J. Korean Math. Soc. **29** (1992), No. 1, pp. 43–54

A GENERALIZATION OF SIMONS' RESULTS ON BEST APPROXIMATIONS

SANGHO KUM

1. Introduction

In [10], Simons gave an existence theorem for certain families of quasiconcave functions on a compact convex set and its application to locally convex, normed, Hilbert and finite dimensional spaces. Bellenger generalized Simons' existence theorem to paracompact setting. Recently, Park and Bae [9] removed the paracompactness assumption in the result of Bellenger, and Park [8] used this extension to generalize results of Simons [10] on fixed points.

In this paper, we are concerned with the results of Simons [10] on best approximations which lead to an extension of the famous Fan's result [3, Theorem 2] in an interesting way. As an application of the existence theorem of Park and Bae [9], we extend various results of Simons to more general cases, mainly to noncompact cases.

Our starting point is Theorem 0 which is a noncompact version of Simons [10, Theorem 2.1]. The usefulness of this theorem fully appears in the rest of the paper. We rely basically on the methods of Simons, however, we refine and simplify several results of Simons by virtue of our own useful observations.

2. Preliminaries

A *convex space* X is a nonempty convex set (in a vector space) with any topology that induces the Euclidean topology on the convex hulls of its finite subsets. Thus, a convex subset X of a topological vector space E with the relative topology is automatically a convex space.

A nonempty subset L of a convex space X is called a *c-compact set* if for each finite subset $S \subset X$ there is a compact convex set $L_S \subset X$

Received January 16, 1991. Revised July 20, 1991.

Supported in part by the Basic Science Research Institute Program, Ministry of Education, 1990.

such that $L \cup S \subset L_S$. It is obvious that every nonempty compact convex set in a Hausdorff topological vector space E is a c-compact subset of E.

Let X be a convex space. We denote by $\hat{X}$ the set of all quasiconcave upper semicontinuous real functions on X. Let L be a c-compact subset and K a nonempty compact subset of X.

We first state the following due to Park [8, Theorem 2], which is a noncompact version of Simons [10, Theorem 2.1].

THEOREM 0. (Park [8, Theorem 2]) *Let X, $\hat{X}$, L, and K be as above. Let B be a nonempty convex subset of $\hat{X}$, and α, $\beta : X \times B \to \overline{R} = [-\infty, +\infty]$ functions such that $\{f \in B \mid \alpha(x,f) > \beta(x,f)\}$ is convex for each $x \in X$. Suppose that, for each $f \in B$,*

(0.1) *$X_f = \{x \in X \mid \alpha(x,f) \leq \beta(x,f)\}$ is closed;*
(0.2) *$X_f \supset M_f = \{x \in K \mid f(x) = \max f(X)\}$; and*
(0.3) *for each $x \in X \setminus K$, $f(x) \geq \sup f(L)$ implies $x \in X_f$.*

Then there exists an $x \in X$ such that $x \in X_f$ for all $f \in B$.

If α is concave and β is convex in their second variables, then $\{f \in B \mid \alpha(x,f) > \beta(x,f)\}$ is convex for each $x \in X$. If α is l.s.c. and β is u.s.c. in their first variables, then (0.1) holds automatically. Threrefore, for $X = L = K$, Theorem 0 reduces to Simons [10, Theorem 2.1].

From now on, we assume that X is a nonempty convex subset of a Hausdorff locally convex topological vector space E with the toplogical dual space E^*.

A multifunction $F : X \to 2^E$ is said to be *upper hemicontinuous* (in short, u.h.c.) or a *CLR map* if for each $f \in E^*$ and each real α, the set $\{x \in X \mid \sup f(Fx) < \alpha\}$ is open in X.

Let m be a continuous seminorm on E. This assumption is different from that of Simons, who assumed that m is continuous with respect to the Mackey topology $\tau(E, E^*)$. In fact, this different assumption was used only to simplify the proof of Theorem 2 in Section 4. Thus our results remain true under the assumption of Simons except Theorem 2.

