

## Resonant Formation Rates of Muonic Molecular Ion in Muon-Catalyzed Fusion

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뮤온 촉매 핵융합에서 뮤온 분자 이온의 공명 형성을

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### ABSTRACT

The resonant formation rates of muonic molecular ion  $dt^\mu$  in the muon-catalyzed fusion are calculated in various fuel temperatures and densities. The elastic scattering cross sections between  $t^\mu$  and deuterons are obtained by making use of the partial wave method. The transition property of the excited compound molecule  $[(dt^\mu)dee]^*$  is derived by the impulse approximation in the form of a bound-state form factor. The radiative, Auger, and collisional deexcitations are considered as the deexcitation mechanisms of the excited  $dt^\mu$ , and each deexcitation width is calculated as well as back decay width. The resultant reaction widths are used to calculate the formation cross sections of resonant  $dt^\mu$ . The resonant formation rates for  $dt^\mu-d$  and  $dt^\mu-t$  collisions are computed as functions of fuel temperature and density. The calculations show that the resonant formation rates increase with fuel densities and have the maximum values at the particular temperatures where the relative collision energies are equal to the resonant ones.

### 요 약

뮤온 촉매 핵융합에서 뮤온 분자이온  $dt^\mu$ 의 공명 형성을 수소밀도와 온도의 함수로서 계산하였다. 부분파 방법을 도입하여  $t^\mu$ 와 중양자 사이의 탄성산란 단면적을 구하였고, 충격근사를 써서 여기상태의  $[(dt^\mu)dee]^*$ 의 천이를 구속상태 형성인자로 도출하였다. 여기상태  $dt^\mu$ 의 탈여기 기작으로는 방사선 방출, 오오저, 그리고 충돌 탈여기를 고려하였고 이들에 대한 탈여기폭과 역붕괴폭을 계산하였다. 계산된 반응폭들로부터 공명상태  $dt^\mu$ 의 형성단면적을 구하고 이것을 써서 수소의 밀도와 온도의 함수로서  $dt^\mu-d$ ,  $dt^\mu-t$ 의 공명 형성을 계산하였다. 계산결과로부터 공명형성은 수소의 밀도와 더불어 증가하고, 상대충돌 에너지가 공명에너지와 같아지는 온도에서 최대값을 갖는다는 것을 알았다.

## I. Introduction

Muon-catalyzed fusion (MCF) is a fusion reaction which occurs in the low-temperature molecular state in which the muon ( $\mu^-$ ) plays a role as a catalyst. The magnetically-confined thermonuclear fusion which occurs in the high-temperature plasma state has had difficulties in overcoming many serious obstacles such as difficult confinement due to various instabilities, high-temperature heating of plasma, and supply of relatively low density fuel. On the contrary, MCF has a lot of advantages such as easy fuel confinement in the neutral state, low operating temperature (a few hundred  $^{\circ}\text{C}$ ), and high fuel density of liquid hydrogen ( $4.22 \times 10^{28}$  atoms/ $\text{m}^3$ ).

If muons are injected into a mixture of deuteriums and tritiums, they replace the orbital electrons of hydrogen isotopes. Then they are captured by the hydrogen isotope nuclei and form muonic atoms. These muonic atoms build up muonic molecules through collisions with hydrogen isotope molecules. In the early stage of MCF study, only the Auger mechanism was considered in explaining the formation and reaction of muonic molecules such as  $\text{pp}\mu$ ,  $\text{pd}\mu$ , and  $\text{pt}\mu$ . Since 1975, the resonant formation mechanism<sup>[1]</sup> which explains the formation of muonic molecules  $\text{dd}\mu$  and  $\text{dt}\mu$  in much shorter formation times has become a major object of MCF research. In the resonant muonic molecule where the distance between the two nuclei is several hundred times smaller than that of normal molecule, a d-t fusion reaction occurs within a few picoseconds. After the fusion reaction most of the muons ( $\geq 99.6\%$ ) are set free and captured by the nuclei of hydrogen isotopes, and repeat the above procedure. The remaining muons are captured by  $\alpha$  particles generated as a fusion product and leave the fusion cycle<sup>[2,3]</sup>. This sticking loss probability has been reported to be less than 0.4%. Consequently, the muons enhance the fusion reactions as a catalyst

without participating in net reactions<sup>[4-7]</sup>.

At present, the essential issues of MCF are the reduction of muon production cost and sticking loss, and the development of resonant formation mechanism. Since the resonant formation mechanism is not completely understood and takes the longest reaction time in an MCF cycle, it is of great importance in MCF research to improve the resonant formation process. In this work the resonant  $\text{dt}\mu$  formation rates by a direct formation mechanism<sup>[8,9]</sup> are calculated in a temperature range of 400–2000 K and a fuel density range of 0.4–1.6  $\rho_0$  ( $\rho_0$  is the liquid hydrogen density) to find out the optimum reaction conditions for MCF in the achievable laboratory environment.

## II. Formation of Muonic Molecule

The muonic molecule formation mechanism is divided into two parts according to the type of emitting the binding energy. One is a non-resonant formation in which the binding energy is delivered to an Auger electron, and the other is a resonant formation in which the binding energy is changed into the form of vibrational and rotational excitation energies of a compound molecule. Table 1 shows the binding energies of various muonic molecules, where  $J$  and  $\nu$  represent rotational and vibrational quantum numbers, respectively.

