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SET–VALUED CONTRACTIBILITY OF HYPERSPACES

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Introduction

Let X be a metric continuum with a metric d. Let 2^X and $C(X)$ be the hyperspaces of nonempty closed subsets and subcontinua respectively and endow with the Hausdorff metric H.

The first result on the contractibility of hyperspace was given by Wojdyslawski [16]. He proved that if X is locally connected then each of 2^X and $C(X)$ is contractible. Kelley in [2] generalized Wojdyslawski's result and, in doing so, he introduced two important notions for the study of hyperspaces: the notion of segment (now it is called order arc) in 2^X and property (3.2) which is called property k. He proved that if X has property k then the hyperspaces of X are contractible. The class of metric continua with property k not only contains locally connected spaces but also it contains homogeneous spaces and hereditarily indecomposable continua [3,10,14]. Property k is a sufficient (but not necessary) condition for the contractibility of hyperspaces. In [7] a necessary and sufficient condition for the hyperspace contractibility in terms of homotopy equivalence was given. This result is not adequet to provide the intrinsic classification of metric spaces whose hyperspaces are contractible.

In [8] it was observed that a necessary condition for the hyperspace contraction is admissibility condition. It was proved in [1,11] that the contractibility of the hyperspaces of X is equivalent to the existence certain set-valued function called c-function on X. Furthermore, the existence of c-function relies on the existence of γ-map on the $\mathcal{M}$-set of X [13].

In this paper we first prove that if X is a space of the type $\mathcal{M}_f$ then its hyperspaces are contractible. Secondly we prove that if a space X

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has its $\mathcal{M}$-set M such that the closure of each component of M belongd to a closed equicontinuous subset of $C(X)$ then the hyperspaces of X are contractible.

1. Preliminary

All spaces are metric continua throughout this paper. We enlist here some basic definitions, known facts, and notion of monotone-connected set-valued maps. For the complete collection of hyperspace terminologies we refer [5].

Let X be a space and $B \subset X$. Let $\epsilon > 0$. Let $N(B, \epsilon) = \{x \in X : d(x, a) < \epsilon \text{ for some } a \in B\}$. The Hausdorff metric H for 2^X or $C(X)$ is given by $H(A, B) = \inf\{\epsilon > 0 : A \subset N(B, \epsilon) \text{ and } B \subset N(A, \epsilon)\}$ for elements A and B. Then each of 2^X and $C(X)$ with this metric is compact and connected.

(1.1) LEMMA [8]. *$H(A \cup B, C \cup D) \leq \max\{H(A, C), H(B, D)\}$ for elements $A, B, C, D, \in 2^X$.*

One useful tool for hyperspace theory is Whitney map [15]. A continuous function $\mu : 2^X \to [0, 1]$ is called a Whitney map for 2^X if the following conditions are satisfied: 1. for $A, B \in 2^X$, $A \subset B$, $A \neq B$, we have $\mu(A) < \mu(B)$.

2. $\mu(\{x\}) = 0$ for each $x \in X$ and $\mu(X) = 1$. We fix one such μ throughout. By a set-valued map $F : X \to Y$ we mean a function F from the space X into 2^X. A set-valued map $F : X \to Y$ is lower semicontinuous (respectively upper semicontinuous) at $x \in X$ if, for each $\epsilon > 0$, there is a neighborhood V of x such that $F(x) \subset N(F(y), \epsilon)$ (respectively $F(y) \subset N(F(x), \epsilon)$) for each $y \in V$. If F is both lower semicontinuous and upper semicontinuous at x, then we say that F is continuous at x. If F is continuous at each point of X then we simply say that F is continuous on X.

(1.2). Suppose $F : X \to Y$ is a set-valued map. Then F is lower semicontinuous at x if and only if, given $\epsilon > 0$ and $a \in F(x)$, there is a neighborhood V of x such that for each $y \in V$ there is an element $b \in F(y)$ such that $d(a, b) < \epsilon$.

A subset $\mathcal{S}$ of $C(X)$ is monotone-connected if, for $A, B \subset \mathcal{S}$ and $A \subset B$, there is an arc $\alpha : I \to \mathcal{S}$ with $\alpha(0) = A$ and $\alpha(1) = B$ which satisfies the condition: $\alpha(s) \subset \alpha(t)$ whenever $s \leq t$. An arc which satisfies the condition is called an order arc.

