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SASAKIAN MANIFOLDS WITH VANISHING CONTACT BOCHNER CURVATURE TENSOR

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1. Introduction

Let M be a $(2m+1)$-dimensional differentiable manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$, where, hear and in the sequel, the indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, 2m+1\}$ and let M admit an almost contact structure, that is, a set $(\phi_i{}^j, \xi^j, \eta_i)$ of a tensor field $\phi_i{}^j$ of type (1,1), a vector field ξ^j and 1-form η_i satisfying

$$(1.1) \qquad \begin{gathered} \phi_i{}^t \phi_t{}^j = -\delta_i{}^j + \eta_i \xi^j, \\ \eta_t \phi_i{}^t = 0, \quad \phi_t{}^j \xi^t = 0, \quad \eta_t \xi^t = 1. \end{gathered}$$

We now assume that M admit an almost contact metric structure, that is, a set $(\phi_i{}^j, \xi^j, \eta_i, g_{ji})$ of $\phi_i{}^j, \xi^j, \eta_i$ and positive definite Riemannian metric g_{ji} satisfying, in addition to (1.1),

$$(1.2) \qquad g_{st} \phi_j{}^s \phi_i{}^t = g_{ji} - \eta_j \eta_i,$$

$$(1.3) \qquad \eta_j = g_{ji} \xi^i, \quad g_{ji} \xi^j \xi^i = 1.$$

In this case, we call M an almost contact metric manifold. Comparing the first equation of (1.1) and (1.2), we see that $\phi_{ji} = \phi_j{}^t g_{ti}$ is skew-symmetric. Since, in an almost contact metric manifold, we have the first equation of (1.3), we shall write η^h instead of ξ^h in the sequel. We denote by $\nabla_i, K_{kji}{}^h, K_{ji}$ and K the operator of covariant differentiation with respect to the Levi-Civita connection, the curvature tensor, the Ricci tensor and the scalar curvature respectively.

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2. Sasakian manifold

In this section, we consider a Sasakian manifold M. In a Sasakian manifold M, we have

$$\nabla_k \phi_j{}^i = -g_{kj}\eta^i + \delta_k{}^i \eta_j, \tag{2.1}$$

$$\nabla_j \eta^i = \phi_j{}^i. \tag{2.2}$$

Now from (2.1), (2.2) and the Ricci identity

$$\nabla_k \nabla_j \eta^h - \nabla_j \nabla_k \eta^h = K_{kjt}{}^h \eta^t, \tag{2.3}$$

we find

$$K_{kjt}{}^h \eta^t = \delta_k{}^h \eta_j - \delta_j{}^h \eta_k \tag{2.4}$$

or

$$K_{kji}{}^t \eta_t = \eta_k g_{ji} - \eta_j g_{ki}, \tag{2.5}$$

from which, by contraction,

$$K_j{}^t \eta_t = 2m\eta_j. \tag{2.6}$$

From equation (2.1), (2.2) and the Ricci identity

$$\nabla_k \nabla_j \phi_i{}^h - \nabla_j \nabla_k \phi_i{}^h = K_{kjt}{}^h \phi_i{}^t - K_{kji}{}^t \phi_t{}^h,$$

we find

$$K_{kjt}{}^h \phi_i{}^t - K_{kji}{}^t \phi_t{}^h = -\phi_k{}^h g_{ji} + \phi_j{}^h g_{ki} - \delta_k{}^h \phi_{ji} + \delta_j{}^h \phi_{ki}, \tag{2.7}$$

from which, by contraction,

$$K_{jt} \phi_i{}^t + K_{tjis} \phi^{ts} = -(2m-1)\phi_{ji}. \tag{2.8}$$

Since

$$K_{tjis}\phi^{ts} = K_{sijt}\phi^{ts} = -K_{tijs}\phi^{ts},$$

we have from (2.8)

$$K_{jt}\phi_i{}^t + K_{it}\phi_j{}^t = 0. \tag{2.9}$$

Since

$$K_{tjis}\phi^{ts} = \frac{1}{2}(K_{tjis} - K_{sjit})\phi^{ts} = -\frac{1}{2}K_{tsji}\phi^{ts}$$