### 1. Non-resonant Formation

This mechanism explains the experimental results of formations of  $\text{pp}\mu$ ,  $\text{pd}\mu$ ,  $\text{pt}\mu$ , and  $\text{tt}\mu$ . The dissociation energy of a hydrogen molecule is approximately 4.5 eV. As all the quantum states of  $\text{pp}\mu$ ,  $\text{pd}\mu$ ,  $\text{pt}\mu$ , and  $\text{tt}\mu$  have much greater energy values than the dissociation energy, the molecular binding is destroyed by the formation of a muonic molecule and the residual energy is delivered to the Auger electron.

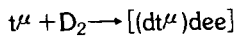
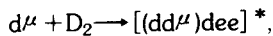
**Table 1. The Binding Energies of Muonic Molecules in Various Quantum States<sup>[2]</sup>**

(unit : eV)

$J \nu$	$pp^\mu$	$pd^\mu$	$pt^\mu$	$dd^\mu$	$dt^\mu$	$tt^\mu$
00	252.95	221.52	213.97	325.04	319.15	362.95
01	—	—	—	35.80	34.87	83.88
10	106.96	97.40	99.01	226.61	232.44	289.15
11	—	—	—	1.91	0.64	45.24
20	—	—	—	86.32	102.54	172.65
30	—	—	—	—	—	48.70

## 2. Resonant Formation

This mechanism is suitable to explain  $dd^\mu$  and  $dt^\mu$  formations. As shown in Table 1, the binding energies of  $(J, \nu)=(1, 1)$  states of  $dd^\mu$  and  $dt^\mu$  are 1.91 and 0.64 eV, respectively. Because these values are smaller than the dissociation energy for hydrogen molecule, the molecular binding is sustained after the muonic molecular formation. Only the compound molecules  $[(dd^\mu)dee]$  and  $[(dt^\mu)dee]$  are transitted from ground states to excited states by the binding energy :



As the energy states of muonic molecules are quantized, the above reactions are possible only in the case that the following relation is satisfied.

$$E_{\text{coll}} = \Delta E + E_{\text{bind}}, \quad (1)$$

where  $E_{\text{coll}}$  which is called resonant energy ( $E_{\text{res}}$ ) is the collision energy between muonic atoms and  $D_2$  or  $DT$ .  $\Delta E$  is the energy difference between the ground state of a hydrogen molecule and the excited state of a compound molecule, and  $E_{\text{bind}}$  is the binding energy of the muonic molecule.

In the case of  $pp^\mu$ ,  $pd^\mu$ ,  $pt^\mu$ , and  $tt^\mu$ , the molecular formation time is  $10^{-6}$  seconds, for which one or less fusion reaction is completed during the muon lifetime of  $\sim 2.15 \times 10^{-6}$  sec.

Since the  $dt^\mu$  molecular formation time is, however,  $10^{-9}$  seconds, it is possible that a muon repeats hundreds of fusion cycles during its life-time<sup>[3,10,11]</sup>. Therefore, from a viewpoint of the practical energy production by MCF, only  $dt^\mu$  are expected to be of most interest and the present analysis for resonant formation rates is thus confined to the muonic molecule  $dt^\mu$ .

## III. Computation of Resonant $dt^\mu$ Formation Rates

### 1. Elastic Scattering Cross Section of $[t^\mu + d]$ Collision

In the partial wave method<sup>[12]</sup> of quantum mechanical collision theory, the partial cross section of resonant scattering with a resonant energy  $E_{\text{res}}$  can be derived as the following Breit-Wigner formula<sup>[12,13]</sup>,

$$\sigma_l^{el} = \frac{4\pi(2l+1)}{k_1^2} \frac{\frac{\Gamma_{el}^2}{4}}{(E_1 - E_{\text{res}})^2 + \frac{\Gamma_{el}^2}{4}} \quad (2)$$

where

$l$  : quantum state of orbital angular momentum

$k_1$  : relative wave number of collision  $[t^\mu + d]$

$E_1$  : relative collision energy between  $t^\mu$  and  $d$

$\Gamma_{el}$  : elastic width.

The subscript 1 denotes the collision between  $t^\mu$

and d, Formation of  $(J, \nu)=(1, 1)$  state  $dt^\mu$  mainly attributes to the p-wave elastic scattering for  $l = 1$ <sup>[9]</sup>. Consequently, for the p-wave elastic scattering, the above formula (2) becomes :

$$\sigma_p^{el} = \frac{12\pi}{k_1^2} \frac{\frac{\Gamma_{el}^2}{4}}{(E_1 - E_{res})^2 + \frac{\Gamma_{el}^2}{4}} \quad (3)$$

If equation (3) is compared with the experimental values of  $t^\mu + d$  scattering cross sections<sup>[14]</sup>, elastic width  $\Gamma_{el}$  can be represented as

$$\Gamma_{el}(E_1) = 0.65 E_1^{1.5} + 0.049 E_1. \quad (4)$$

By inserting equation (4) into equation (3) [ $t^\mu + d$ ] elastic scattering cross sections can be computed as a function of relative collision energy  $E_1$ . Fig. 1 shows  $\sigma_p^{el}$  as a function of  $E_1$ .

