터널과 지하공간, 한국암반역학회
Vol. 2, 1992, pp. 164~176

Rock Stress Measurement and Numerical Approach for Cavern Designing

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ABSTRACT

The conical-ended borehole technique and hemispherical-ended borehole technique are proposed, for the accurate stress measurement within a rock mass. Theory of stress tensor determination and in situ measurement system are presented with successful case examples, and the characteristics of stress distribution within rock masses are examined by the multiple times measurement in a single borehole. Subsequently, the problem in relation to the numerical approach for cavern designing is discussed on the basis of the dependency of the stress discontinuity on the geological discontinuities and so on.

Introduction

Rock stress is of fundamental importance for the construction of rock structures, such as underground openings, since the mechanical behavior of surrounding rock masses is mainly dominated by rock stress. Therefore, the rock stress measurement is considered to be an important field measurement in rock engineering. Particularly, recent developments in relation to the prediction of structural stability of underground openings have indicated the need for systematic stress measurements, e.g. the systematic initial stress measurement, the systematic measurement of stress changes under construction, the systematic induced stress measurement

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and so on.

In order to determine the complete state of stress at a certain point within a rock mass, the deformation of a borehole, which is a function of the rock stress, needs to be measured by applying a stress relief technique, i. e. the overcoring operation. Various types of devices have been developed to measure the changes in length of one or more diameters of the borehole [1~3], or to measure the changes in strain on the cylindrical wall or bottom surface of the borehole[4~7]. However, the methods which require two or more boreholes to determine the complete state of stress are not desirable because of the time, effort and cost involved. For the systematic stress measurement in rock engineering, it is desired that the stress tensor can be computed from the measurement in a single borehole with high accuracy. For this purpose, the combination of the conical-ended borehole technique and a compact overcoring method is proposed in this paper, as well as the hemispherical-ended borehole technique. The hemispherical-ended borehole technique (HEBT)developed by Sugawara et al.[7~15] and the conical-ended borehole technique (CEBT)first proposed by Kobayashi et al. [16]will be refined and expanded in this paper.

For the construction of large rock caverns, such as underground power houses, the initial stress should be given the first priority in consideration. The initial stress is usually in the 3-dimensional state, and this dominates the induced stress distribution and the deformation profile of the cavern. Accordingly, the 3-dimensional stress analysis is essentially necessary to determine the cavern geometry and the support system. For this purpose, various methods for numerical stress analysis have been applied, for example the linear boundary element method[17] to calculate the stress distribution around the cavern. These numerical analyses are usually carried out under the assumption that the initial stress is uniform in a macroscopic view. In this paper, the limitation of this assumption will be discussed by analyzing the case examples of rock stress measurement.

Theory of Rock stress Measurement

For the determination of the rock stress, which exists prior to boring, the latitudinal strain ε_θ and the longitudinal strain ε_m on the bottom surface of a borehole are measured by means of the overcoring. As shown in Fig.1, the cylindrical co-ordinates(γ,θ,z), the spherical coordinates$(\rho,\theta,\phi)$and the Cartesian co-ordinates$(x,y,z)$are respectively defined, and the rock stress which exists prior to boring can be represented as

$$\{\sigma\} = \{\sigma_x,\sigma_y,\sigma_z,\tau_{yz},\tau_{zx},\tau_{xy}\}^T \tag{1}$$

in the Cartesian co-ordinates.

The strains to be measured by HEBT are illustrated in Fig.1(a), and those by CEBT are as in Fig.1(b). These strains are respectively measured by means of the 16-elements strain cells made from epoxy resin with eight cross-type strain gauges. The center points of the strain gauges are arranged axisymmetrically along a strain measuring circle by rotating 45° at a step. As shown in Fig.1, the radius of the strain measuring circle of HEBT is $0.766R$ and that of CEBT is $0.5R$, where R is the radius of the borehole. The radius of the strain measuring circle of HEBT has been determined from the analysis of the theoretical accuracy in stress tensor determination. On the other hand, the radius of the strain measuring circle of CEBT has been made smaller to save the cost for the overcoring operation.

The relation between the strains and the rock stress is given from the theory of elasticity as follows.