Let D be a subset of X and $B \in C(X)$ such that $D \subset B$. Let $C(B)$ be the collection of nonempty subcontinua of B. A fiber function of D into $C(B)$ is a set-valued map $F : D \to C(B)$ such that, for each $x \in D$, $\{(x\}, B\} \subset F(x)$ and each element of $F(x)$ contains x. A fiber function $F : D \to C(B)$ is said to be monotone-connected if $F(x)$ is monotone-connected for each $x \in D$. A monotone-connected fiber function F is called an m-map if F is lower semicontinuous at each point of D with respect to the subspace topologies of D and $C(B)$.

(1.3) [2]. Let $D, B \in C(X)$ such that $D \subset B$. Then the set $P(D, B) = \{C \in C(B) : D \subset C\}$ is a monotone-connected closed subset of $C(X)$. If $D = \{x\}$ we write $P(\{x\}, B) = P(x, B)$. For a metric continuum X, let $T : X \to C(X)$ be the set-valued map defined by $T(x) = P(x, X)$, $x \in X$. Then T is monotone-connected and upper semicontinuous on X. We call T the total fiber map for X.

(1.4). Let $D, B \in C(X)$ and $D \subset B$. Let $F : D \to C(B)$ be an m-map. Then, for each $N \in C(X)$ containing B, the set-valued map $F' : D \to C(N)$ defined by $F'(x) = F(x) \cup P(B, N)$, $x \in D$, is an m-map.

Let $T : X \to C(X)$ be the total fiber map for X. An element $A \in T(x)$ is said to be admissible at x in X if, for each $\epsilon > 0$, there is a $\delta > 0$ such that each point y in the δ-neighborhood of x has an element $B \in T(y)$ such that $H(A, B) < \epsilon$. For each $x \in X$, let $\mathcal{A}(x)$ be the collection of all elements of $T(x)$ which are admissible at x in X. Then $\mathcal{A} : X \to C(X)$ is a fiber map. If N is a subcontinuum of X and $x \in N$, we denote $\mathcal{A}(x, N)$ to be the set of all elements of $P(x, N)$ which are admissible at x in N under the subspace topologies of N and $C(N)$. Elements of $\mathcal{A}(x, N)$ may not be admissible at x in X. Also some elements of $\mathcal{A}(x) \cap C(N)$ may not be elements of $\mathcal{A}(x, N)$. We define the $\mathcal{M}$-set M of X as follows:

$$M = \{x \in X : \mathcal{A}(x) \neq T(x)\}$$

The set M is called the $\mathcal{M}$-set of X. The points of $X \backslash M$ are called k-points of X. If $M = \emptyset$, then the space X is said to have property k.

In general the total fiber map $T : X \to C(X)$ is upper semicontinuous on X and continuous at x if and only if $T(x) = \mathcal{A}(x)$. Thus if T is continuous on X then $C(X)$ is contractible [14]. We may note that if X is locally connected at the point $x \in X$ then x is a k-point.

A space X is said to be T-admissible if, for each point $(x, t) \in X \times I$, the following condition is satisfied: for each $A \in \mathcal{A}(x) \cap \mu^{-1}(t)$ and $t' \in [t, 1]$, there is an element $B \in \mathcal{A}(x) \cap \mu^{-1}(t')$ such that $A \subset B$. Since the singleton set $\{x\}$ is in $\mathcal{A}(x)$, the T-admissibility of X implies that $\mathcal{A}(x) \cap \mu^{-1}(t) \neq \emptyset$ for each $t \in I$.

Let D be a subset of X and B a subcontinuum of the space X such that $D \subset B$. An m-map $F : D \to C(B)$ is called a γ-map if $F(x) \subset \mathcal{A}(x)$ for each $x \in D$. If $D = B = X$ then F is called c-function. In [11] it was shown that, for a space X which admits a c-function, the contractibility of $C(X)$ is equivalent to the existence of a continuous c-function. Subsequently in [1] it was proved that if the space X admits a c-function then it admits a continuous c-function. Thus we state

(1.5) THEOREM. *$C(X)$ is contractible if and only if X admits a c-function.*

(1.6) [8]. Let X be a metric continuum. Let $x, y \in X$. Then

(i) $\mathcal{A}(x)$ is closed in $C(X)$ and $\{\{x\}, X\} \subset \mathcal{A}(x)$.
(ii) For each $B \in T(x), \cup\{A \in \mathcal{A}(x) : A \subset B\} \in \mathcal{A}(x)$.
(iii) If $A \in \mathcal{A}(y)$, $B \in \mathcal{A}(x)$, and $y \in A \cap B$ then $A \cup B \in \mathcal{A}(x)$.