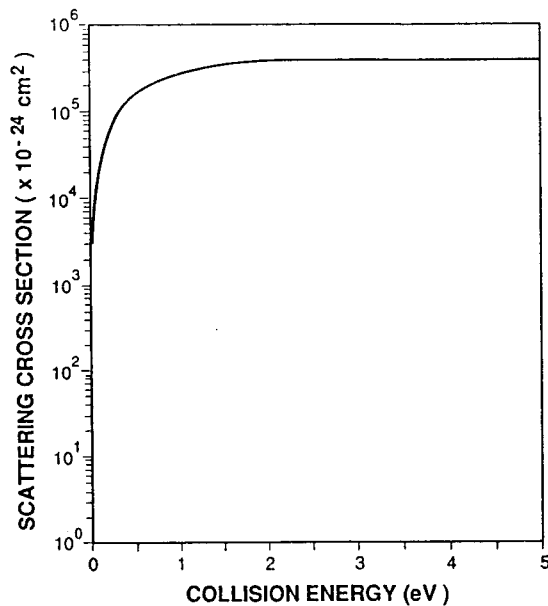


Fig. 1. [ $t^\mu + d$ ] Elastic Scattering Cross Sections as a Function of  $E_1$

## 2. Impulse Approximation

Since the molecular sizes of  $t^\mu$  and  $dt^\mu$  are considerably small,  $t^\mu$  can be treated as a neutron, and  $dt^\mu$  as one of the two nuclei in the compound molecule [ $(dt^\mu)dde$ ].

A transition to the excited state of a compound molecule after collision between  $t^\mu$  and  $D_2$  molecules can be assumed as a transit from ground state to excited state in a molecule by collision, although one of the two nuclei is replaced by  $dt^\mu$ . The impulse approximation<sup>[15,16]</sup> gives the theoretical basis on these assumptions. The followings are the basic assumptions of impulse approximation.

- 1) The incident particle reacts with only one particle of the molecule at a time.
- 2) The amplitude of wave incident on each constituent (nucleon) is assumed to be almost the same as if that constituent were alone in the molecule.
- 3) The binding force of the constituents of  $D_2$  molecule is negligible as compared with the collision force between  $t^\mu$  and  $D_2$ .
- 4) The vibrational motion of a diatomic molecule is considered as a harmonic oscillation.

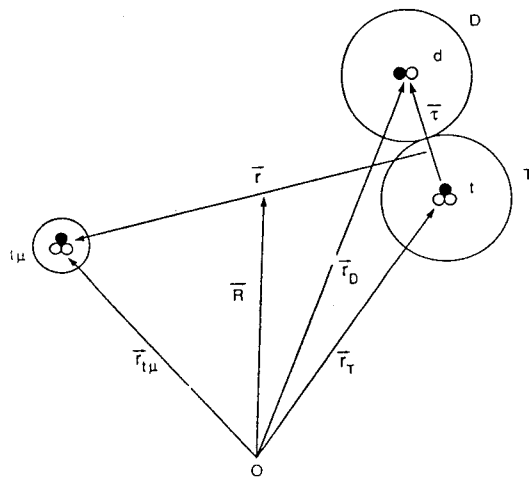


Fig. 2. The  $t^\mu + DT$  Collision System

Fig. 2 depicts the  $t^\mu$  and DT collision system to be considered in this calculation. Schrödinger equation for relative motion is given as follows.

$$\left\{ -\frac{\hbar^2}{2M_2} \nabla_2^2 - \frac{\hbar^2}{2\mu_\tau} \nabla_\tau^2 + V_D + V_T + V_{DT} \right\} \psi(\mathbf{r}, \tau) = E_2 \psi(\mathbf{r}, \tau) \quad (5)$$

where

$$M_2 = \frac{m_{t^\mu}(m_D + m_T)}{m_{t^\mu} + m_D + m_T}$$

$$\mu_\tau = \frac{m_D m_T}{m_D + m_T}$$

$V_D, V_T, V_{DT}$ : potential energies of molecules D, T, DT, respectively

$E_2$ : relative collision energy between  $t^\mu$  and DT

$\hbar$ : Planck's constant divided by  $2\pi$

$\psi(\mathbf{r}, \tau)$ : wave function for relative motion of  $t^\mu$  and DT

The subscript 2 indicates the collision between  $t^\mu$  and DT. Potential energies  $V_D, V_T,$  and  $V_{DT}$  are assumed as the sum of two-body potentials. For a scattering of  $t^\mu$  by DT in the  $z$ -direction in a state  $\nu K m$ , the solution of equation (5) can be written as the following integral equation<sup>[16]</sup>:

$$\psi(\mathbf{r}, \tau) = \exp(ik_2 z) \Psi_{\nu_0 K_0 m_0}(\tau) + \sum_{\nu, K, m} \psi_{\nu K m}(\mathbf{r}, \tau)$$

$$\psi_{\nu K m} =$$

$$-\frac{M_2}{4\pi\hbar^2} \int d\mathbf{r}' \int d\tau' \left\{ \Psi_{\nu K m}(\tau) \frac{\exp(ik_2' |\mathbf{r}' - \mathbf{r}|)}{|\mathbf{r}' - \mathbf{r}|} \Psi_{\nu K m}^*(\tau') \right. \\ \left. \times [V_D(|\mathbf{r}' + \beta_D \tau'|) + V_T(|\mathbf{r}' + \beta_T \tau'|)] \psi(\mathbf{r}', \tau') \right\}$$

$$\Psi_{\nu K m}(\tau) =$$

$$\frac{\left[ \frac{\omega_e \mu_\tau}{\hbar \pi 2^{2\nu} (\nu!)^2} \right]^{\frac{1}{4}} H_\nu(\xi) \exp\left(-\frac{\xi^2}{2}\right) Y_{K m}(\theta_\tau, \phi_\tau)}{\tau} \quad (6)$$