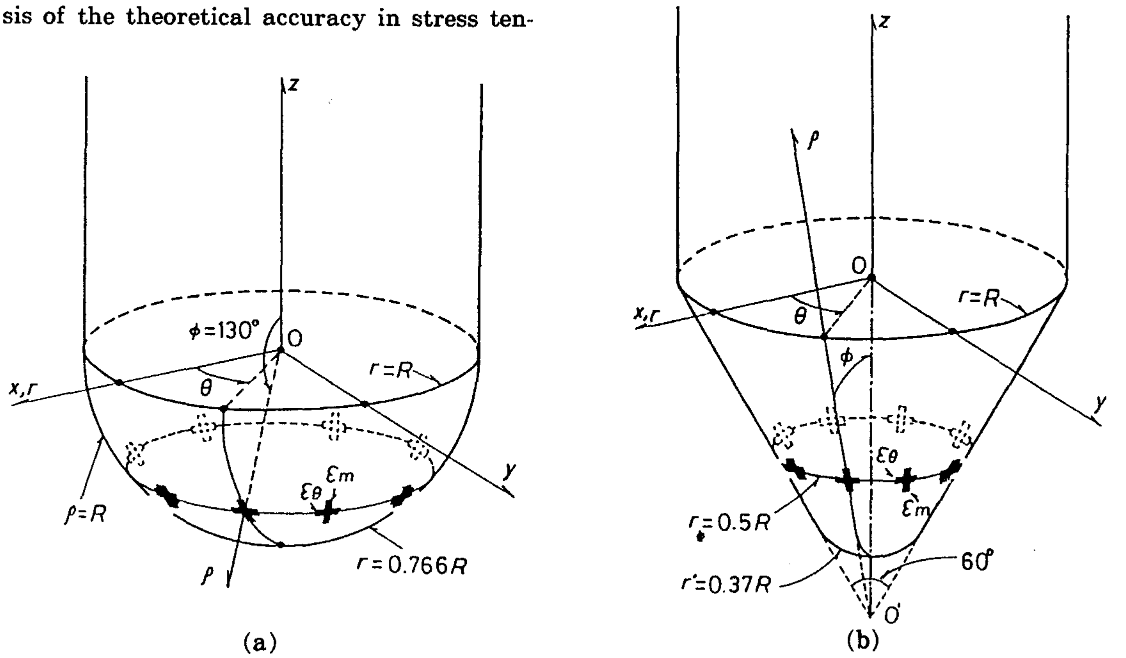


Fig.1 Suggested gauge arrangements,(a) HEBT;(b)CEBT

Table 1 Strain coefficients for HEBT.

Poisson's ratio : ν	A_{11}	A_{12}	A_{21}	A_{22}	C_1	C_2	D_1	D_2
0.10	0.857	−1.360	0.223	0.629	−0.243	0.971	−0.053	2.311
0.20	0.830	−1.392	0.162	0.624	−0.333	0.939	−0.062	2.459
0.25	0.814	−1.398	0.134	0.615	−0.375	0.920	−0.066	2.524
0.30	0.793	−1.386	0.107	0.598	−0.412	0.892	−0.069	2.570
0.40	0.751	−1.363	0.055	0.564	−0.487	0.837	−0.076	2.664

Table 2 Strain coefficients for CEBT.

Poisson's ratio : ν	A_{11}	A_{12}	A_{21}	A_{22}	C_1	C_2	D_1	D_2
0.10	1.002	−1.762	0.109	0.343	−0.155	0.655	0.082	1.524
0.20	1.000	−1.752	0.022	0.365	−0.263	0.641	0.095	1.627
0.25	0.999	−1.733	−0.021	0.373	−0.317	0.636	0.101	1.673
0.30	0.997	−1.704	−0.065	0.380	−0.371	0.632	0.108	1.716
0.40	0.989	−1.611	−0.154	0.386	−0.481	0.630	0.123	1.787

$$\begin{Bmatrix} \varepsilon\theta \\ \varepsilon_m \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} A_{11}+A_{12}\cos2\theta, & A_{11}-A_{12}\cos2\theta, & C_1, & D_1\sin\theta, & D_1\cos\theta, & 2A_{12}\sin2\theta \\ A_{21}+A_{22}\cos2\theta, & A_{21}-A_{22}\cos2\theta, & C_2, & D_2\sin\theta, & D_2\cos\theta, & 2A_{22}\sin2\theta \end{bmatrix} \cdot \{\sigma\} \quad (2)$$

where E is the Young's modulus of rock and A_{11}, A_{12},·········,A_{22} are the strain coefficients which depend on the Poisson's ratio of rock and the geometry of the borehole bottom.

The strain coefficients for HEBT are summarized in Table 1, and those for CEBT in Table 2. These have been evaluated from FEM analysis, and for the approximation of the strain coefficients corresponding to a certain value of the Poisson's ratio, the linear interpolation for each section can be recommended[15,18].

When the measured strains, number of n, are denoted by

$$\{\beta\} = \{\beta_1, \beta_2 \cdots\cdots, \beta_n\}^T, \quad (3)$$

the observation equation, i.e. the relation between $\{\sigma\}$ and $\{\beta\}$, is obtained as follows:

$$[A]\cdot\{\sigma\} = E\cdot\{\beta\}, \quad (4)$$

where[A]is the n by 6 coefficient matrix, elements of which are calculated according to eqn.(2). The normalized version of eqn.(4) is

$$[B]\cdot\{\sigma\} = E\cdot\{\beta^*\}, \quad (5)$$

where$[B] = [A]^T\cdot[A]$ and $\{\beta^*\} = [A]^T\cdot\{\beta\}$. Then, the most probable stress tensor $\{\sigma^*\}$is

$$\{\sigma^*\} = E\cdot[C]\cdot\{\beta^*\}, \quad (6)$$

where [C] is the inverse matrix of [B], and its variance $\{\xi^2\sigma\} = \{\xi_1^2, \xi_2^2, \cdots \xi_6^2\}$ can be given by the least squares method as follows:

$$\{\xi_i^2\} = c_{ii}\cdot E^2\cdot\xi_\beta^2, \quad (7)$$

where ξ_β^2 is the variance of the measured strains and c_{ii} is the diagonal elements of [C].