We define two sets B_m and A_m as follows:

$$B_m = \{f \in E^* \mid \quad |f(x)| \leq m(x) \text{ for all } x \in E\},$$
$$A_m = \left\{f \in B_m \mid \sup_{\substack{x\in E \\ m(x)\leq 1}} |f(x)| = 1\right\}.$$

And also, we put

$$\|f\| = \sup_{x\in E,\ m(x)\leq 1} |f(x)| \quad \text{for each } f \in B_m.$$

We say that $T : X \to 2^E \setminus \{\emptyset\}$ is a *m-upper hemicontinuous map* (simply *m-CLR map* as in [10]) if for all $f \in A_m$, the map $x \to \sup f(Tx)$ is u.s.c. on X. An upper hemicontinuous (or CLR in [10]) map is clearly m-CLR.

Throughout this paper, $cc(E)$ always denotes the set of nonempty closed convex subsets of E.

3. Good approximation theorems

We use the same notations and circumstances of the previous section. We begin with the following.

PROPOSITION A. (Hirano *et al.* [4, Theorem 1]) *Let p be a sublinear functional on a vector space E, C a nonempty convex subset of E, and f a concave functional on C such that $f(x) \leq p(x)$ for all $x \in C$. Then there exists a linear functional f_0 on E such that*

$$f(x) \leq f_0(x) \quad \text{for } x \in C,$$

$$f_0(y) \leq p(y) \quad \text{for } y \in E.$$

THEOREM 1. *Let P, $Q : X \to cc(E)$ and $g : X \to R^+$ a nonnegative real function. For each $f \in B_m$, define X_f and M_f as follows:*

$$X_f = \{x \in X \mid \inf f(Qx - Px) \leq g(x)\},$$
$$M_f = \{x \in K \mid f(x) = \max f(X)\}.$$

Suppose that,

(1.1) *for each* $f \in B_m$, X_f *is closed in* X;

(1.2) *for each* $f \in A_m$, $M_f \subset X_f$; *and*

(1.3) *for each* $x \in X \setminus K$ *and for each* $f \in B_m$, $f(x) \geq \sup f(L)$ *implies* $x \in X_f$.

Then there exists an $x_0 \in X$ *such that* $\mathrm{dist}_m(Px_0, Qx_0) \leq g(x_0)$, *where*

$$\mathrm{dist}_m(Px_0, Qx_0) := \inf m(Qx_0 - Px_0).$$

Proof. Observe that (1.2) is actually equivalent to the condition that for all $f \in B_m$, $M_f \subset X_f$. Indeed, let $f \in B_m \setminus A_m$ and $x \in M_f$ be given (we may assume that $0 < \|f\| < 1$). Then $\frac{f}{\|f\|} \in A_m$ and $x \in M_{\frac{f}{\|f\|}}$. By (1.2), we have

$$\inf \frac{f}{\|f\|}(Qx - Px) = \frac{1}{\|f\|} \inf f(Qx - Px) \leq g(x).$$

Since $g(x) \geq 0$ and $0 < \|f\| < 1$,

$$\inf f(Qx - Px) \leq \|f\| g(x) \leq g(x),$$

hence, $M_f \subset X_f$. Taking $B = B_m$, $\alpha(x, f) = \inf f(Qx - Px)$ and $\beta(x, f) = g(x)$, we can easily check that all the requirements of Theorem 0 are satisfied. Thus there exists an $x_0 \in X$ such that for all $f \in B_m$,

$$\inf f(Qx_0 - Px_0) \leq g(x_0). \tag{1.4}$$

It remains to show that

$$\mathrm{dist}_m(Px_0, Qx_0) \leq g(x_0).$$

Suppose the contrary, i.e., $\inf m(Qx_0 - Px_0) > g(x_0)$. Then there is an $\varepsilon > 0$ such that $\inf m(Qx_0 - Px_0) > g(x_0) + \varepsilon$. From Proposition A, with $C = Qx_0 - Px_0$, $f(x) = g(x_0) + \varepsilon$ for all $x \in C$ and $p(x) = m(x)$ for all $x \in E$, there is a linear functional f_0 on E such that $f_0(x) \geq f(x)$ for all $x \in C$ and $|f_0(x)| \leq m(x)$ for all $x \in E$. This f_0 belongs to B_m since m is continuous (see Treves [11, Corollary, p.64]). Since $f_0(x) \geq f(x)$ for all $x \in C$,

$$\inf f_0(Qx_0 - Px_0) \geq g(x_0) + \varepsilon.$$

This contradicts (1.4), because $f_0 \in B_m$. This completes the proof.