For a T-admissible space X with the nonempty $\mathcal{M}$-set M, we have the following:

(1.7). (i) The components of M are nondegenerate.
(ii) For each $x \in M$, there is a positive number $t(x)$ such that $\mathcal{A}(x) \cap \mu^{-1}(s) \subset \{A \in \mathcal{A}(x) : A \subset M\}$ for $0 \leq s < t(x)$.

(1.8). Let $\overline{M}_\alpha$ be the closure of the component M_α and $x \in \overline{M}_\alpha$. Then

(i) for each $A \in \mathcal{A}(x) \cap C(\overline{M}_\alpha)$ and each $t \in [\mu(A), \mu(\overline{M}_\alpha)]$, there is an element $B \in \mathcal{A}(x) \cap \mu^{-1}(t)$ such that $A \subset B \subset \overline{M}_\alpha$.
(ii) if B is an element of $T(x)$ and $M_\alpha \subset B$ then $B \in \mathcal{A}(x)$.

The proofs of (1.7) and (1.8) can be found in [12].

Let M_α be a component of a T-admissible space X and let B and N be subcontinua of X such that $M_\alpha \subset B \subset N$. Suppose $F : \overline{M}_\alpha \to C(B)$ is a γ-map. Let $F' : \overline{M}_\alpha \to C(N)$ be defined by $F'(x) = F(x) \cup P(B, N)$ for each $x \in \overline{M}_\alpha$. Then

(1.9). $F' : \overline{M}_\alpha \to C(N)$ is a γ-map.

Since $\overline{M}_\alpha \subset B \subset N$, we have $P(B, N) \subset \mathcal{A}(x)$ for each $x \in \overline{M}_\alpha$ by (1.8).

(1.10) [12]. Let X be a T-admissible space with its $\mathcal{M}$-set M. Then $C(X)$ is contractible if and only if there is a subcontinuum Z of X containing M and a γ-map $F : \overline{M} \to C(Z)$.

2. Main Theorems

Let X be a T-admissible space with nonempty $\mathcal{M}$-set M. We define $\mathcal{M}_0, \mathcal{M}_1, \ldots$ inductively as follows:

Let $\mathcal{M}_0 = \{\overline{M}_\alpha : M_\alpha$ is a component of $\{x \in X : T(x) \neq \mathcal{A}(x)\}\}$.

$\mathcal{M}_1 = \{\overline{N}_\alpha : N_\alpha$ is a component of $\{x \in \overline{M}_\alpha : P(x, \overline{M}_\alpha) \neq \mathcal{A}(x) \cap C(\overline{M}_\alpha)\}, \overline{M}_\alpha \in \mathcal{M}_0\}$.

Suppose $\mathcal{M}_n$ is defined and $\mathcal{M}_n \neq \emptyset$. Let

$\mathcal{M}_{n+1} = \{\overline{N}_\alpha : N_\alpha$ is a component of $\{x \in \overline{N}_\beta : P(x, \overline{N}_\beta) \neq \mathcal{A}(x) \cap C(\overline{N}_\beta)\}, \overline{N}_\beta \in \mathcal{M}_n\}$.

The element $\overline{N}_\alpha \in \mathcal{M}_{i+1}$ obtained from the element $\overline{N}_\beta \in \mathcal{M}_i$ is called a derived set of $\overline{N}_\beta$.

(2.1) LEMMA. *Let $N \in \mathcal{M}_i$ and $0 \leq i$. Then*

(i) *for each $A \in \mathcal{A}(x) \cap C(N)$ and each $t \in [\mu(A), \mu(N)]$, there exists an element $B \in \mathcal{A}(x) \cap C(N)$ such that $\mu(B) = t$ and $A \subset B$.*

(ii) *if $M = \{x \in N : P(x, N) \neq \mathcal{A} \cap C(N)\} \neq \emptyset\}$, then each component N_α of M is nondegenerate and such that $\overline{N}_\alpha$ also satisfies the condition (i).*

(iii) *if $\overline{N}_\alpha$ is a derived set of N, then $P(\overline{N}_\alpha, N) \subset \mathcal{A}(x)$ for each $x \in \overline{N}_\alpha$. The proofs are almost identical with those of (1.7) and (1.8).*