Table 3 Comparison of the value of c_{max} in eqn.(8).

Method	Radius of the strain measuring circle	Number of strains to be measured : n	Poisson's ratio : ν	Maximum value of c_{ii} : c_{max}
Strain measurements on the cylindrical wall of a borehole	1.000R	9	1/4	0.280
			1/3	0.344
Strain measurements on the conical bottom of a borehole (CEBT)	0.589R	12	1/6	0.378
			1/3	0.467
	0.500R	16	1/6	0.291
			1/4	0.316
			1/3	0.360
	0.500R	24	1/6	0.167
			1/4	0.211
			1/3	0.240
Strain measurements on the hemispherical bottom of a borehole (HEBT)	0.766R	16	1/6	0.127
			1/4	0.133
			1/3	0.148

Eqn.(7) shows that the error of each stress component is proportional to the magnitude of the corresponding diagonal element of [C].

Replacing c_{ii} in eqn.(7) by the maximum value c_{max} of c_{ii}, the maximum variance ξ^2_{max} of stress can be estimated by

$$\xi^2_{max} = c_{max} \cdot E^2 \cdot \xi^2_{\beta}, \tag{8}$$

where c_{max} is in inverse proportion to the value of n. Eqn.(8) shows that ξ^2_{β} is important for the estimation of the accuracy, as well as the value of c_{max}. The magnitude of ξ^2_{β} may depend on the system of strain measurement and etc., then it has to be evaluated experimentally. However, if it is constant, c_{max} is a rational parameter for choosing the better of the methods. The analysis of c_{max}, in Table 3, shows that the highest theoretical accuracy is given by HEBT[18].

In Situ Measurement System

The in situ measurement procedure of HEBT is illustrated in Fig.2. Firstly, a borehole having a diameter 146～210㎜ is drilled to the stress measurement station. Then, a pilot coring of a diameter of 75㎜ is performed, making the axis coincide with that of the former. The bottom of the pilot borehole is formed into a hemispherical shape with a spherical borz crown bit in Fig.3(a), and the bottom surface is ground smooth with a spherical impregnated bit in Fig.3(b). After cleaning the bottom with water and acetone, it is confirmed that there is no crack at the borehole bottom using a borehole bottom scope. Then, the 16-elements spherical strain cell in Fig.3(c) is directly bonded to the hemispherical borehole bottom with glue using a spring type bonding device. Finally, the stresses around the pilot borehole are relieved by the overcoring operation having an outer diameter of 146㎜～210㎜. During this operation, the changes in strain are measured and the data are automatically stored in a computer by a signal from a displacement transducer attached to the boring machine. For this purpose, the cable from the strain cell is linked to the strain meter through the boring rods and the water swivel.

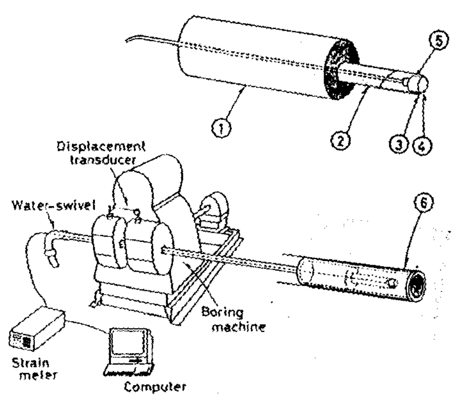


Fig.2 In situ measurement system and procedure of HEBT: 1: large diameter boring; 2: pilot boring ; 3: bottom reforming; 4: bottom cleaning; 5: bonding a strain cell; 6: large diameter overcoring.

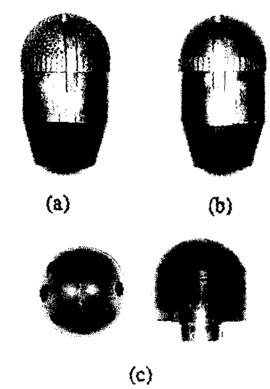


Fig.3 Devices for HEBT,
(a) borz crown bit;
(b) impregnated bit;
(c) 16-elements spherical strain cell

As mentioned above, HEBT requires the overcoring of large diameter. This is a disadvantage of HEBT. To solve this disadvantage, the radius of the strain measuring circle is made smaller in CEBT, then the stress relief is performed by a coring of small diameter, that is equal to the diameter of the pilot boring, as illustrated in Fig.4. By this reformation, the expense of measurement has been drastically cut down. In the present paper, this coring of small diameter for the stress relief is termed a compact overcoring.