REMARKS. 1. If P and Q are m-CLR and $g(x)$ is u.s.c., (1.1) is automatically true. In this case, for $X = L = K$, Theorem 1 reduces to Simons [10, Theorem 4.1].

2. Simons derived his theorem from the theorem of Mazur and Orlicz applied to the seminorm m and the convex set $Qx_0 - Px_0$. We gave an easy proof by using Proposition A.

We strengthen the continuity conditions on P and Q in the same manner as in Simons [10]. We say that P, $Q : X \to 2^E$ are *m-continuous* if they are both u.s.c. and l.s.c. into the topology defined by the seminorm m. Simons stated the following lemma without proof. We give a detailed proof to improve Simons' results slightly.

LEMMA B. *The function $x \mapsto \mathrm{dist}_m(Px, Qx)$ is continuous.*

Proof. Claim 1. The function $x \mapsto \mathrm{dist}_m(Px, Qx)$ is u.s.c.

Fix $x \in X$ and $\varepsilon > 0$. Take $p \in Px$ and $q \in Qx$ arbitrarily. We define U_p and V_q as follows:

$$U_p := \{y \in E \mid m(x - p) < \frac{\varepsilon}{2}\},$$

$$V_q := \{y \in E \mid m(x - q) < \frac{\varepsilon}{2}\}.$$

Since P and Q are l.s.c., there is an open neighborhood W of x in X such that for each $z \in W$, $Pz \cap U_p \neq \emptyset$ and $Qz \cap V_q \neq \emptyset$. For $p_\alpha \in Pz \cap U_p$ and $q_\alpha \in Qz \cap V_q$, we have

$$m(p_\alpha - q_\alpha) \leq m(p_\alpha - p) + m(p - q) + m(q_\alpha - q).$$

Thus

$$m(p_\alpha - q_\alpha) \leq m(p - q) + \varepsilon$$

and so,

$$\mathrm{dist}_m(Pz, Qz) \leq m(p - q) + \varepsilon.$$

Since p and q are arbitrary,

$$\mathrm{dist}_m(Pz, Qz) \leq \mathrm{dist}_m(Px, Qx) + \varepsilon \text{ for all } z \in W.$$

This implies that the function $x \to \mathrm{dist}_m(Px, Qx)$ is u.s.c.

Claim 2. The function $x \mapsto \text{dist}_m(Px, Qx)$ is l.s.c.

Define two sets U and V as follows:

$$U := \bigcup_{p \in Px} U_p, \quad V := \bigcup_{q \in Qx} V_q$$

where

$$U_p = \{y \in E \mid m(x-p) < \frac{\varepsilon}{2}\} \quad \text{and} \quad V_q = \{y \in E \mid m(x-q) < \frac{\varepsilon}{2}\}.$$

Then U [resp. V] is an open neighborhood of the set Px [resp. Qx]. Since P and Q are u.s.c., there exists an open neighborhood W of x in X such that for each $z \in W$,

$$Pz \subset U \quad \text{and} \quad Qz \subset V.$$

For each $p_\alpha \in Pz$ and $q_\alpha \in Qz$ there exist $p \in Px$ and $q \in Qx$ so that

$$m(p_\alpha - p) < \frac{\varepsilon}{2} \quad \text{and} \quad m(q_\alpha - q) < \frac{\varepsilon}{2}.$$

Now consider the inequality

$$\begin{aligned} m(p-q) &\leq m(p_\alpha - p) + m(p_\alpha - q_\alpha) + m(q_\alpha - q) \\ &\leq m(p_\alpha - q_\alpha) + \varepsilon. \end{aligned}$$

Since $p_\alpha \in Pz$ and $q_\alpha \in Qz$ are arbitrary, we have

$$\text{dist}_m(Px, Qx) \leq m(p-q) \leq \text{dist}_m(Pz, Qz) + \varepsilon \text{ for all } z \in W.$$

Thus the function $x \mapsto \text{dist}_m(Px, Qx)$ is l.s.c. as desired. This completes the proof.

Now we can state the following.