A T-admissible space is called a continuum of type $\mathcal{M}_f$ if it satisfies the following conditions: there is a non-negative integer n satisfying $\mathcal{M}_n \neq \emptyset$ and $\mathcal{M}_{n+1} = \emptyset$ such that, for each $0 \leq i \leq n$,

(i) *$\mathcal{M}_i$ is finite,*

(ii) *the elements of* $\mathcal{M}_i$ *are pairwise disjoint,*
(iii) *for each* $N \in \mathcal{M}_i$, $P(x, N) = \mathcal{A}(x, N)$ *for each* $x \in N$.

(2.2) THEOREM. *The hyperspaces of the continuum* X *of type* $\mathcal{M}_f$ *are contractible.*

Proof. Since the contractibility of 2^X is equivalent to that of $C(X)$ by Lemma 3.1 in [2], we prove the contractibility of $C(X)$.

Let n be the non-negative integer such that $\mathcal{M}_n \neq \emptyset$ and $\mathcal{M}_{n+1} = \emptyset$. We define a set-valued map α_n on $\cup\mathcal{M}_n$ as follows: for each $N \in \mathcal{M}_n$, $\alpha_n(x) = P(x, N)$ for each $x \in N$. Let α_N denote the restriction of α_n on N. Then $\alpha_N : N \to C(N)$ is monotone-connected by (1.3) and is continuous by the condition (iii) and the condition $\mathcal{M}_{n+1} = \emptyset$ ensures the map to be a γ-map. Now the condition (ii) provides the continuity of α_n on $\cup\mathcal{M}_n$.

Now assume for $0 < i < n$ that a lower semicontinuous set-valued map α_i is defined on $\cup\mathcal{M}_i$ such that the restriction of α_i on each element $N \in \mathcal{M}_i$ is a γ-map $\alpha_N : N \to C(N)$. We define a set-valued map α_{i-1} on $\cup\mathcal{M}_{i-1}$ whose restriction on each element M of $\mathcal{M}_{i-1}$ is given as follows: let $N_1, N_2, \ldots, N_s$ be the collection of the derived sets of M. (It may be that M has no derived set). Define $\alpha_M : M \to C(M)$ by

$$\alpha_M(x) = \begin{cases} \alpha_i(x) \cup P(N_i, M), & x \in N_i \\ P(x, M), & x \in M \setminus \cup_{i=1}^{s} N_i \end{cases}$$

We note that if M has no derived set then $\alpha_M(x) = P(x, M)$ for each $x \in M$. Now define α_{i-1} as follows: for each $M \in \mathcal{M}_{i-1}$, $\alpha_{i-1}(x) = \alpha_M(x)$ for each $x \in M$.

Let M be an element of $\mathcal{M}_{i-1}$. We show that $\alpha_M : M \to C(M)$ is a γ-map. Certainly α_M is an m-map by (1.3) and (1.4). Also α_M restricted on the closed set $\cup_{i=1}^{s} N_i$ is an γ-map by part (iii) of (2.1). Suppose x is a point of $M \setminus \cup_{i=1}^{s} N_i$. Then the condition $P(x, M) = \alpha(x) \cap C(M)$ and (iii) for $\mathcal{M}_{i-1}$ imply that α_M is continuous at x. Now suppose x_0 is a limit point of $M \setminus \cup_{i=1}^{s} N_i$ such that $x_0 \in N_j$ for some j. Let $\epsilon > 0$ and $A \in \alpha_M(x_0)$. Then the condition (iii) for M, (i) and (ii) for $\mathcal{M}_i$ to gether with the lower semicontinuity of the restriction of α_M on N_j at x_0 provide the existence of a neighborhood V of x_0 in M such that $V \cap N_i = \emptyset$ for all N_i different from N_j and each point y of V has an element $B \in \alpha_M(y)$ (if $y \in N_j$) or $B \in P(y, M) \subset \alpha(y) \cap C(M)$

(if $y \in M \setminus \cup_{i=1}^{s} N_i$) such that $H(A,B) < \epsilon$. This proves the lower semicontinuity of α_M on M. Therefore α_M is a γ-map. Since the set $\mathcal{M}_{i-1}$ is finite and the elements in the set are closed and disjoint, we see that α_{i-1} is lower semicontinuous on $\cup\mathcal{M}_{i-1}$.