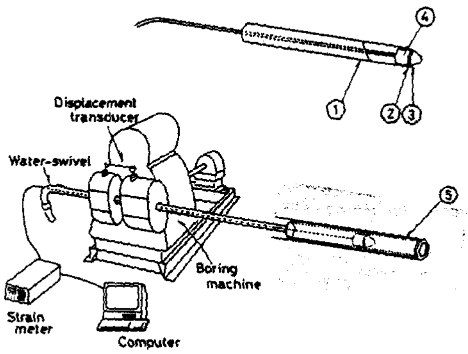
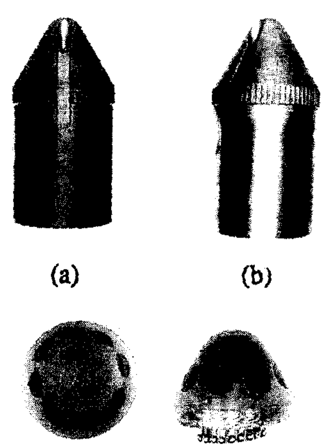


Fig.4. In situ measurement system and procedure of CEBT: 1: pilot boing; 2: bottom reforming; 3: bottom cleaning; 4: bonding a strain cell; 5: compact overcoring.



(a) (b)

(c)

Fig.5 Devices for CEBT,

(a) borz crown bit;

(b) impregnated bit;

(c) 16-elements conical strain cell.

In the case of CEBT, a pilot borehole of a diameter of 76mm is firstly drilled to the stress measurement station. Then, the bottom of the borehole is formed into a conical shape with a conical borz crown bit in Fig.5 (a) and the bottom surface is polished with a conical impregnated bit in Fig.5(b). After the bottom cleaning procedure and the confirmation of the no-crack condition, the 16-elements conical strain cell in Fig.5(c) is directly bonded to the bottom surface. Finally, the stresses on the bottom surface are relieved by the compact overcoring, that is a thin-wall core boring having an outer diameter of 76mm. During this operation, the changes in strain are measured by the continuous strain measurement system.

Fig.6 shows a series of changes in strain on the conical bottom surface during the compact overcoring. The lateral axis in Fig.6 is the distance between the head of the overcoring and the section of the strain measuring circle. The changes in strain are rapid, in all cases, after the overcoring passed through the section of the strain measuring circle. The strains on the bottom surface need to be determined after examining the convergence to a stable terminal value, respectively, Such a monitoring of the changes in strain by the continuous strain measurement system is useful to improve the reliability of the measurement.

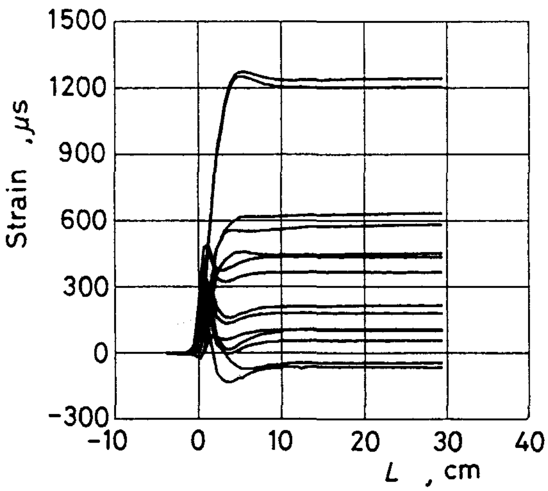


Fig.6 Strain changes measured during the compact overcoring operation(CEBT).

Case Examples of Rock Stress Measurement

Rock stress distribution within rock masses is able to be measured by the multiple times measurement of rock stress. In this paragraph, the three case examples of the multiple times measurement in a single borehole will be shown to examine the stress continuity within a rock mass.

The multiple times measurement in a horizontal borehole, in Fig.7, has been carried out by HEBT, within isotropic and homogeneous quartz diorite involving very tight joints of a joint interval of 0.3～1.0m[15]. The borehole was drilled from a road tunnel, 11.8m in width and 7.8m in height, at a depth of 820m. The seven measurement stations are ranging from 4.6m to 13.4m from the lateral wall of the tunnel. The measured results show that the rock stress is under the 3-dimensional compression, and the maximum compressive stress is nearly in the cross section of the tunnel. The second and third principal stresses are almost in the horizontal plane. In the region up to 6m from the lateral wall, the measured stresses are affected by the excavation. In the region more than 10m from the lateral wall, the measured stresses at the neighboring stations agree well with each other, then the stress in this region is considered to be corresponding to the initial stress. This case example suggests that the initial stress within a homogeneous hard rock mass is comparatively uniform in the macroscopic view.