COROLLARY 1. *Let P, $Q : X \to cc(E)$ be m-CLR and l.s.c. with respect to the topology of E defined by the seminorm m. Given $\eta \in [0, 1)$, we assume that for each $x \in X \setminus K$ and $f \in B_m$, $f(x) \geq \sup f(L)$ implies*

$$\inf f(Qx - Px) \leq \eta \, \text{dist}_m(Px, Qx).$$

Then there exist an $f \in B_m$ and an $x \in M_f$ such that

$$\inf f(Qx - Px) \geq \eta \ \mathrm{dist}_m(Px, Qx).$$

Proof. Define $g : X \to R^+$ by $g(x) = \eta \ \mathrm{dist}_m(Px, Qx)$ for each $x \in X$. Note that g is u.s.c. with the aid of the proof of Lemma B because P and Q are l.s.c. We consider two cases.

Case 1. $\mathrm{dist}_m(Px, Qx) > 0$ for all $x \in X$.

Then $\mathrm{dist}_m(Px, Qx) > g(x)$ for all $x \in X$. Assume that for any $f \in B_m$ and $x \in M_f$,

$$\inf f(Qx - Px) < \eta \ \mathrm{dist}_m(Px, Qx) = g(x).$$

Since g is u.s.c. and P and Q are m-CLR,

$$X_f = \{x \in X \mid \inf f(Qx - Px) \leq g(x)\}$$

is closed for each $f \in B_m$. It is not hard to see that the remaining requirements of Theorem 1 are also satisfied. Hence there is an $x \in X$ such that $\mathrm{dist}_m(Px, Qx) \leq g(x)$, a contradiction.

Case 2. $\mathrm{dist}_m(Px, Qx) = 0$ for some $x \in X$.

Taking $f = 0$, the zero functional, we trivially obtain the result. This completes our proof.

REMARK. Clearly, m-continuity implies m-CLR and m-l.s.c.. Hence, Corollary 1 improves Simons' result in the sense that the condition on the domain X and the continuity conditions on P and Q are weakened.

4. Best approximation theorems

THEOREM 2. *Let P, $Q : X \to cc(E)$ be m-continuous. Suppose that for each $x \in X \setminus K$ and $f \in B_m$, $f(x) \geq \sup f(L)$ implies*

$$\inf f(Qx - Px) \leq \frac{1}{2} \ \mathrm{dist}_m(Px, Qx).$$

Then there exist an $f \in B_m$ and an $x \in M_f$ such that

$$\inf f(Qx - Px) = \mathrm{dist}_m(Px, Qx).$$

Further if for all $x \in X$,

$$\mathrm{dist}_m(Px, Qx) > 0, \tag{2.1}$$

then $f \in A_m$.

Proof. From Corollary 1, for all $k \geq 2$, there are $f_k \in B_m$ and $x_k \in M_{f_k} \subset K$ such that

$$\inf f_k(Qx_k - Px_k) \geq (1 - \frac{1}{k}) \,\mathrm{dist}_m(Px_k, Qx_k), \tag{2.2}$$

Note that the set $\{k \in R \mid k \geq 2\}$ is a net ordered by the usual order in R. Since K is compact, by passing to an appropriate subnet $x_{k'}$, we may suppose that there exists an $x \in K$ such that $x_{k'} \to x$. Let

$$U = \{x \in E \mid m(x) \leq 1\},$$
$$U^0 = \{f \in E^* \mid |f(x)| \leq 1 \text{ for all } x \in U\}.$$

Recall that U^0 is the polar of U. The Banach-Alaoglu theorem states that U^0 is weak* compact. Actually, it is compact in the topology E^{*k} of uniform convergence on each compact subset of E (See [5, Exercise 18.E] or [6, Theorem 2.2]). In this case, B_m is a closed subset of U^0 in the topology E^{*k}. Hence there exists an $f \in B_m$ such that a subnet of $f_{k'}$ converges to f in the topology E^{*k}. In fact, we may assume without loss of generality that $\{f_{k'}\}$ satisfies this property.

Claim 1. $x \in M_f$.

Since $f_{k'} \to f$ uniformly on the compact set K, the dual pairing $\langle\ ,\ \rangle$ on $B_m \times K \to R$, defined by $\langle f, y\rangle = f(y)$, for each $f \in B_m$ and $y \in X$, is continuous with respect to the product topology on $B_m \times K$. Therefore $f_{k'}(x_{k'})$ converges to $f(x)$. Since $x_{k'} \in M_{f_{k'}}$,

$$f_{k'}(y) \leq f_{k'}(x_{k'}) \text{ for all } y \in X$$

and so, by letting $k' \to \infty$, we obtain that

$$f(y) \le f(x) \text{ for all } y \in X.$$

This forces us to get the result.