we have from (2.8)

$$K_{tsji}\phi^{ts} = 2K_{jt}\phi_i{}^t + 2(2m-1)\phi_{ji}. \tag{2.10}$$

Transvecting (2.7) with $\phi_l{}^i$ and using (2.5), we find

$$-K_{kjl}{}^h - K_{kji}{}^t\phi_l{}^i\phi_t{}^h = -\phi_k{}^h\phi_{lj} + \phi_j{}^h\phi_{lk} - \delta_k{}^h g_{jl} + \delta_j{}^h g_{kl},$$

or

$$K_{kjlt}\phi_i{}^l\phi_h{}^t = K_{kjih} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - g_{kh}g_{ji} + g_{jh}g_{ki}. \tag{2.11}$$

Transvecting (2.11) with $\phi_s{}^h\phi_m{}^j$, we find

$$\begin{aligned} &K_{kjsl}\phi_m{}^j\phi_i{}^l - K_{kjih}\phi_m{}^j\phi_s{}^h \\ &= -g_{ks}(g_{mi} - \eta_m\eta_i) + g_{ki}(g_{ms} - \eta_m\eta_s) + \phi_{ks}\phi_{mi} - \phi_{ki}\phi_{ms}, \end{aligned} \tag{2.12}$$

or

$$K_{ktjs}\phi_m{}^t\phi_i{}^s - K_{ktis}\phi_m{}^t\phi_j{}^s = \Lambda_{kjmi}, \tag{2.13}$$

where we have put

$$\Lambda_{kjmi} = -g_{kj}(g_{mi} - \eta_m\eta_i) + g_{ki}(g_{mj} - \eta_m\eta_j) + \phi_{kj}\phi_{mi} - \phi_{ki}\phi_{mj}. \tag{2.14}$$

From (2.13), we have easily

$$K_{jtks}\phi_i{}^t\phi_m{}^s - K_{itks}\phi_j{}^t\phi_m{}^s = \Lambda_{kjmi}. \tag{2.15}$$

From

$$\begin{aligned} K_{kjlt}\phi_i{}^l\phi_h{}^t &= -(K_{lkjt} + K_{jlkt})\phi_i{}^l\phi_h{}^t \\ &= K_{kljt}\phi_i{}^l\phi_h{}^t - K_{ktjl}\phi_i{}^l\phi_h{}^t = K_{kljt}(\phi_i{}^l\phi_h{}^t - \phi_i{}^t\phi_h{}^l) \end{aligned}$$

and (2.11), we have

$$K_{kljt}(\phi_i{}^l\phi_h{}^t - \phi_h{}^l\phi_i{}^t) = \Omega_{kjih}, \tag{2.16}$$

where we have put

$$\Omega_{kjih} = K_{kjih} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - g_{kh}g_{ji} + g_{jh}g_{ki}. \tag{2.17}$$

Moreover we have

$$K_{ksjt}\phi_i{}^s\phi_h{}^t = K_{jskt}\phi_h{}^s\phi_i{}^t. \tag{2.18}$$

3. Sasakian manifold with vanishing contact Bochner curvature tensor

We consider the section determined by ϕX and $\phi^2 X$ which are orthogonal each other ([3]). We call this a C-holomorphic section, and the sectional curvature determined by such a section is said to be the C-holomorphic sectional curvature:

$$K(X) = -\frac{K_{kjih}(\phi X)^k(\phi^2 X)^j(\phi X)^i(\phi^2 X)^h}{g_{kj}(\phi X)^k(\phi X)^j g_{ih}(\phi^2 X)^i(\phi^2 X)^h}, \tag{3.1}$$

from which, using (2.5), we have

$$K(X) = -\frac{[K_{ksir}\phi_u{}^k\phi_t{}^i + \eta_s\eta_r\gamma_{ut}]X^uX^sX^tX^r}{\gamma_{us}\gamma_{tr}X^uX^sX^tX^r}, \tag{3.2}$$

where we have put

$$\gamma_{kj} = g_{kj} - \eta_k\eta_j.$$

Now, suppose that the contact Bochner curvature tensor $B_{kji}{}^h$ ([4]) of the Sasakian manifold M of dimension $2m+1 > 3$ ([2]) vanishes, then we have