Now let $i = 1$. Then we have α_0 is a lower semicontinuous set-valued map on $\cup\mathcal{M}_0$ such that the restriction on each element $N \in \mathcal{M}_0$, $\alpha_N : N \to C(N)$ is a γ-map. Let M be the $\mathcal{M}$-set of X. Then $\overline{M} = \cup\mathcal{M}_0$. So we define $F : \overline{M} \to C(X)$ by

$$F(x) = \begin{cases} \alpha_0(x) \cup P(N,X), & x \in N \in \mathcal{M}_0 \\ P(x,X), & x \in X \setminus \overline{M} \end{cases}$$

Then it is easily seen that F is a γ-map. Thus by (1.10) $C(X)$ is contractible.

Let $\hat{H}$ be the Hausdorff metric for the hyperspace $2^{C(X)}$ of all nonempty closed subsets of $C(X)$. A subset $\mathcal{B}$ of $C(X)$ is said to be equicontinuous if, for each $\epsilon > 0$, there is $\delta > 0$ such that whenever $d(x_1,x_2) < \delta$, $x_i \in \cup\mathcal{B}$, $x_i \in B_i \in \mathcal{B}$, $i = 1,2$, we have $\hat{H}(P(x_1,B_1), P(x_2,B_2)) < \epsilon$.

Suppose $\mathcal{B}$ is equicontinuous. Then for each $B \in \mathcal{B}$, the set consists of B alone is also equicontinuous. Hence, for each $B \in \mathcal{B}$, we have $P(x,B) = \mathcal{A}(x,B)$ for each $x \in B$.

(2.3) LEMMA. *Let $x_i \in A_i \in C(X)$, $i = 1,2$.*

Let $\epsilon > 0$. If $\hat{H}(P(x_1,A_1), P(x_2,A_2)) < \epsilon$, then $H(A_1,A_2) < \epsilon$.

Proof. $\hat{H}(P(x_1,A_1), P(x_2,A_2)) < \epsilon$ implies that $\hat{H}(A_1, P(x_2,A_2)) < \epsilon$ and $\hat{H}(A_2, P(x_1,A_1)) < \epsilon$. Hence there exist $C_i \in P(x_i,A_i)$, $i = 1,2$, such that $H(A_1,C_2) < \epsilon$ and $H(A_2,C_1) < \epsilon$. So that $A_1 \subset N(C_2,\epsilon)$ and $A_2 \subset N(C_1,\epsilon)$. Since $C_i \subset A_i$ for $i = 1,2$, we have $A_1 \subset N(A_2,\epsilon)$ and $A_2 \subset N(A_1,\epsilon)$. Therefore $H(A_1,A_2) < \epsilon$.

(2.4) LEMMA. *Let $\mathcal{B}$ be an equicontinuous subset of $C(X)$. Then elements of $\mathcal{B}$ are pairwise disjoint.*

Suppose A and B are distinct elements of $\mathcal{B}$ such that there is a point $x \in A \cap B$. Let $H(A,B) = \epsilon$. Then equicontinuity of $\mathcal{B}$ implies that $\hat{H}(P(x,A), P(x,B)) < \epsilon$ for all $\epsilon > 0$. Hence by (2.3) $H(A,B) < \epsilon$ for all $\epsilon > 0$, which is a contracdition.

(2.5) THEOREM. *Let X be a T-admissible space. Let $\mathcal{B}$ be an equicontinuous closed subset of $C(X)$ such that, for each component M_α of the $\mathcal{M}$-set M of X, $\overline{M}_\alpha \in \mathcal{B}$ and $P(x, \overline{M}_\alpha) \subset \mathcal{A}(x)$ for each $x \in \overline{M}_\alpha$. Then $C(X)$ is contractible.*

Proof. Let $D = \cup\mathcal{B}$. Since $\mathcal{B}$ is closed in $C(X)$, D is closed in X. Applying the hypotheses and (2.4) we see that if an element B of $\mathcal{B}$ does not contain a point of M, then each point of B is a k-point of X, which implies that $P(x, B) \subset \mathcal{A}(x)$ for each $x \in B$. Now we define a set-valued map $F : D \to C(\cup\mathcal{B})$ as follows: for each $B \in \mathcal{B}$, let $F(x) = P(x, B)$ for each $x \in B$. Then equicontinuity of $\mathcal{B}$ implies the lower semicontinuity of F on $\cup\mathcal{B}$. Furthermore the restriction of F on an element B of $\mathcal{B}$, $F_B : B \to C(B)$, is monotone-connected for each $x \in B$ and lower semicontinuous on B by the equicontinuity of $\{B\}$. Thus F_B is a γ-map on B.