Claim 2. $\inf f(Qx - Px) = \mathrm{dist}_m(Px, Qx)$.

We follow the fashion of Simons. Fix $p \in Px$, $q \in Qx$ and $\varepsilon > 0$. Since P and Q are m-l.s.c. and $x_{k'} \to x$, there are two nets $\{p_{k'}\}$ and $\{q_{k'}\}$ such that $p_{k'} \in Px_{k'}$ [resp. $q_{k'} \in Qx_{k'}$] and $p_{k'} \to p$ [resp. $q_{k'} \to q$]. Hence there is an k_0 such that for any $k' \ge k_0$,

$$m(p_{k'} - p) < \frac{\varepsilon}{2} \quad \text{and} \quad m(q_{k'} - q) < \frac{\varepsilon}{2}. \tag{2.3}$$

From (2.2) and (2.3), for $k' \ge k_0$,

$$f_{k'}(q_{k'} - p_{k'}) \ge (1 - \frac{1}{k'})\ \mathrm{dist}_m(Px_{k'}, Qx_{k'}),$$

$$f_{k'}(-p) \ge f_{k'}(-p_{k'}) - \|f_{k'}\|\ m(p_{k'} - p) > f_{k'}(-p_{k'}) - \frac{\varepsilon}{2}$$

and

$$f_{k'}(q) \ge f_{k'}(q_{k'}) - \|f_{k'}\|\ m(q_{k'} - q) > f_{k'}(q_{k'}) - \frac{\varepsilon}{2}$$

hence

$$f_{k'}(q - p) \ge (1 - \frac{1}{k'})\ \mathrm{dist}_m(Px_{k'}, Qx_{k'}) - \varepsilon.$$

Thus for all $p \in Px$ and $q \in Qx$,

$$\liminf_{k'} f_{k'}(q - p) \ge \mathrm{dist}_m(Px, Qx)$$

from which

$$f(q - p) \ge \mathrm{dist}_m(Px, Qx).$$

Since this holds for all $p \in Px$ and $q \in Qx$,

$$\inf f(Qx - Px) \ge \mathrm{dist}_m(Px, Qx).$$

Moreover, the reverse inequality is trivial. Therefore, the conclusion follows.

Suppose, finally, (2.1) is true. From Claim 2, $f \neq 0$. Let $g = \frac{f}{\|f\|}$. Then $g \in A_m$ and $\|g\| = 1$, hence,

$$\begin{aligned}\mathrm{dist}_m(Px, Qx) &\geq \inf g(Qx - Px) \\ &= \frac{1}{\|f\|} \inf f(Qx - Px) \\ &= \frac{1}{\|f\|} \mathrm{dist}_m(Px, Qx) > 0.\end{aligned}$$

Thus $\|f\| \geq 1$, from which it follows that $\|f\| = 1$, i.e., $f \in A_m$, as desired. This completes the proof.

For a normed vector space E, we have the following consequence. However, we can simplify the proof in this case, which consequently gives a short proof for Simons' result.

THEOREM 3. *Let P, $Q : X \to cc(E)$ be continuous. Let L be a compact convex subset, and K a nonempty compact subset of E. Suppose that for each $x \in X \setminus K$ and $f \in E^*$ with $\|f\| \leq 1$, $f(x) \geq \sup f(L)$ implies*

$$\inf f(Qx - Px) \leq \frac{1}{2} \mathrm{dist}(Px, Qx).$$

Then there exist an $f \in E^*$ with $\|f\| \leq 1$ and an $x \in M_f$ such that $\inf f(Qx - Px) = \mathrm{dist}(Px, Qx)$. Further, if for all $x \in X$,

$$\mathrm{dist}(Px, Qx) > 0$$

then $\|f\| = 1$.