$$\begin{aligned} K_{kjih} = {} & -\gamma_{kh}L_{ji} + \gamma_{jh}L_{ki} - L_{kh}\gamma_{ji} + L_{jh}\gamma_{ki} - \phi_{kh}M_{ji} + \phi_{jh}M_{ki} \\ & -M_{kh}\phi_{ji} + M_{jh}\phi_{ki} + 2(M_{kj}\phi_{ih} + \phi_{kj}M_{ih}) \\ & -(\phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - 2\phi_{kj}\phi_{ih}), \end{aligned} \tag{3.3}$$

where

$$L_{ji} = -\frac{1}{2(m+2)}[K_{ji} + (L+3)g_{ji} - (L-1)\eta_j\eta_i], \quad L = g^{ji}L_{ji},$$

$$M_{ji} = -L_{jt}\phi_i{}^t,$$

and consequently the C-holomorphic sectional curvature $K(X)$ with respect to a C-holomorphic section spanned by ϕX and $\phi^2 X$ is given by

$$K(X) = -\frac{1}{\gamma_{su}X^sX^u}8(L_{tr} + \eta_t\eta_r)X^tX^r - 3, \tag{3.4}$$

where we have used

$$L_{st}\phi_j{}^s\phi_i{}^t = L_{ji} + \eta_j\eta_i, \tag{3.5}$$

$$L_{ji}\eta^i = -\eta_j. \tag{3.6}$$

Conversely suppose that the sectional curvature $K(X)$ of M defined by (3.1) is equal to the right-hand side member of (3.4). Then from (3.2) and (3.4), we have

$$(3.7)\quad [K_{ksir}\phi_u{}^k\phi_t{}^i + \eta_s\eta_r\gamma_{ut} - \{8\gamma_{sr}(L_{ut}+\eta_u\eta_t) + 3\gamma_{sr}\gamma_{ut}\}]X^uX^sX^tX^r = 0,$$

or

$$(3.8)\quad K_{ktis}\phi_j{}^t\phi_h{}^s X^kX^jX^iX^h = (8\gamma_{ki}L_{jh} + 7\gamma_{ki}\eta_j\eta_h + 3\gamma_{ki}\gamma_{jh})X^kX^jX^iX^h,$$

where $L_{kj}X^kX^j$ is a certain quadratic form whose coefficients satisfy (3.5).

This place X's are arbitrary, therefore by (2.18) and the symmetry of L_{jh}, we have from (3.8), ([1])

$$(3.9)\quad \begin{aligned} &2(K_{ktis}\phi_j{}^t\phi_h{}^s + K_{kths}\phi_j{}^t\phi_i{}^s + K_{ktis}\phi_h{}^t\phi_j{}^s + K_{htis}\phi_j{}^t\phi_k{}^s \\ &+K_{ktjs}\phi_i{}^t\phi_h{}^s + K_{kths}\phi_i{}^t\phi_j{}^s + K_{ktjs}\phi_h{}^t\phi_i{}^s + K_{htjs}\phi_i{}^t\phi_k{}^s \\ &+K_{iths}\phi_j{}^t\phi_k{}^s + K_{jtis}\phi_k{}^t\phi_h{}^s + K_{jtis}\phi_h{}^t\phi_k{}^s + K_{jths}\phi_i{}^t\phi_k{}^s) \\ &= 4[\gamma_{ki}(8L_{jh}+7\eta_j\eta_h+3\gamma_{jh}) + \gamma_{kj}(8L_{ih}+7\eta_i\eta_h+3\gamma_{ih}) \\ &+\gamma_{kh}(8L_{ji}+7\eta_j\eta_i+3\gamma_{ji}) + \gamma_{ij}(8L_{hk}+7\eta_h\eta_k+3\gamma_{hk}) \\ &+\gamma_{ih}(8L_{kj}+7\eta_k\eta_j+3\gamma_{kj}) + \gamma_{jh}(8L_{ki}+7\eta_k\eta_i+3\gamma_{ki})], \end{aligned}$$