where

$$\beta_D = -\frac{m_T}{m_D + m_T}$$

$$\beta_T = \frac{m_D}{m_D + m_T}$$

$$\xi = \frac{\tau - \tau_e}{\left(\frac{\hbar}{\omega_e \mu_\tau}\right)^{1/2}}$$

$$k_2'^2 = \frac{2M_2(E_2 - E_{\nu K})}{\hbar^2}$$

$\nu, K, m$ : quantum numbers of DT after collision

$\omega_e$ : vibrational frequency of DT

$\tau_e$ : equilibrium internuclear distance of DT

$H_\nu(\xi)$ : Hermite polynomial

$Y_{K m}(\theta_\tau, \phi_\tau)$ : spherical harmonics

In equation (6), if  $|\mathbf{r}'| \gg |\mathbf{r}''|$ ,

$$\text{then } \frac{1}{|\mathbf{r}' - \mathbf{r}|} \approx \frac{1}{r}$$

$\Psi_{\nu K m}$  reduces to

$$\Psi_{\nu K m}(\mathbf{r}, \tau) \approx \frac{\exp(ik_2' r)}{r} \Psi_{\nu K m}(\tau)$$

$$\times \left\{ -\frac{2M_2}{4\pi\hbar^2} \int d\mathbf{r}' \int d\mathbf{r}'' \exp(-i\mathbf{k}'_2 \cdot \mathbf{r}') \Psi_{\nu K m}^*(\mathbf{r}'') [V_D + V_T] \Psi(\mathbf{r}', \mathbf{r}'') \right\} \quad (7)$$

The expression in the braces is a scattering amplitude,  $f_{\nu K m}(\mathbf{k}'_2, \mathbf{k}_2)$ , which can be expressed separately for  $V_D$  and  $V_T$  in the following manner,

$$\begin{aligned} f_{\nu K m}(\mathbf{k}'_2, \mathbf{k}_2) &= -\frac{2M_2}{4\pi\hbar^2} \int d\mathbf{r}' \\ &\int d\mathbf{r}'' \exp(-i\mathbf{k}'_2 \cdot \mathbf{r}') \Psi_{\nu K m}^*(\mathbf{r}'') [V_D + V_T] \Psi(\mathbf{r}', \mathbf{r}'') \\ &= -\frac{2M_2}{4\pi\hbar^2} \int d\mathbf{r}' \int d\mathbf{r}'' \exp(-i\mathbf{k}'_2 \cdot \mathbf{r}') \Psi_{\nu K m}^*(\mathbf{r}'') V_D \Psi(\mathbf{r}', \mathbf{r}'') \\ &\quad - \frac{2M_2}{4\pi\hbar^2} \int d\mathbf{r}' \int d\mathbf{r}'' \exp(-i\mathbf{k}'_2 \cdot \mathbf{r}') \Psi_{\nu K m}^*(\mathbf{r}'') V_T \Psi(\mathbf{r}', \mathbf{r}'') \\ &= f_D(\mathbf{k}'_2, \mathbf{k}_2) + f_T(\mathbf{k}'_2, \mathbf{k}_2) \end{aligned} \quad (8)$$

According to the impulse approximation assumptions 1), 2), and 3),  $\Psi_D(\mathbf{r}', \mathbf{r}'')$  instead of true

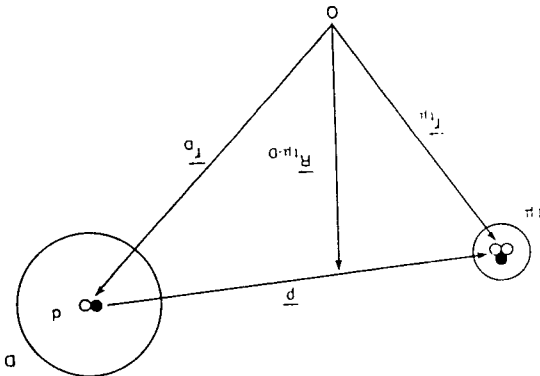


Fig. 3. The  $t^\mu + D$  Collision System

wave function  $\Psi(\mathbf{r}', \mathbf{r}'')$ , is taken as an approximate wave function, in which the effect of T in DT on the  $dt^\mu$  formation is neglected in a collision of  $t^\mu$  with D in DT. Fig. 3 shows the  $t^\mu + D$  collision system.

In order to separate the relative motion and the center of mass motion, the following variables are introduced.

$$\mathbf{R}_{t^\mu-D} = \frac{m_{t^\mu} \mathbf{r}_{t^\mu} + m_D \mathbf{r}_D}{m_{t^\mu} + m_D},$$

$$\mathbf{\beta} = \mathbf{r}_{t^\mu} - \mathbf{r}_D,$$

$$\mathbf{k}_R = \mathbf{k}_{t^\mu} + \mathbf{k}_D,$$

$$\mathbf{k}_p = \frac{m_D \mathbf{k}_{t^\mu} - m_{t^\mu} \mathbf{k}_D}{m_{t^\mu} + m_D}$$