Proof. We have only to modify Claim 2 in Theorem 2 as follows:

Fix $p \in Px$, $q \in Qx$ and $\varepsilon > 0$. Since P and Q are l.s.c. and $x_{k'} \to x$, there are two subsequences $\{p_{k'}\}$ and $\{q_{k'}\}$ such that $p_{k'} \in Px_{k'}$, $q_{k'} \in Qx_{k'}$, $p_{k'} \to p$, and $q_{k'} \to q$. Hence there is an k_0 such that for any $k' \geq k_0$,

$$\|p_{k'} - p\| < \frac{\varepsilon}{2} \quad \text{and} \quad \|q_{k'} - q\| < \frac{\varepsilon}{2}. \tag{3.1}$$

From (2.2) and (3.1), for $k' \geq k_0$,

$$f_{k'}(q_{k'} - p_{k'}) \geq (1 - \frac{1}{k'}) \operatorname{dist}(Px_{k'}, Qx_{k'}). \tag{3.2}$$

Since $p_{k'} \to p$, $q_{k'} \to q$ and $f_{k'} \to f$, we have $f_{k'}(q_{k'} - p_{k'}) \to f(q-p)$. By Lemma B we see that $\operatorname{dist}(Px_{k'}, Qx_{k'}) \to \operatorname{dist}(Px, Qx)$. Hence we obtain from (3.2) that

$$f(q - p) \geq \operatorname{dist}(Px, Qx).$$

Since this holds for all $p \in Px$ and $q \in Qx$,

$$\inf f(Qx - Px) \geq \operatorname{dist}(Px, Qx).$$

Moreover, the reverse inequality is trivial. Therefore, the conclusion follows.

REMARK. Theorem 3 is a noncompact version of Simons [10, Theorem 5.1]. Simons showed how Theorem 5.1 leads to an extension of Fan [3, Theorem 2]. For more discussion, refer to Simons [10, Remark 5.3]. On the other hand, we can also generalize Simons [10, Corollary 5.2]. We omit it here.

In what follows we suppose that $(E, \|\cdot\|)$ is a normed vector space and X is a nonempty weakly convex subset of E. Let E have the $\|\cdot\|$ topology and X the weak topology. We only state our final result without proof which is easily obtained by employing a similar process in Claim 1 of Theorem 2 and Simons' argument.

THEOREM 4. *Suppose that E^* is locally uniformly convex. Let P, $Q : X \to cc(E)$ be continuous. Let L be a weakly compact convex subset, and K a nonempty weakly compact subset of E. Suppose that for each $x \in X \setminus K$ and $f \in E^*$ with $\|f\| \leq 1$, $f(x) \geq \sup f(L)$ implies*

$$\inf f(Qx - Px) \leq \frac{1}{2} \operatorname{dist}(Px, Qx).$$

Then there exist an $f \in E^*$ with $\|f\| \leq 1$ and an $x \in M_f$ such that $\inf f(Qx - Px) = \operatorname{dist}(Px, Qx)$. If, further for all $x \in X$,

$$\operatorname{dist}(Px, Qx) > 0$$

then $\|f\| = 1$.

REMARK. We can easily obtain a Corollary to Theorem 4 which is an improvement of Simons [10, Corollary 6.2]. We omit it here.

References

1. J. P. Aubin and I. Ekeland, *Applied Nonlinear Analysis*, John Wiley and Sons, New York 1984.
2. J. C. Bellenger, *Existence of maximizable quasiconcave functions on paracompact convex spaces*, J. Math. Anal. Appl. **123**(1987), 333-338.
3. Ky Fan, *Extensions of two fixed point theorems of F. E. Browder*, Math. Z. **112**(1969), 234-240.
4. N. Hirano, H. Komiya and W. Takahashi, *A generalization of the Hahn-Banach theorem*, J. Math. Anal. Appl. **88**(1982), 333-340.
5. J. Kelley and I. Namioka, *Linear Topological Spaces*, Springer-Verleg, Berlin/ New York, 1976.
6. S. H. Kye, *Several reflexivities in topological vector spaces*, J. Math. Anal. Appl. **139**(1989), 477-482.
7. M. Lassonde, *On the use of KKM multifunctions in fixed point theory and related topics*, J. Math. Anal. Appl. **97**(1983), 151–201.
8. Sehie Park, *Fixed point theory of multifunctions in topological vector spaces*, to appear.
9. ______, and J. S. Bae, *Existence of maximizable quasiconcave functions on Convex Spaces*, J. Korean Math. Soc. **28**(1991), 285–292.
10. S. Simons, *An existence theorem for quasiconcave functions with applications*, Nonlinear Anaysis, TMA **10**(1986), 1133-1152.
11. F. Treves, *Topological Vector Spaces, Distributions and Kernels*, Academic Press, New York/London 1967.

Department of Mathematics
Seoul National University
Seoul 151–742, Korea