from which, taking account of (2.13), (2.15) and (2.16), we have

$$(3.10)\quad \begin{aligned} &(8K_{ktis}\phi_j{}^t\phi_h{}^s + 2\wedge_{khji} + 4\wedge_{ijhk} + 2\Omega_{jihk}) \\ &+(8K_{kths}\phi_i{}^t\phi_j{}^s + 4\wedge_{hijk} + 2\wedge_{kjih} + 2\Omega_{hikj}) \\ &+(8K_{ktjs}\phi_h{}^t\phi_i{}^s + 2\wedge_{kihj} + 4\wedge_{jhik} + 2\Omega_{hjik}) \\ &= 4[8(\gamma_{ki}L_{jh} + \gamma_{kj}L_{ih} + \gamma_{kh}L_{ji} + \gamma_{ij}L_{hk} + \gamma_{ih}L_{kj} + \gamma_{jh}L_{ki}) \\ &+7(\gamma_{ki}\eta_j\eta_h + \gamma_{kj}\eta_i\eta_h + \gamma_{kh}\eta_j\eta_i + \gamma_{ij}\eta_k\eta_h + \gamma_{ih}\eta_k\eta_j + \gamma_{jh}\eta_k\eta_i) \\ &+6(\gamma_{ki}\gamma_{jh} + \gamma_{kj}\gamma_{ih} + \gamma_{kh}\gamma_{ji})]. \end{aligned}$$

Taking into consideration of the definition of the tensors $\wedge$ and Ω, we see that

$$(3.11)\quad 2(\wedge_{khji} + \wedge_{kjih} + \wedge_{kihj}) = 4(\phi_{kh}\phi_{ji} + \phi_{kj}\phi_{ih} + \phi_{ki}\phi_{hj}),$$

$$(3.12)\quad \begin{aligned} 4(\wedge_{ijhk} + \wedge_{hijk} + \wedge_{jhik}) &= 4(g_{hi}\eta_j\eta_k - g_{jk}\eta_i\eta_h + g_{ji}\eta_k\eta_h \\ &-g_{hk}\eta_j\eta_i + g_{jh}\eta_i\eta_k - g_{ik}\eta_h\eta_j) + 8(\phi_{hi}\phi_{jk} + \phi_{jh}\phi_{ik} + \phi_{ji}\phi_{kh}) \end{aligned}$$

and

$$(3.13)\qquad 2(\Omega_{jihk}+\Omega_{hikj}+\Omega_{hjik})=4(\phi_{kh}\phi_{ij}+\phi_{kj}\phi_{hi}+\phi_{ki}\phi_{jh}),$$

where we have used

$$K_{hkji}+K_{kjhi}+K_{jhki}=0.$$

Substituting (3.11), (3.12) and (3.13) into (3.10), we find

$$\begin{aligned}
&2(K_{ktis}\phi_j{}^t\phi_h{}^s+K_{kths}\phi_i{}^t\phi_j{}^s+K_{ktjs}\phi_h{}^t\phi_i{}^s)\\
&\quad+(g_{hi}\eta_j\eta_k-g_{jk}\eta_i\eta_h+g_{ji}\eta_k\eta_h-g_{hk}\eta_j\eta_i+g_{jh}\eta_i\eta_k-g_{ik}\eta_j\eta_h)\\
&\quad+2(\phi_{hi}\phi_{jk}+\phi_{jh}\phi_{ik}+\phi_{ji}\phi_{kh})\\
&=8(\gamma_{ki}L_{jh}+\gamma_{kj}L_{ih}+\gamma_{kh}L_{ji}+\gamma_{ij}L_{hk}+\gamma_{ih}L_{kj}+\gamma_{jh}L_{ki})\\
&\quad+7(\gamma_{ki}\eta_j\eta_h+\gamma_{kj}\eta_i\eta_h+\gamma_{kh}\eta_j\eta_i+\gamma_{ij}\eta_k\eta_h+\gamma_{ih}\eta_k\eta_j+\gamma_{jh}\eta_k\eta_i)\\
&\quad+6(\gamma_{ki}\gamma_{jh}+\gamma_{kj}\gamma_{ih}+\gamma_{kh}\gamma_{ji}),
\end{aligned}$$