With these variables  $\psi_D$  can be written as

$$\Psi_D(\mathbf{r}', \mathbf{r}'') = \psi_D(\mathbf{R}_{t^\mu-D}) \cdot \chi_D(\mathbf{\beta})$$

where the wave function  $\chi_D(\mathbf{\beta})$  is the solution of the equation of relative motion:

$$\left\{ -\frac{\hbar^2}{2\mu_{t^\mu-D}} \nabla_p^2 + V_{t^\mu-D}(\mathbf{\beta}) \right\} \chi_D(\mathbf{\beta}) = \frac{\hbar^2 k_p^2}{2\mu_{t^\mu-D}} \chi_D(\mathbf{\beta})$$

$$\mu_{t^\mu-D} = \frac{m_{t^\mu} \cdot m_D}{m_{t^\mu} + m_D} = M_1$$

Subsequently  $f_D(\mathbf{k}'_2, \mathbf{k}_2)$  in equation (8) can be calculated as

$$\begin{aligned}
f_D(\vec{k}'_2, \vec{k}_2) &= \left( \frac{M_2}{M_1} \right) \tilde{f}_D \left( \frac{M_2}{M_1} \vec{k}'_2 + \vec{q}, \frac{M_2}{M_1} \vec{k}_2 \right) \times \\
&F_{\nu K m; \nu_0 K_0 m_0} \left( -\frac{m_T}{m_D + m_T} \vec{q} \right), \quad (9) \\
&= \left( \frac{M_2}{M_1} \right)^2 \left( \int d\Omega \tilde{f}_D^2 \right) \left( |F_{\nu K m; \nu_0 K_0 m_0}| \right)^2 \\
&= \left( \frac{M_2}{M_1} \right)^2 \tilde{\sigma}_p^{el} \left( |F_{\nu K m; \nu_0 K_0 m_0}| \right)^2 \quad (10)
\end{aligned}$$

where  $\vec{q} = \vec{k}'_2 - \vec{k}_2$  and the free scattering amplitude  $\tilde{f}_D$  and the bound-state form factor  $F_{\nu K m; \nu_0 K_0 m_0}$  for the transition  $(\nu_0, K_0, m_0) \rightarrow (\nu, K, m)$  are expressed as

$$\tilde{f}_D(\vec{k}'_a, \vec{k}_b) \equiv -\frac{2M_1}{4\pi\hbar^2} \int d\vec{p} \exp(-i\vec{k}'_a \cdot \vec{p}) V_D(\vec{p}) \chi_D(\vec{p})$$

$$F_{\nu K m; \nu_0 K_0 m_0} \left( -\frac{m_T}{m_D + m_T} \vec{q} \right) =$$

$$\int d\vec{r} \exp \left[ -i \frac{m_T}{m_D + m_T} \vec{q} \cdot \vec{r} \right] \Psi_{\nu K m}^*(\vec{r}) \Psi_{\nu_0 K_0 m_0}(\vec{r})$$

The differential cross section for the transition  $(\nu_0, K_0, m_0) \rightarrow (\nu, K, m)$  is obtained by<sup>[12,16]</sup>

$$\begin{aligned}
d\sigma(\theta, \phi) &= \left| f_{\nu K m}(\vec{k}'_2, \vec{k}_2) \right|^2 \\
&= \left| f_D(\vec{k}'_2, \vec{k}_2) + f_T(\vec{k}'_2, \vec{k}_2) \right|^2
\end{aligned}$$

The interference term  $f_D f_T$  assumes to nearly vanish<sup>[16]</sup>. In addition, because the  $t^\mu + D$  collision gives rise to the formation of  $dt^\mu$ ,  $f_D$  is the only term that contributes to  $\sigma_{dt^\mu}$ . Accordingly,

$$\sigma_{dt^\mu} = \int d\sigma(\theta, \phi) d\Omega = \int \left| f_D(\vec{k}'_2, \vec{k}_2) \right|^2 d\Omega$$

$\tilde{\sigma}_p^{el}$  is obtained by adding inelastic scattering effects of deexcitation and back decay to  $\sigma_p^{el}$  for the p-wave elastic scattering.

$$\begin{aligned}
\tilde{\sigma}_p^{el} &= \\
&\frac{12\pi}{k_1^2} \frac{\frac{\Gamma_{el} \Gamma_{deex}}{4}}{(E_2 - E_{res})^2 + \frac{(\Gamma_{el} + \Gamma_{deex} + \Gamma_{back})^2}{4}} \quad (11)
\end{aligned}$$

where  $\Gamma_{deex}$  and  $\Gamma_{back}$  are the deexcitation and back decay widths, respectively.

### 3. Deexcitation Width

At first  $dt^\mu$  is formed in a  $(J, \nu) = (1, 1)$  state, but the d-t fusion takes place either in a  $(J, \nu) = (0, 1)$  state (84%) or a  $(J, \nu) = (1, 0)$  state (16%)<sup>[18]</sup>. In other words, the fusion must be accompanied by the deexcitations in the form of emissions of radiation, Auger electron, and excitation energy by collision with other molecules.

#### 3.1 E<sub>1</sub> Radiative deexcitation ( $\Gamma_{rad}$ )

E<sub>1</sub> radiative deexcitation results in the radiation emitted from an electric dipole transition. E<sub>1</sub> radiative deexcitation rate is written as<sup>[18]</sup>

$$\frac{\Gamma_{rad}}{\hbar} = \frac{4e^2\omega^3}{9\hbar c^3} \langle R \rangle^2 \sim 10^6 \text{ sec}^{-1} \quad (12)$$

where  $\hbar\omega$ , and  $\langle R \rangle$  are the transition energy and the average internuclear distance of  $dt^\mu$ , respectively.