We define a set-valued map $G : D \to C(X)$ as follows: for each $B \in \mathcal{B}$, $G(x) = F(x) \cup P(B, X)$ for each $x \in B$. Then $G(x)$ is monotone-connected for each $x \in D$ by (1.4). If points of B are k-points then $G(x) \subset \mathcal{A}(x)$ for each $x \in B$. If B is the closure of a component of M then $G(x) \subset \mathcal{A}(x)$ for each $x \in B$ by (1.9).

We now show the lower semicontinuity of G at $x \in D$. Let $x \in B_x \in \mathcal{B}$. Since F is lower semicontinuous at x, it suffices to show the lower semicontinuity of G at x for an element $A \in P(B_x, X)$. Let $\epsilon > 0$. Then the equicontinuity of $\mathcal{B}$ and admissibility of A at x in X imply that there exists $\delta > 0$ such that if $d(x, y) < \delta$, for $x, y \in D$ and $y \in B_y \in \mathcal{B}$, then $\hat{H}(P(x, B_x), P(y, B_y)) < \epsilon$ and $P(y, B_y)$ has an element C such that $H(A, C) < \epsilon$. We observe that $B_x \subset A$ and $C \cup B_y \in P(y, X)$. So that $H(A, C \cup B_y) = H(A \cup B_x, C \cup B_y)$. Also $H(A \cup B_x, C \cup B_y) \leq \max\{H(A, C), H(B_x, B_y)\}$ by (1.1) and (2.3). And hence by (1.2) G is lower semicontinuous at x. The set-valued map $G : D \to C(X)$ is therefore a γ-map.

If X contains its $\mathcal{M}$-set M then $\overline{M} \subset D$. In this case, the restriction of G on $\overline{M}$, $G_{\overline{M}} : \overline{M} \to C(X)$, is a γ-map. Hence by (1.10) $C(X)$ is contractible. If $M = \emptyset$ then we extend G to $G' : X \to C(X)$ by $G'(x) = G(x)$ if $x \in D$ and $G'(x) = P(x, X)$ if $x \in X \backslash D$. Then one can easily see that G' is a c-function. Hence $C(X)$ is contractible by (1.5). This proves the theorem.

EXAMPLES. Our first example is a continuum X which is given in [4, Example 4, page 191]. The continuum is the union of disjointed family of curves of the shapes $\cup$, $\cap$, and vertical line segments J in the plane. The set $\mathcal{M}_0$ of the $\mathcal{M}$-set of X consists of the vertical parts of $\cup$ and $\cap$. Let $\mathcal{B}$ be the collection of all vertical segments of X. If B is a vertical part of either $\cup$ or $\cap$, then $P(x, B) \subset \mathcal{A}(x)$ for each $x \in B$. Also any subcontinuum of X containing B is admissible at each point of B in X. Clearly $\mathcal{B}$ is an equicontinuous closed subset of $C(X)$. One can easily see that X is T-admissible.

Our second example is a subcontinuum X of the plane.

Let X be the space consistion of the closures of the sets $\{(x, \sin \frac{1}{x}) : 0 < x \leq 1\}$, $\{(x, \frac{1}{2} \sin \frac{1}{x}) : -1 \leq x < 0\}$, and $\{(x, 4+\sin \frac{1}{x}) : 0 < x \leq 1\}$ together with the vertical segment J joining the points $(0, -1)$ and $(0,6)$.

Let M_1 be the segment joining the points $(0, -1)$ and $(0, 1)$, M_2 the segment joining the points $(0, 3)$ and $(0, 5)$, and N the segment joining the points $(0, 1/2)$ and $(0, -1/2)$. Then one can check that the $\mathcal{M}$-set of X consists of two components, namely, M_1 and M_2. So that $\mathcal{M}_0 = \{M_1, M_2\}$. M_1 has one derived set, namely N, and M_2 and N do not have any derived set. Hence $\mathcal{M}_1 = \{N\}$ and $\mathcal{M}_2 = \emptyset$. X is T-admissible as well. Therefore by (2.2) $C(X)$ is contractible.

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