or

$$\begin{aligned}
(3.14)\quad &2(K_{ktis}\phi_q{}^t\phi_p{}^s+K_{ktps}\phi_i{}^t\phi_q{}^s+K_{ktqs}\phi_p{}^t\phi_i{}^s)\\
&\quad+(g_{pi}\eta_q\eta_k-g_{qk}\eta_i\eta_p+g_{qi}\eta_k\eta_p-g_{pk}\eta_q\eta_i+g_{qp}\eta_i\eta_k-g_{ik}\eta_q\eta_p)\\
&\quad+2(\phi_{pi}\phi_{qk}+\phi_{qp}\phi_{ik}+\phi_{qi}\phi_{kp})\\
&=8(\gamma_{ki}L_{qp}+\gamma_{kq}L_{ip}+\gamma_{kp}L_{qi}+\gamma_{iq}L_{pk}+\gamma_{ip}L_{kq}+\gamma_{qp}L_{ki})\\
&\quad+7(\gamma_{ki}\eta_q\eta_p+\gamma_{kq}\eta_i\eta_p+\gamma_{kp}\eta_q\eta_i+\gamma_{iq}\eta_p\eta_k+\gamma_{ip}\eta_k\eta_q+\gamma_{qp}\eta_k\eta_i)\\
&\quad+6(\gamma_{ki}\gamma_{qp}+\gamma_{kq}\gamma_{ip}+\gamma_{kp}\gamma_{qi}),
\end{aligned}$$

from which, transvecting with $\phi_j{}^q\phi_h{}^p$, we have

$$\begin{aligned}
(3.15)\qquad &2(K_{kjih}-K_{ktpj}\phi_i{}^t\phi_h{}^p-K_{khqs}\phi_j{}^q\phi_i{}^s\\
&\quad-g_{hk}\eta_j\eta_i+g_{ki}\eta_j\eta_h-g_{ji}\eta_k\eta_h+g_{ih}\eta_k\eta_j)\\
&\quad+\gamma_{jh}\eta_i\eta_k+2(\gamma_{kj}\gamma_{hi}-\phi_{ik}\phi_{hj}-\gamma_{kh}\gamma_{ij})\\
&=8[\gamma_{ki}(L_{jh}+\eta_j\eta_h)-\phi_{jk}M_{ih}-\phi_{hk}M_{ij}\\
&\quad-\phi_{ji}M_{kh}-\phi_{hi}M_{kj}+\gamma_{jh}L_{ki}]\\
&\quad+7\gamma_{jh}\eta_k\eta_i+6(\gamma_{ki}\gamma_{jh}+\phi_{jk}\phi_{hi}+\phi_{ji}\phi_{hk}),
\end{aligned}$$

where we have used (2.5) and (3.5) and have put

$$(3.16)\qquad M_{ji}=-L_{jt}\phi_i{}^t,$$

or

$$\begin{aligned}
&K_{kjih} - K_{ktpj}\phi_i{}^t\phi_h{}^p - K_{khji} + \phi_{hi}\phi_{kj} + g_{ki}g_{hj} - g_{kh}g_{ij} \\
&\quad + g_{ki}\eta_j\eta_h - g_{kj}\eta_h\eta_i \\
&= 4[\gamma_{ki}(L_{jh} + \eta_j\eta_h) - \phi_{jk}M_{ih} - \phi_{hk}M_{ij} - \phi_{ji}M_{kh} - \phi_{hi}M_{kj} + \gamma_{jh}L_{ki}] \\
&\quad + 3(\gamma_{jh}\eta_k\eta_i + \gamma_{ki}\gamma_{jh} + \phi_{jk}\phi_{hi} + \phi_{ji}\phi_{hk}),
\end{aligned}$$

where by (2.11), we have used

$$K_{khqs}\phi_j{}^q\phi_i{}^s = K_{khji} + \phi_{ki}\phi_{hj} - \phi_{hi}\phi_{kj} - g_{ki}g_{hj} + g_{hi}g_{kj}.$$