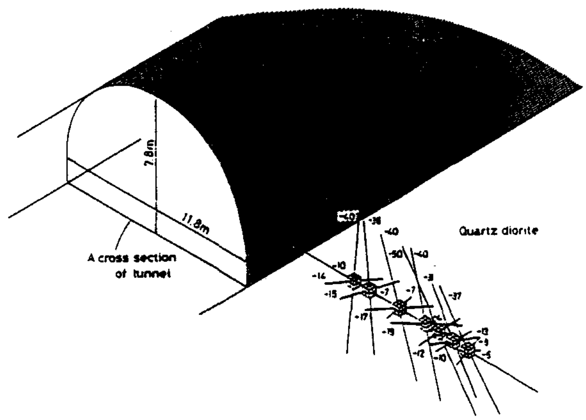


Fig.7 Rock stress around a road tunnel measured by HEBT(Numerals in figure represent the magnitude of stress in MPa).

The case example in Fig.8 is the multiple times measurement carried out by HEBT within homogeneous fine sandstone[15]. The borehole has been drilled normal to the lateral wall of a rectangular gallery, 3.0m high by 5.0m wide, at the depth of 330m. The nine stations for the stress measurement are ranging from 0.85m to 8.8m from the lateral wall of the gallery. In this site, there are three predominant systems of geological discontinuities. One is the bedding planes forming the roof and floors of the gallery, the others are gapped joint systems dipping about 80°. One joint system is roughly parallel to the borehole and unable to be observed in the borehole, but the other intersects the borehole with almost right angles at 4m and 6.3m from the lateral wall of the gallery, as shown in Fig.8. The measured results show that the rock stress in this field is nearly under the uniaxial compression, and the stress distribution is strongly influenced by the geological discontinuities. In the distribution of the maximum compressive stress, there is a clear discontinuity at 4m. This means the slip on the joint. Then, it can be concluded that the unstable joint plays an important role in determining the stress distribution around the cavern.

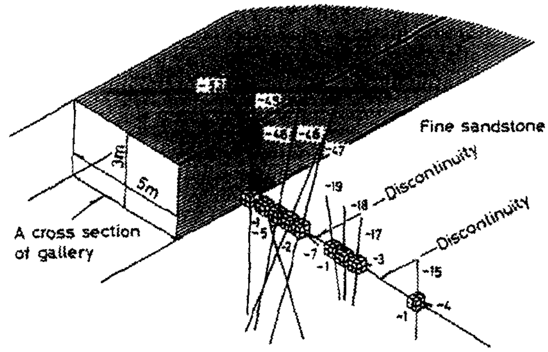


Fig.8 Rock stress around a rectangular gallery measured by HEBT(Numerals in figure represent the magnitude of stress in MPa).

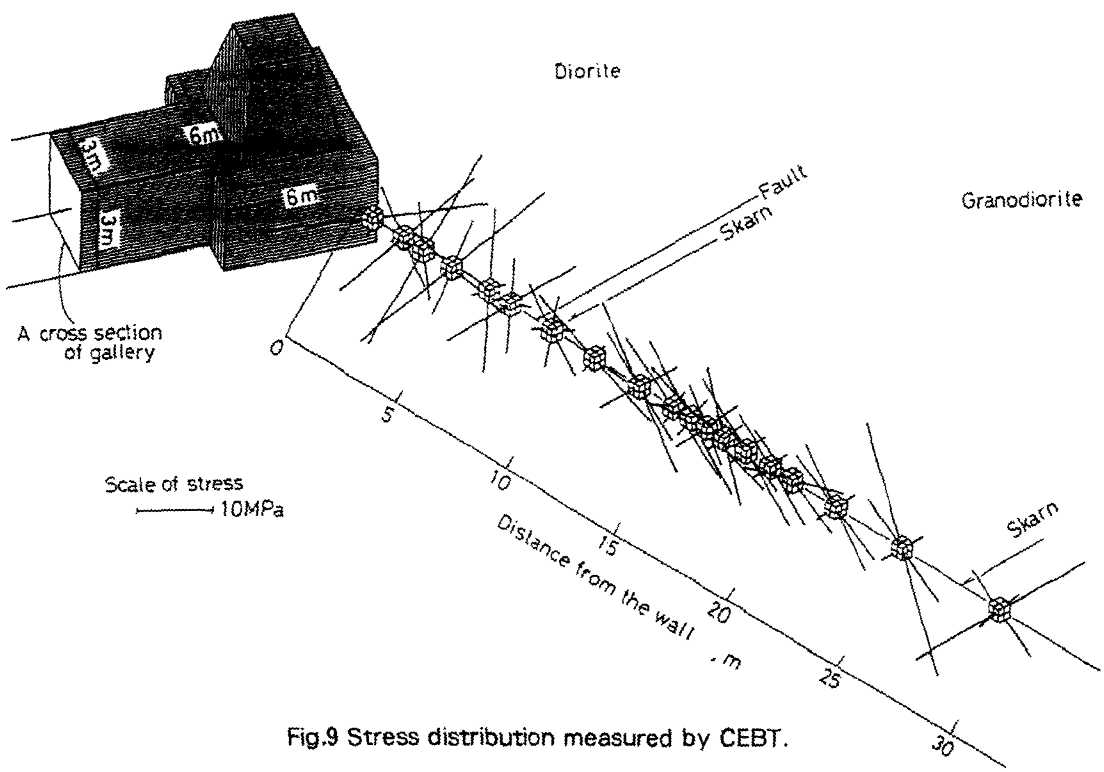


Fig.9 Stress distribution measured by CEBT.

