transition rates of units is then rather simple.

Another problem arises in standby redundant system tasksa proper choice of units. The function of a unit in the system should considered, not only its state, ie its level of operating ability or its failure mode.

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