Taking the skew-symmetric part of this equation with respect to k and j and taking account of

$$\begin{aligned}
K_{ktjp}\phi_i{}^t\phi_h{}^p - K_{jtkp}\phi_i{}^t\phi_h{}^p &= K_{ktjp}(\phi_i{}^t\phi_h{}^p - \phi_i{}^p\phi_h{}^t) \\
&= \Omega_{kjih},
\end{aligned}$$

where Ω_{kjih} are defined by (2.17), we find

$$\begin{aligned}
(3.17)\quad &2K_{kjih} + K_{kjih} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} + 2\phi_{hi}\phi_{kj} - K_{khji} + K_{jhki} \\
&-3g_{kh}g_{ji} + 3g_{jh}g_{ki} + g_{ki}\eta_j\eta_h - g_{ji}\eta_k\eta_h \\
&= 4[\gamma_{jh}L_{ki} - \gamma_{kh}L_{ji} + \gamma_{ki}L_{jh} - \gamma_{ji}L_{kh} - \phi_{hk}M_{ij} + \phi_{hj}M_{ik} - \phi_{ji}M_{kh} \\
&+\phi_{ki}M_{jh} - \phi_{hi}M_{kj} + \phi_{hi}M_{jk} - 2\phi_{jk}M_{ih} + (\gamma_{ki}\eta_j - \gamma_{ji}\eta_k)\eta_h] \\
&+3[(\gamma_{jh}\eta_k - \gamma_{kh}\eta_j)\eta_i + \gamma_{ki}\gamma_{jh} - \gamma_{ji}\gamma_{kh} + \phi_{ji}\phi_{hk} - \phi_{ki}\phi_{hj} + 2\phi_{jk}\phi_{hi}].
\end{aligned}$$

Transvecting (3.5) with $\phi_k{}^j$ and taking account of (3.6), we have

$$-L_{kt}\phi_i{}^t = L_{ji}\phi_k{}^j,$$

thus we have from (3.16)

$$-M_{ki} = M_{ik}.$$

From (3.17), we have

$$\begin{aligned}
(3.18)\quad K_{kjih} &= -\gamma_{kh}L_{ji} + \gamma_{jh}L_{ki} - L_{kh}\gamma_{ji} + L_{jh}\gamma_{ki} - \phi_{kh}M_{ji} + \phi_{jh}M_{ki} \\
&\quad -M_{kh}\phi_{ji} + M_{jh}\phi_{ki} + 2(M_{kj}\phi_{ih} + \phi_{kj}M_{ih}) \\
&\quad -(\phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - 2\phi_{kj}\phi_{ih}).
\end{aligned}$$

Transvecting (3.18) with g^{kh}, we find

$$K_{ji} + 2mL_{ji} - (\gamma_j{}^hL_{hi} + \gamma_i{}^hL_{jh}) + L\gamma_{ji} - 3(\phi_j{}^hM_{hi} + \phi_i{}^hM_{jh}) - 3\phi_j{}^h\phi_{hi} = 0,$$

where

$$L = g^{kh} L_{kh},$$

from which, substituting (3.16),

$$K_{ji} + (2m+4)L_{ji} + (L+3)\gamma_{ji} + 4\eta_j\eta_i = 0,$$

that is,

$$L_{ji} = -\frac{1}{2(m+2)}[K_{ji} + (L+3)g_{ji} - (L-1)\eta_j\eta_i],$$

and

$$L = -\frac{K + 2(3m+2)}{4(m+1)}.$$

Thus, (3.18) gives

$$B_{kji}{}^h = 0.$$

Thus, we have the following.

Theorem. *In order that contact Bochner curvature tensor of a* $(2m+1)$*-dimensional Sasakian manifold* $(2m+1>3)$ *vanishes, it is necessary and sufficient that*

$$K(X) = -\frac{1}{(g_{kj} - \eta_k\eta_j)X^kX^j} 8(L_{ih} + \eta_i\eta_h)X^iX^h - 3,$$

where $K(X)$ *is the* C*-holomorphic sectional curvature with respect to a section spanned by vectors* ϕX *and* $\phi^2 X$ *and* $L_{ih}X^iX^h$ *is a certain quadratic form whose coefficients satisfy*

$$L_{st}\phi_j{}^s\phi_i{}^t = L_{ji} + \eta_j\eta_i.$$

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