### 3.2. Auger deexcitation ( $\Gamma_{Auger}$ )

Auger deexcitation is a mechanism which emits an Auger electron with deexcitation energy due to the Coulomb interaction between orbital electrons and  $dt^\mu$ . From the golden-rule formula, Auger deexcitation rate is given as<sup>[18,19]</sup>:

$$\frac{\Gamma_{Auger}}{\hbar} = \frac{4m_e k}{\hbar^3} \left| \langle u_f(\rho) \phi^f | V | u_i(\rho) \phi^{(1,1)} \rangle \right|^2 \quad (13)$$

$$\sim 8 \times 10^{10} \text{ sec}^{-1}$$

where

$u_i(\rho)$ ,  $u_f(\rho)$ : electron state functions before and after Auger electron emission

$\phi^{(1,1)}$ ,  $\phi^f$ :  $dt^\mu$  state functions before and after Auger electron emission

$V$ : Coulomb interaction potential between orbital electron and  $dt^\mu$

$k$ : electron wave number after emission

### 3.3. Collisional deexcitation ( $\Gamma_{coll}$ )

Collisional deexcitation takes place by the collision of compound molecule  $[(dt^\mu)dee]^*$  or  $[(dt^\mu)tee]^*$  with  $D_2$  or  $DT$ . Collisional deexcitation rate  $\Gamma_{coll}/\hbar$  is the same as a collision frequency  $1/\tau_{coll}$ , which is given as<sup>[20]</sup>:

$$\frac{\Gamma_{coll}}{\hbar} = \frac{1}{\tau_{coll}} = \text{density} \times k_{deex}(T) \quad (14)$$

In a temperature range to be considered in this work, the deexcitation constant  $k_{deex}(T)$  follows the Landau-Teller variation<sup>[20]</sup> in the form of

$$\log_{10} k_{deex}(T) = A + BT^{-1/3}$$

Constants  $A$  and  $B$  can be obtained from the experimental values of deexcitation rate<sup>[20]</sup> for  $D_2$ - $D_2$  collision, where the empirical values are  $A = -8.312$ , and  $B = -58.605$ . In the temperature range from 400 to 2000 K, the resultant collisional deexcitation rates become  $10^6 \sim 10^{11} \text{ sec}^{-1}$ .

### 4. Back Decay Width ( $\Gamma_{back}$ )

Occasionally back decay processes happen in which the formed  $dt^\mu$  divides into its original constituents  $t^\mu$  and  $d$  during deexcitation of muonic molecule. Consequently this back decay brings about a reducing fusion rate. Assuming the Maxwellian energy distribution of hydrogen molecules, the back decay rate is derived as<sup>[18]</sup>

$$\frac{\Gamma_{back}}{\hbar} = \frac{\lambda_{res}}{\rho \left( \frac{2\pi\hbar^2}{M_2 k_B T} \right)^{3/2} \exp\left(-\frac{E_{res}}{k_B T}\right)} \quad (15)$$

where  $E_{res} = 0.219 \text{ eV}$  in the case of  $t^\mu$ - $D_2$  collision, and  $0.107 \text{ eV}$  in the case of  $t^\mu$ - $DT$  collision. The back decay rate is calculated to be approximately  $3 \times 10^{10} \text{ sec}^{-1}$ .

### 5. Bound State Form Factor

The bound state form factor  $F_{\nu K m; \nu_0 K_0 m_0}$  in equation (9) is needed to be known only by the square of its absolute value. Since the magnetic



quantum numbers  $m$  and  $m_0$  do not appear in the rotational-vibrational motion, they must be summed up. Also since the rotational motion is very weak in comparison with the vibrational one, summing up over the vibrational quantum number  $K$  is done for simplicity of calculation. The bound state form factor is then obtained as<sup>[16]</sup>:

$$\sum_{K,m,m_0} |F_{\nu K m; \nu_0 K_0 m_0}|^2 = \frac{\gamma \left( \nu + \frac{1}{2}, \xi^2 d^2 \right)}{2\nu! \xi d}$$

where

$$\xi = \left| -\frac{m_T}{m_D + m_T} q \right|$$

$$d = \left( \frac{h}{2\omega_e \mu_r} \right)^{\frac{1}{2}}$$

$\gamma(\nu + 1/2, \xi^2 d^2)$  is the incomplete gamma function.  $\nu$  is the vibrational quantum numbers of compound molecule, among which the  $\nu = 3$  state is dominant.

#### 6. Resonant Formation Rate $\lambda_{res}$

Combining equation (10) with equations (11)-(15),  $\sigma_{dt^\mu}$  is written as

$$\sigma_{dt^\mu}(E_2) =$$

$$\left( \frac{M_2}{M_1} \right)^2 \frac{12\pi}{k_1^2} \frac{|F_{\nu K m; \nu_0 K_0 m_0}|^2 \frac{\Gamma_{et} \Gamma_{deex}}{4}}{(E_2 - E_{res})^2 + \frac{\Gamma_{tot}^2}{4}} \quad (16)$$

Assuming the Maxwellian energy distribution of molecules in D-T mixture the average value  $\langle \sigma_{dt^\mu}(E_2) \cdot v_{rel} \rangle$  over Maxwellian distribution gives the resonant formation rate  $\lambda_{res}$ .

$$\lambda_{res} = \rho \langle \sigma_{dt^\mu}(E_2) \cdot v_{rel} \rangle$$

where  $\rho$  is the fuel density, and  $v_{rel}$  is the relative velocity between  $t^\mu$  and  $D_2$  or  $DT$ .