The multiple times measurement in Fig.9 has been performed by CEBT within diorite and granodiorite. The borehole for the measurement was drilled horizontally from the wall of a gallery, 6m in width and 6m in height, at a depth of 520m. The nineteen measurement stations are ranging from 0.6 m to 29.5m from the wall of the gallery. The borehole passed through the geological boundary between the hard diorite and the comparatively soft granodiorite, at 8.7m from the wall of the gallery. On this geological boundary, there were a fault of 0.25m in width dipping about 80 degrees and an immediate skarn of 1.5m in width. The measured results indicate clearly that the initial stress in this field varies with the region bounded by the fault. Accordingly, it can be pointed out that the macroscopic uniformity of the initial stress occasionally disappears in the faulted rock formations, then the initial stress has to be evaluated for each rock mass bounded by faults.

Numerical Approach for Cavern Designing

In relation to the construction of the large rock cavern, such as underground power houses, various methods of numerical stress analysis have been applied, and the induced stress has been analyzed as well as the deformation profile of the cavern. It can be noted that FEM[19] is the most popular method in this field. However, FEM is not powerful for the 3-dimensional problems. The boundary element method[17] is considered to be more effective for the estimation of the 3-dimensional induced stress around the cavern.

Fig.10 shows an example of stress analysis by the linear boundary element method [8, 12]. In such a 3-dimensional analysis, the initial stress is usually assumed to be uniform within the analytic domain, but this assumption is unconditionally not permissible in the faulted rock formations, as discussed previously.

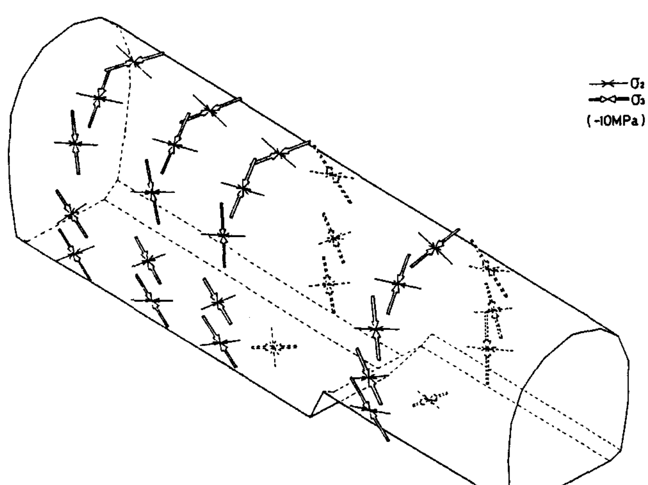


Fig.10 Elastic tangential stresses on the cavern wall analyzed by BEM.