#### IV. Results and Discussion

In the temperature range 400~2000 K, the most probable values  $v = \sqrt{2k_B T/M_2}$  of the relative velocity are in a range of  $2 \times 10^3 \sim 4 \times 10^3$  m/sec, and the relative collision energies  $E_2$  are in

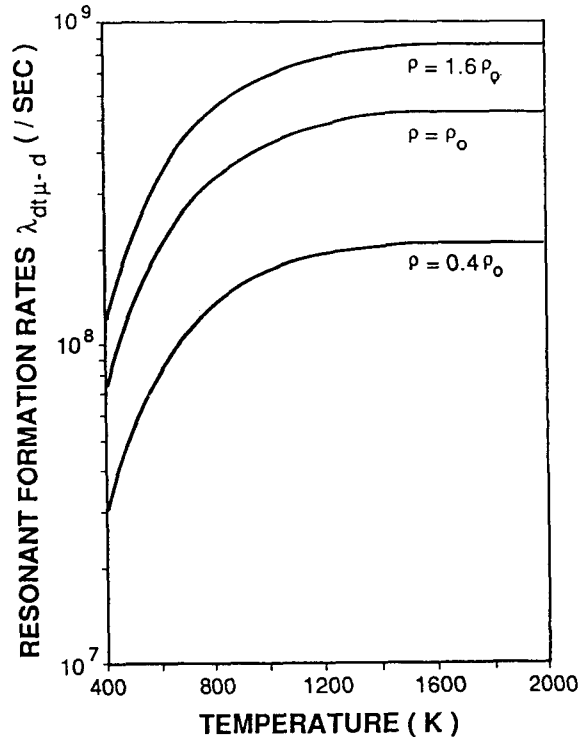


Fig. 4.  $\lambda_{dt^\mu-d}$  in Various Fuel Temperatures and Densities

0.03~0.2 eV. By use of these values the calculation of  $\langle \sigma_{dt} \mu \cdot v_{rel} \rangle$  is done in the relative collision energy range 0~3 eV. The computed resonant formation rates  $\lambda_{dt} \mu-d$  for  $t^\mu-D_2$  collision and  $\lambda_{dt} \mu-t$  for  $t^\mu-DT$  collision are shown in Fig. 4 and Fig. 5, respectively, at various fuel densities  $\rho = 0.4\rho_0, \rho_0, 1.6\rho_0$  where  $\rho_0$  is the liquid hydrogen density. A comparison of  $\lambda_{dt} \mu-d$  with  $\lambda_{dt} \mu-t$  at  $\rho = \rho_0$  is presented in Fig. 6.

From the calculated results the following conclusions can be deduced.

- 1) In a liquid hydrogen density  $1.2 \times 10^8 \leq \lambda_{res} \leq 4.2 \times 10^8 \text{ sec}^{-1}$  is obtained. The comparison with other results is shown in table 2 at the same ratio of D to T in the mixture. A wider temperature range is taken into account in this study. The present results calculated in the

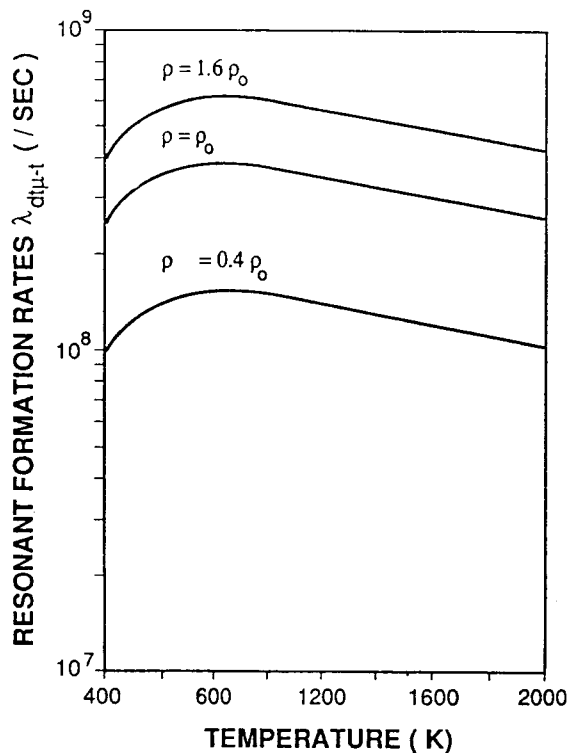


Fig. 5.  $\lambda_{dt} \mu-t$  in Various Fuel Temperatures and Densities

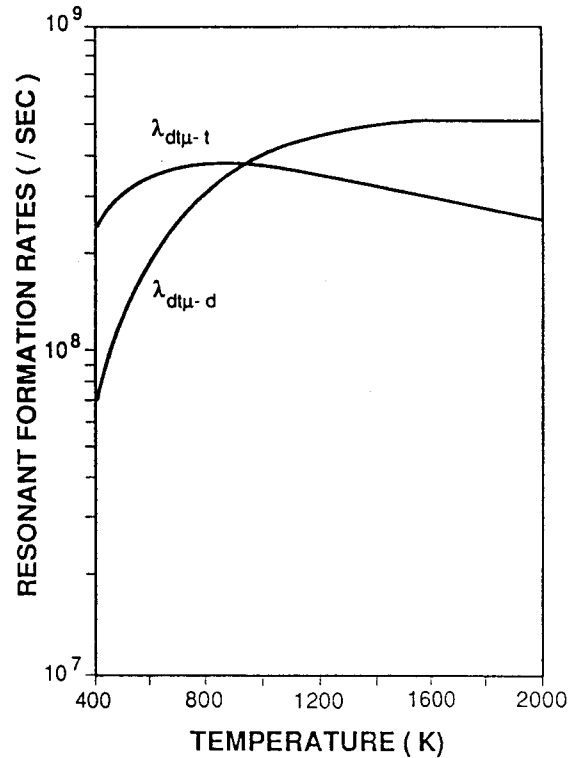


Fig. 6. A Comparison of  $\lambda_{dt} \mu-d$  with  $\lambda_{dt} \mu-t$  at  $\rho = \rho_0$