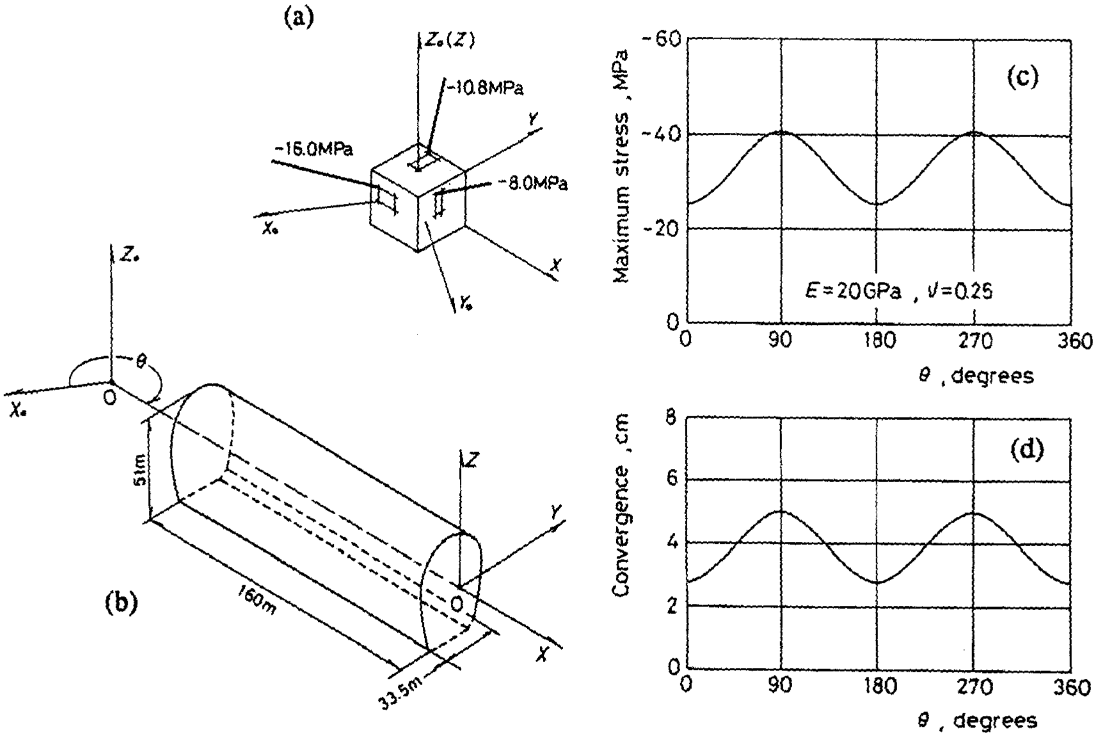


Fig.11 Elastic stress concentration depending on the cavern orientation, analyzed by BEM, (a) initial stress condition; (b) cavern orientation; (c) maximum tangential stress; (d) convergence of opposite lateral walls.

The 3-dimensional stress analysis in Fig. 11 demonstrates the fact that the elastic stress concentration around the cavern is minimized as well as the convergence in the case when the longitudinal axis of the cavern is parallel to the maximum horizontal stress orientation. This leads us to the practical concept that minimizing the elastic stress concentration is a rational procedure for determining the cavern geometry and orientation. In some projects of cavern construction, this concept has been adopted to design the cavern. The successful results have been reported by Mimakiv et al.[20].

In regard to the support system, such as rock bolts, rock anchors, shotcrete and so on, the forecast of deformation profile is much important, as well as the estimation of the extent of the failure zone around the cavern. As shown in Fig.12, the maximum displacement on the lateral wall of the underground power house increases with increasing the ratio of the overburden pressure to the uniaxial compressive strength of rock mass, and the larger displacement is always connected with unstable rock movements and some damages of shotcrete and so on. Therefore, it is clear that the control of the maximum displacement by the support becomes an important subject.

In order to estimate the deformation profile, the extent of the failure zone and the maximum displacement, the 2-dimensional elasto-plastic analysis is usually performed under the plane strain condition. As a matter of course, this condition is reasonable in a case when the longitudinal axis of the cavern coincides with one of the principal axes of the initial stress.

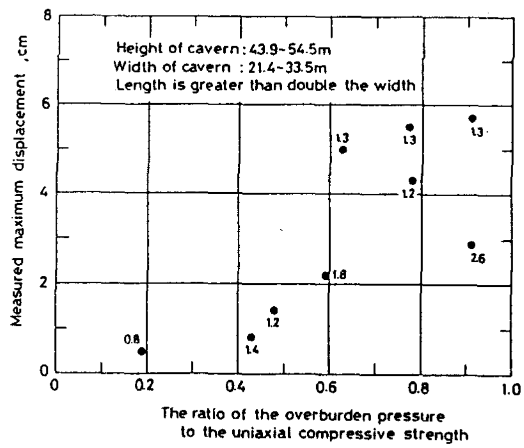


Fig.12 Maximum displacement of cavern wall measured at the underground power houses in Japan, depending on the ratio of the overburden pressure to the uniaxial compressive strength and the supporting pressure by rock anchors(Numerals in figure are the supporting pressure in MPa).

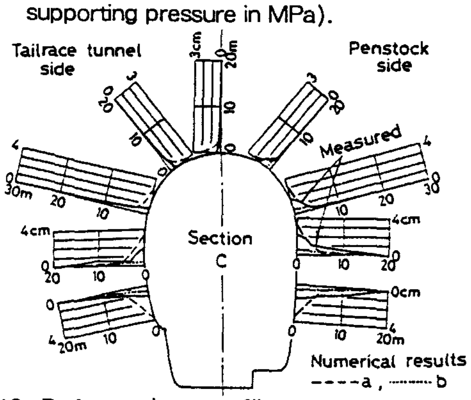


Fig.13 Deformation profiles compared with the analyzed results by BECM[21].

Fig.13 shows an example of the 2-dimensional elasto-plastic analysis carried out by the coupled boundary element-characteristics method(BECM)[21]. The mean deformability constants determined from the jack tests are used in this analysis, with the yield function of two cases evaluated from the rock shear tests. Broken curves a in Fig.13 represent the relative displacement corresponding with the pessimistic case of the yield function, and broken curves b are the analyzed results by the optimistic yield function. Comparing the analyses with the measurement shown by solid curves, it is pointed out that the pessimistic evaluation is in a good approximation in the penstock side, but over estimation in the tailrace tunnel side. On the other hand, the optimistic analysis agrees well with the measurement in the tailrace tunnel side, but it is too small in quantity in the penstock side. Since such an asymmetric deformation should be produced by the non-uniformity of geomechanical condition, the estimation having a certain range is considered to be more practical in the numerical approach for the cavern designing.

Conclusions

The conical-ended borehole technique (CEBT) has been proposed as a promising method presently available for rock stress measurement in hard rock. The theory of stress tensor determination from the 16 strains on the bottom surface has been presented with the in situ strain measurement system. It has been discussed that theoretical accuracy in stress tensor determination of CEBT is not better than that of the hemispherical-ended borehole technique(HEBT), however the cost and time for measurement are drastically saved by combining CEBT with the compact overcoring method.