broader temperature range than the previous ones of other work are in good agreement with previously known results. The previous theoretical and experimental research are referred to Refs. [11], [21], [22], and [8]. In Ref. [8], only the Auger deexcitation was considered as deexcitation process of muonic molecule, and back decay process was not taken into account.

- 2) For a constant fuel density,  $\lambda_{dt} \mu-d$  and  $\lambda_{dt} \mu-t$  have maxima in particular temperature ranges, respectively. This phenomenon has not appeared in other results where the temperature ranges were considered in lower temperature ranges than the temperature at which the maxima appear.

i) In the case of  $\lambda_{dt} \mu-d$  the maxima appear in

**Table 2. Comparison of Results with the Other Works**

	$\lambda_{dt^{\mu}}$ (sec <sup>-1</sup> )	Temperature Range
Results of this study	$1.2\sim 4.2\times 10^8$	400~2000 K
Bystritsky et al. <sup>[21]</sup>	$\geq 10^8$	93~613 K
Leon <sup>[8]</sup>	$\sim 3\times 10^8$	70~620 K
Jones et al. <sup>[11]</sup>	$6\times 10^8$	< 130 K
Jones et al. <sup>[22]</sup>	$3.2\sim 5.0\times 10^8$	350~550 K

a range of 1700~1850 K. This temperature range is consistent with the temperature satisfying the relation  $E_2=0.219$  eV (resonant collision energy of  $t^{\mu}-D_2$ )= $3/2k_B T$ , which explains that the resonant formation cross sections  $\sigma_{dt^{\mu}}$  have large values when the relative collision energy,  $E_2$  approaches the resonant energy  $E_{res}$ .

ii) In the case of  $\lambda_{dt^{\mu}-t}$  the extrema appear at about 850 K. This temperature is about the same as the temperature satisfying the relation  $E_2=0.107$  eV (resonant collision energy of  $t^{\mu}-DT$ )= $3/2k_B T$  with the similar reason as described above.

3) The higher the density is, the greater  $\lambda_{res}$  is. This behavior is resulted from increasing the deexcitation rate and decreasing the back decay rate while increasing the density.

If the fuel contains equal ratio of D and T,  $\lambda_{res}$  are expected to have its extremum at about 1400 K. Since the extremum of  $\lambda_{res}$  appears at about 1400K which is higher than the temperature 1000 K, presently achievable in a laboratory, efforts in increasing the fuel temperature should be made in the future MCF experiment.

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#### References

- [1] S. S. Gershtein and L. P. Ponomarev, *Phys. Lett.* **72B**, 80 (1977)
- [2] L. Bracci and G. Fiorentini, *Physics Reports* **86**, 170 (1982)
- [3] S. E. Jones, *Nature* **321**, 127 (1986)
- [4] B. Brunelli, *Fus. Tech.* **12**, 324 (1987)
- [5] A. A. Harms, *Fus. Tech.* **14**, 381 (1988)
- [6] W. Stacey, *Fus. Tech.* **16**, 268 (1989)
- [7] G. Preparata, *Fus. Tech.* **20**, 82 (1991)
- [8] M. Leon, *Phys. Rev. Lett.* **52**, 605 (1984)
- [9] J. S. Cohen and M. Leon, *Phys. Rev. Lett.* **55**, 52 (1985)
- [10] R. M. Kulsrud, *Nucl. Fusion* **27**, 1347 (1987)
- [11] S. E. Jones et al., *Phys. Rev. Lett.* **56**, 588 (1986)
- [12] H. S. Song, "Quantum Mechanics," Ch. 13, Kyohaksa, Seoul (1984)
- [13] E. Serge, "Nuclei and Particles," Ch. 11, W. A. Benjamin Inc., Reading (1977)
- [14] V. S. Melezhik, L. I. Ponomarev and M. P. Faifman, *Sov. Phys. JETP* **58**, 254 (1983)
- [15] G. F. Chew and G. C. Wick, *Phys. Rev.* **85**, 636 (1952)
- [16] A. Bogan, *Phys. Rev.* **A9**, 1230 (1974)
- [17] L. N. Bogdanova, L. Bracci, G. Fiorentini and S. S. Gershtein, *Nucl. Physics* **A454**, 653 (1983)
- [18] A. M. Lane, *Phys. Lett.* **98A**, 337 (1983)

- [19] C. Cohen, B. Diu and F. Laloe, "Quantum Mechanics," Ch XIII, John Wiley & Sons, N. Y. (1977)
- [20] J. Lucasik and J. Ducuing, *Chem. Phys. Lett.* **27**, 203 (1974)
- [21] V. M. Bystritsky et al., *Sov. Phys. JETP* **53**, 877 (1981)
- [22] S. E. Jones et al., *Phys. Rev. Lett.* **51**, 1757 (1983)