The multiple times measurement of rock stress in a single borehole by HEBT or CEBT has been presented and discussed as an economical procedure to evaluate the stress distribution. From the three case examples of the multiple times measurement, it has been demonstrated that the macroscopic uniformity of the initial stress occasionally disappears in association with the presence of faults.

Numerical approach for cavern designing has been discussed, and it has been concluded that the initial stress condition for the stress analysis has to be evaluated for each region bounded by faults. Additionally, it has been discussed that the minimization of the elastic stress concentration around the cavern is a rational concept for determining the cavern geometry and orientation.

References

1. Leeman E.R., Measurement of stress in abutments at depth, In Proc. Strata Control Conf., Paper D.5, 1960, pp.295-311.
2. Obert L., Merrill R.H. and Morgan T.A., Borehole deformation gauge for determining the stress in mine rock, U.S. Bureau of Mines, R.I.5978, 1962.
3. Kanagawa T., Hibino S. and Ishida T., In-situ stress measurements by over-coring method, Development of 8-elements gauge for 3-dimensional estimation, Central Research Institute of Electric Power Industry Report, 385033, 1986.
4. Leeman E.R., The determination of the complete state of stress in rock in a single borehole-laboratory and underground measurement, Int. J. Rock Mech. Min. Sci., 5, 1968, pp.31-56.
5. Dancan Fama M.E. and Pender M.J., Analysis of the hollow inclusion technique for measuring in situ rock stress, Int. J. Rock Mech. Min. Sci.& Geomech. Abstr., 17, 1980, pp.137-146.
6. Oka Y. and Bain I., A means of determining the complete state of stress in a single borehole. Int.J.Rock Mech. Min. Sci., 7, 1970, pp.503-515.
7. Sugawara K., Obara Y., Okamura H. and Wang Y., The determination of the complete state of stress in rock by the measurement of strains on a hemispherical borehole-bottom. J. Min.& Metall. Inst. Japan, 101, 1167,1985, pp.227－282.
8. Sugawara K., Kaneko K., Obara Y. and Okamura H., Determination of the state of stress in rock by the measurement of strains on the hemispherical borehole bottom. In Proc. Int. Symp. of Large Rock Caverns, Helsinki, 1986,pp.1039-1050.
9. Sugawara K., Obara Y., Okamura H. and Aoki T., Measurement of strains on a hemispherical borehole bottom by the stress relief technique. J. Min.& Metall. Inst.Japan,102, 1182, 1982, pp.463-468.

10. Sugawara K. and Obara Y., Measurement of in situ rock stress by hemispherical-ended borehole technique, J. Mining Science & Technology, 3, 1986, pp.287-300.

11. Sugawara K., Obara Y., Kaneko K. and Aoki T., Hemispherical-ended borehole techique for measurement of absolute rock stress. In Proc. Int. Symp. of Rock Stress and Rock Stress Measurement, Stockholm, 1986, pp.207-216.

12. Sugawara K., Obara Y. Akimoto M. and Aoki T., Stability estimation of large rock cavern by in situ stress measurements. In Proc. Int. Symp. on Engineering in Complex Rock Formations, Beijing, 1986, pp.135-141.

13. Obara Y. and Sugawara K., Field stress measurements in jointed rock. In Proc. of Int. Conf. on Mech. of Jointed and Faulted Rock, Vienna, 1990, pp.827-834.

14. Obara Y., Sugawara K., Sakaguchi K. and Mizuochi Y., Application of hemispherical-ended borehole technique to hot rock, In Proc. 7th Int. Cong. ISRM, Aachen, 1991, pp.587-590.

15. Sugawara K., Obara Y. and Sakaguchi K., Hemispherical-ended borehole technique for rock stress measurement and its application, In Proc. Int. Symp. on Residual Stress, Tokushima, 1991.

16. Kobayashi S., Nishimura N. and Matsumoto K., Displacements and strains around a non-flatend borehole, In Proc. Int. Symp. of Field Measurements in Geomech., Kobe, 1987, pp.1079-1084.

17. Banajee P.K. and Butterfield R., Boundary element methods in engineering science, McGraw-Hill, 1981.

18. Sakaguchi K., Obara Y., Nakayama T. and Sugawara K., Accuracy of rock stress by means of conicalended borehole technique, J. Min.& Metall. Inst. Japan, to be appeared.

19. Zienkiewicz O.C., The Finite Element Method. 3rd ed., McGraw-Hill, 1977.

20. Mimaki Y. and Matsuo K., Investigation of asymmetrical deformation behavior at the horseshoe-shaped large cavern opening, In Proc.Int. Symp. of Large Rock Caverns, Helsinki, 2, 1986, pp.1337-1348.

21. Sugawara K., Aoki T. and Suzuki Y., A coupled boundary element-characteristics method for elasto-plastic analysis of rock caverns, In proc. Int. Symp. on Rock Mech. and Power House, Madrid, 1988, pp.249-258.